
DEPARTMENT OF COMPUTER SCIENCE & SOFTWARE ENGINEERING
COMP232 MATHEMATICS FOR COMPUTER SCIENCE
FALL 2017

Assignment 2. Due date: October 13

1. Let $P(x, y, z)$ denote the statement $x + y \leq z$, where $x, y, z \in \mathbb{Z}^+$. What is the truth value of each of the following? Explain your answers.

(a) $\forall x \exists y \exists z P(x, y, z)$.

(b) $\forall y \exists x \forall z P(x, y, z)$.

(c) $\exists z \exists y \forall x P(x, y, z)$.

2. For each of the premise-conclusion pairs below, give a valid step-by-step argument (proof) along with the name of the inference rule used in each step. For examples, see pages 73 and 74 in textbook.

(a) Premise: $\{\neg p \vee q \rightarrow r, s \vee \neg q, \neg t, p \rightarrow t, \neg p \wedge r \rightarrow \neg s\}$, conclusion: $\neg q$.

(b) Premise: $\{\neg p \rightarrow r \wedge \neg s, t \rightarrow s, u \rightarrow \neg p, \neg w, u \vee w\}$, conclusion: $\neg t \vee w$.

(c) Premise: $\{p \vee q, q \rightarrow r, p \wedge s \rightarrow t, \neg r, \neg q \rightarrow u \wedge s\}$, conclusion: t .

3. For each of the following, determine whether the argument is valid. You may use a counterexample or equivalence transformations to justify your answer.

(a) $p \rightarrow q$

$$\frac{\neg p}{\therefore \neg q}$$

(b) $\frac{\neg p \rightarrow \neg q}{\therefore (\neg p \rightarrow q) \rightarrow p}$

(c) $p \rightarrow r$
 $q \rightarrow r$
 $\frac{\neg(p \vee q)}{\therefore \neg r}$

(d) $p \rightarrow q$
 $\frac{p \rightarrow (q \rightarrow \neg p)}{\therefore \neg p}$

4. For each of the arguments below, indicate whether it is valid or invalid.
- (a) All healthy people eat an apple a day.
Helen eats an apple a day.
 \therefore Helen is a healthy person.
 - (b) All healthy people eat an apple a day.
Herbert is not a healthy person.
 \therefore Herbert does not eat an apple a day.
 - (c) If a product of two real numbers is 0, then at least one of the numbers is 0.
For a particular real number x , neither $(x - 1)$ nor $(x + 1)$ equals 0.
 \therefore The product $(x - 1)(x + 1)$ is not 0.
5. Use rules of inference to show that if $\forall x(P(x) \rightarrow Q(x))$, $\forall x(Q(x) \rightarrow R(x))$, and $\exists x(\neg R(x))$ are true, then $\exists x(\neg P(x))$ is true.
6. (a) Give a direct proof of: “If x is an odd integer and y is an even integer, then $x + y$ is odd.”
 (b) Give a proof by contradiction of: “If n is an odd integer, then n^2 is odd.”
 (c) Give an indirect proof of: “If x is an odd integer, then $x + 2$ is odd.”
 (d) Use a proof by cases to show that there are no solutions in positive integers to the equation $x^4 + y^4 = 100$.
 (e) Prove that given a nonnegative integer n , there is a unique nonnegative integer m , such that $m^2 \leq n < (m + 1)^2$.
7. For each of the statements below state whether it is True or False. If True then give a proof. If False then explain why, e.g., by giving a counterexample.
- (a) The difference of any two odd integers is odd.
 - (b) Let a and b be integers. If $a + b$ is even, then either a or b is even.
 - (c) For all positive integers n , it holds that n is even if and only if $3n^2 + 8$ is even.
 - (d) For all positive $x, y \in \mathbb{R}$, if x is irrational and y is irrational then $x + y$ is irrational.
 - (e) $\forall x, y \in \mathbb{R}$, if x is irrational and y is rational then xy is irrational.