PART A: MULTIPLE CHOICE QUESTIONS.

8

[5 × 2 marks each = 10 marks]

A1. $\lim_{x\to 0} \frac{\cancel{3x^5} - \cancel{2x^3} + \cancel{7x} + 1}{\cancel{x^6} + 5}$ is

(a) $\frac{1}{5}$ (b) 3 (c) 0 (d) ∞ (e) None of the above.

A2. $\lim_{x\to 3} \frac{9 - x^2}{x - 3}$ is

$\frac{(\cancel{3-x})(3+x)}{\cancel{x-3}}$

(a) 6 (b) -6 (c) 0 (d) ∞ (e) None of the above.

[forgot to factor out the negative in this equation]

A3. $\lim_{x\to -\infty} \frac{7x^4 + x^3 + 3^2x - 3}{x^7 + 3x^2 + 1}$ is

(a) 7 (b) 0 (c) -3 (d) ∞ (e) None of the above.

A4. $\lim_{x\to\infty} \frac{3x^5 + x^3 - 4x^2 - 11}{x^2 + x + 1}$ is

(a) 3 (b) 0 (c) -11 (d) ∞ (e) None of the above.

A5. The function $g(x) = \frac{x-3}{x^4+1}$ is discontinuous at

(a) $x = \pm 1$ (b) $x = 3$ (c) $x = -1$ (d) continuous everywhere

(e) None of the above.

$x^4 + 1 = 0$

$x^4 = -1$

4. $$\frac{\frac{3x^5 + x^3 - 4x^2 - 11}{x^2}}{\frac{x^2 + x + 1}{x^2}}$$

$$= \frac{3x^3 + x - 4}{1 + \cancel{\frac{1}{x}}_{0} + \cancel{\frac{1}{x^2}}_{0}}$$

5 **[5 marks] B3.** Use implicit differentiation to find $\frac{dy}{dx}$ if $2x^3 + 3xy - y^2 = 5$.

$$\frac{d}{dx}\left(2x^3 + 3xy - y^2\right) = \frac{d}{dx}(5)$$

$$6x^2 + \left(3y + 3x\frac{dy}{dx}(1)\right) - \frac{dy}{dx}2y = 0$$

$$3y + 3x\frac{dy}{dx} - 2y\frac{dy}{dx} = -6x^2$$

$$\frac{dy}{dx} = \frac{-6x^2 - 3y}{3x - 2y}$$

$$\therefore \frac{dy}{dx} = \frac{-6x^2 - 3y}{3x - 2y}$$

10 **[10 marks] B4.** Find the derivative of the following functions using the appropriate rules of differentiation:

[3] (a) $f(x) = \frac{4x^2}{2x^3 - 7}$. (Simplify the numerator.)

[2] (b) $g(x) = 3^{1-4x}$.

[2] (c) $h(x) = 4(x^5 + 2x^2 - 1)^{1/4}$.

[3] (d) $k(x) = \log_5(2x^3 + e^x)$.

a) $f(x) = \frac{4x^2}{2x^3 - 7}$

$$f'(x) = \frac{8x(2x^3-7) - 6x^2(4x^2)}{(2x^3-7)^2}$$

$$= \frac{16x^4 - 56x - 24x^4}{(2x^3-7)^2}$$

$$= \frac{-(8x^4+56x)}{(2x^3-7)^2}$$

b) $g(x) = 3^{1-4x}$

$$g'(x) = 3^{1-4x}(\ln 3)(-4)$$

$$= (-4\ln 3)\,3^{1-4x}$$

c) $h(x) = 4(x^5 + 2x^2 - 1)^{1/4}$

$$h'(x) = (x^5 + 2x^2 - 1)^{-3/4}(5x^4 + 4x)$$

$$= \frac{5x^4 + 4x}{(x^5 + 2x^2 - 1)^{3/4}}$$

d) $K(x) = \log_5(2x^3 + e^x)$

$$K'(x) = \frac{6x^2 + e^x}{(2x^3 + e^x)\ln 5}$$

$\therefore$ a) $f'(x) = \frac{-(8x^4 + 56x)}{(2x^3 - 7)^2}$

b) $g'(x) = (-4\ln 3)\,3^{1-4x}$

c) $h'(x) = \frac{5x^4 + 4x}{(x^5 + 2x^2 - 1)^{3/4}}$

d) $K'(x) = \frac{6x^2 + e^x}{(2x^3 + e^x)\ln 5}$

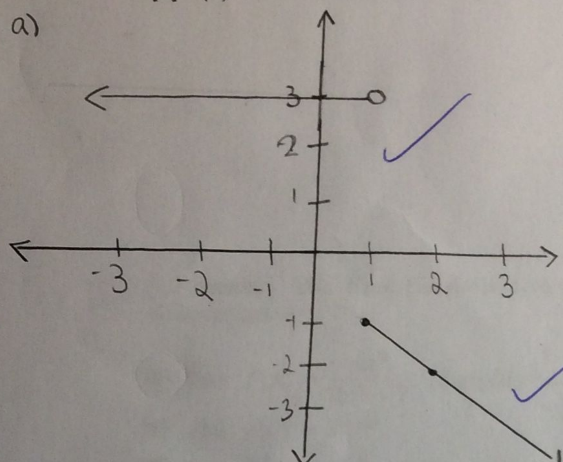
7 [7 marks] **B1**. Let $f(x) = \begin{cases} 3, & x < 1, \\ -x, & x \geq 1. \end{cases}$

[1] (a) Sketch the graph of f.

[5] (b) Find the following limits: (i) $\lim_{x \to 1+} f(x)$; (ii) $\lim_{x \to 1-} f(x)$; (iii) $\lim_{x \to 1} f(x)$; (iv) $\lim_{x \to -15} f(x)$; (v) $\lim_{x \to 6} f(x)$;

[1] (c) Is the function f continuous at $x = 1$? Explain.

a)



b) i) -1 ii) 3 iii) DNE

iv) 3 v) -6

c) The function isn't continuous at $x = 1$ because when the limit was taken from both sides it gave two different numbers. Also when looking at the graph you can tell that the group jumps from point one to point two. ∴ this function isn't continuous.

6 [8 marks] **B2**. Use the Intermediate Value Theorem to show that the equation has at least one solution in the designated interval. Explain why the theorem is applicable.

[4] (a) $x^3 + 2x^2 - 5 = 0$, in $(1, 2)$

$f(1) = (1)^3 + 2(1)^2 - 5$

$= 1 + 2 - 5$

$= -2$

$\therefore f(1) = -2$

$f(2) = (2)^3 + 2(2)^2 - 5$

$= 11$

$\therefore f(2) = 11$

∴ the give values of intervals gave different signs. ∴ there is at least one solution.

[4] (b) $4x - 1 = 2^x$, in $(0, 1)$.

$f(0) = 4(0) - 1 - 2^0$

$= -1 - 1$

$= -2$

$\therefore f(0) = -2$

$f(1) = 4(1) - 1 - 2^1$

$= 4 - 1 - 2$

$= 1$

$\therefore f(1) = 1$

The given intervals gave different signs (positive & negative) which means that this function has at least one solution.

Have to mention (because it is continous and that's why you can use it)

(That why lost two marks.)