

## SOCI 213 - STATISTICS II

### ASSIGNEMENT 3

#### Exercise 1

Question 1 : Proportion of scores

a)  $Z = \frac{50-47}{10} = 0,3$

—>  $p = 0,12$  or 12%

b) 50 % + 12% = 62 %

c) 50 % + 12 % = 62%

d)  $Z = \frac{35-50}{10} = -1,5$

—>  $p = 0,43$  or 43%

$$Z = \frac{65-50}{10} = 1,5$$

—>  $p = 0,43$  or 43%

Therefore, between 35 and 65 : 43% + 43% = 86%

e)  $Z = \frac{40-50}{10} = -1$

—>  $p = 0,34$  or 34%

$$Z = \frac{47-50}{10} = -0,30$$

—>  $p = 0,12$  or 12%

Therefore, between 40 and 47 : 34% - 12% = 22%

f) 
$$Z = \frac{72-50}{10} = 2,2$$

—>  $p = 0,40$  or 40%

Above 72 = 50% - 40% = 10%

g) 
$$Z = \frac{31-50}{10} = -1,9$$

—>  $p = 0,47$  or 47%

Below 31 = 50% - 47% = 3%

h) 
$$Z = \frac{55-50}{10} = 0,5$$

—>  $p = 0,19$  or 19%

$$Z = \frac{62-50}{10} = 1,2$$

—>  $p = 0,38$  or 38%

Therefore, between 55 and 62 : 38% - 19% = 19%

Question 2 : Score to be in the top 15% of the population distribution

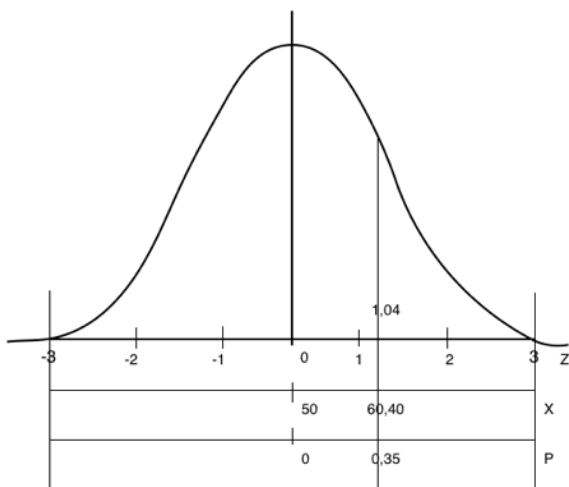
$p \rightarrow Z \rightarrow X$

$$p = 0,50 - 0,15 = 0,35$$

$$Z = 1,04$$

$$\begin{aligned} X &= \mu \pm Z\sigma \\ &= 50 + 1,04(10) \\ &= 60,4 \end{aligned}$$

Therefore, we need a minimum of 60,4 to be in the top 15% of the population distribution.



Question 3 : Score to be in the bottom 5% of the population distribution

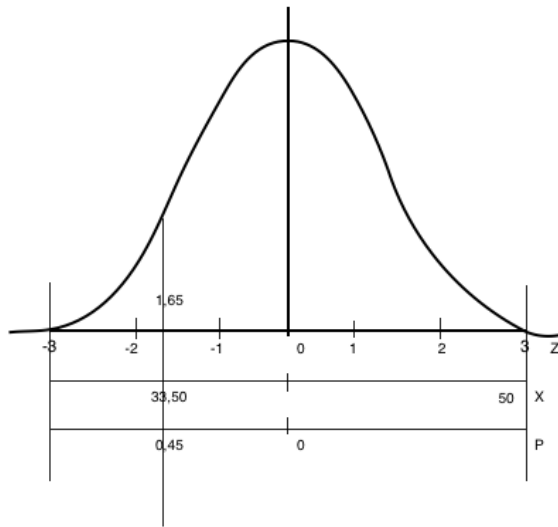
$$p \rightarrow Z \rightarrow X$$

$$p = 0,50 - 0,05 = 0,45$$

$$Z = -1,65$$

$$\begin{aligned} X &= \mu \pm Z\sigma \\ &= 50 - 1,65(10) \\ &= 33,5 \end{aligned}$$

Therefore, we need a maximum of 33,5 to be in the bottom 5% of the population distribution.



Question 4 : Probability of selecting a score less than 55

$$Z = \frac{x - \mu}{\sigma} = \frac{55 - 50}{10} = 0,5$$

$$\rightarrow p = 0,19 \text{ or } 19\%$$

$$\text{So, Less than 55 : } 50\% + 19\% = 69\%$$

Therefore, the probability of selecting a score less than 55 is 69%.

## Exercise 2

### Question 1 : Probability of the sample mean

a) For less than 98%

$$Z = \frac{102 - 100}{0,35} = -5,71$$

—> p = 0,50 or 50%

For less than 98 : 50% - 50 % = 0%

b) For more than 102

$$Z = \frac{98 - 100}{0,35} = -5,71$$

—> p = 0,50 or 50%

For more than 102 : 50% - 50% = 0%

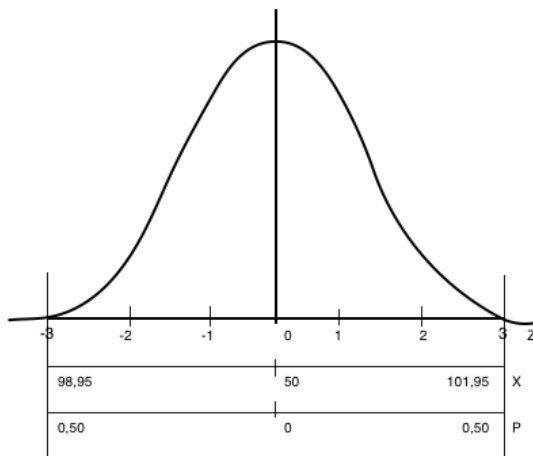
c) Between 98 and 102 we find : 50% + 50% = 100%

### Question 2 : Highest and lowest sample in the sampling distribution

We know that the highest and lowest same mean is equal to  $Z = +3$  and  $-3$   
Limits for  $X = \mu \pm Z\sigma_X$

The highest sample mean :  $100 + 3 (0,35) = 101,05$

The lowest sample mean :  $100 - 3 (0,35) = 98,95$



### Exercise 3

#### Question 1 : Proportion

a) Between 1 and 1,3

$$Z = \frac{1-1,3}{1,7} = -0,18$$

—>  $p = 0,07$  or 7%

b) Below 2

$$Z = \frac{2-1,3}{1,7} = 0,41$$

—>  $p = 0,16$  or 16%

Therefore below 2 : 16% + 50 % = 66%

c) Between 2 and 3

$$Z = \frac{3-1,3}{1,7} = 0,34$$

$$Z = \frac{2-1,3}{1,7} = 0,41$$

—>  $p = 0,16$  or 16%

—>  $p = 0,34$  or 34%

Therefore between 2 and 3 : 34% - 16% = 18%

d) Greater than 1

$$Z = \frac{1-1,3}{1,7} = -0,18$$

—>  $p = 0,07$  or 7%

1 = 50% + 7% = 57%

#### Question 2 : Score to be on top of the 10% of the sample distribution?

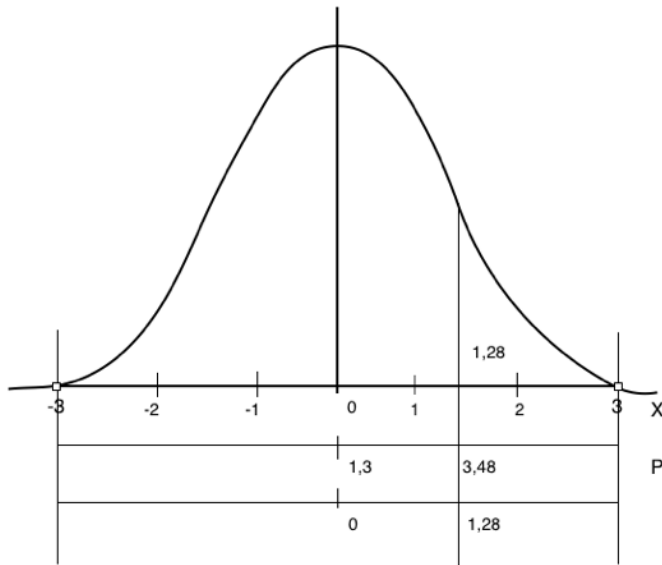
$p \rightarrow Z \rightarrow X$

$p = 0,50 - 0,10 = 0,40$

$Z = 1,28$

$$\begin{aligned}
 X &= \bar{X} \pm ZS \\
 &= 1,3 + 1,28 (1,7) \\
 &= 3,48
 \end{aligned}$$

Therefore, we need a minimum of 3,48 to be in the top 10% of the sample distribution



Question 3 : Probability of selecting another sample with a mean

We know that sample parameters with interference :

$$Z = \frac{x - \mu}{\sigma \bar{X}} = \frac{x - 1,3}{0,24}$$

$$\sigma \bar{X} = \frac{S}{\sqrt{N-1}} = \frac{1,7}{\sqrt{50-1}} = 0,24$$

a) Less than 1,1

$$Z = \frac{1,1 - 1,3}{0,24} = -0,83$$

—> p = 0,30 or 30%

Less than 1 = 50% - 30% = 20%

b) More than 1,9

$$Z = \frac{1,9 - 1,3}{0,24} = 2,5$$

—>  $p = 0,49$  or 49%

More than 1,9 = 50% - 49% = 1%

c) Between 1 and 2

Between 1 and 2 = 39% + 50% = 89%

$$Z = \frac{1 - 1,3}{0,24} = -1,25$$

—>  $p = 0,39$  or 39%

$$Z = \frac{2 - 1,3}{0,24} = 2,92$$

—>  $p = 0,50$  or 50%

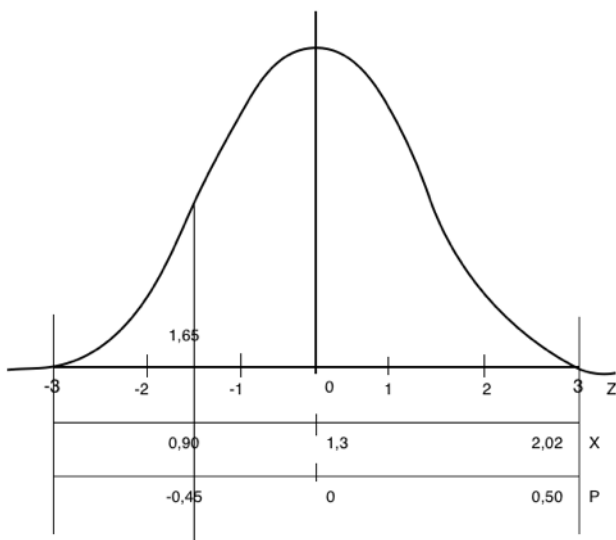
Therefore between 1 and 2 : 39% + 50% = 89%

Question 4 : score to be in the bottom 5% of the sampling distribution

$$\begin{aligned} p &= Z = X \\ p &= 0,50 - 0,05 = 0,45 \\ Z &= -1,65 \end{aligned}$$

$$\begin{aligned} X &= \mu \pm Z\sigma_x \\ &= 1,3 - 1,65(0,24) \\ &= 0,90 \end{aligned}$$

Therefore, we need a maximum of 0,90 to be in the bottom 5% of the population distribution



Question 5 : Highest and lowest sample means in the sampling distribution

We know that the highest and lowest same mean is equal to  $Z = +3$  and  $-3$   
Limits for  $X = \mu \pm Z\sigma_X$

The highest sample mean :  $1,3 + 3 (0,24) = 1,3 + 0,72 = 2,02$

The lowest sample mean :  $1,3 - 3 (0,24) = 1,3 - 0,72 = 0,58$

