Economics 302: Intermediate Microeconomics II

Assignment 4

(Due 4:00 pm Thursday ~~April 8~~, in my office H1155-71)
31 March

1. Consider the following game played between players 1 and 2. Player 1 chooses A, B or C while player 2 chooses from a,b,c. Choices are made independently and simultaneously. Payoffs are presented below. (30 pts)

			2	
		a	b	c
	A	6,7	5,4	0,0
1	B	4,5	7,6	0,-1
	C	0,0	-1,0	1,1

a) Find all of the pure strategy Nash equilibria for this game.
b) If we allow these players to talk to each other before the game what would you anticipate to be the outcome(s) in this game. Explain.
c) If player 2 is given the ability to move first what would you anticipate to be the outcome(s) in this game? Explain.

2. Consider the following entry deterrence model. An entrant makes an initial decision either to enter or not into some industry. The incumbent observes this decision and subsequently decides either to compete aggressively with the entrant, or to accommodate the entrant and not compete aggressively. Suppose that aggressive competition pays both firms a profit of -1. Non aggressive competition pays each firm a profit of 2. If the entrant stays out then the entrant earns zero profit elsewhere and the incumbent earns a monopoly profit of 5. In this framework, entry deterrence can be thought of as an initial threat from the incumbent to compete aggressively should entry arise. (35 pts)

a) Present this game information in an extensive form.
b) Can entry deterrence arise in equilibrium in this game? Explain.
c) Now suppose that we repeat this sequential game once. After the first period the entrant decides to either be in the industry or not. Then the incumbent decides how to react if the entrant enters or stays in. Now there can be a reward for aggressive competition, because if the entrant exits between periods the incumbent earns monopoly profit in the final period. Can we have an equilibrium outcome where firm 1 competes aggressively in the initial period in reaction to entry, and threatens to continue competing aggressively if the entrant stays in? Explain.

3. Consider the following game. (35 pts)

		Player 2	
		l	r
Player 1	t	4,3	1,2
	b	2,1	3,4

a) Solve for all of the pure and mixed strategy equilibria for this game if the players choose strategies simultaneously.

b) Now suppose that player 2 moves first and player 1 moves second. Show the game information in an extensive form. What is the pure strategy Nash equilibrium in this case.

c) Would player 1 prefer the mixed strategy equilibrium in a) or the sequential move equilibrium in b)? Why?

ECON 302
Assignment 4, winter 2011
Solutions

Question (1)

		Player 2		
		a	b	c
Player 1	A	6,7	5,4	0,0
	B	4,5	7,6	0,-1
	C	0,0	-1,0	1,1

(a) NE in pure strategies = { (A,a), (B,b), (C,c) }

Explanation:

* If PL 2 plays a ⇒ PL 1 will plays A as $A > B > C$ (6, 4, 0)

& if PL 1 plays A ⇒ PL 2 will play a as $a > b > c$ (7, 4, 0)

∴ (A,a) is NE as both players have no tendency to deviate

* If PL 2 plays b ⇒ PL 1 will play B as $B > A > C$ (7, 5, -1)

& if PL 1 plays B ⇒ PL 2 will play b as $6 > 5 > -1$ or $b > a > c$

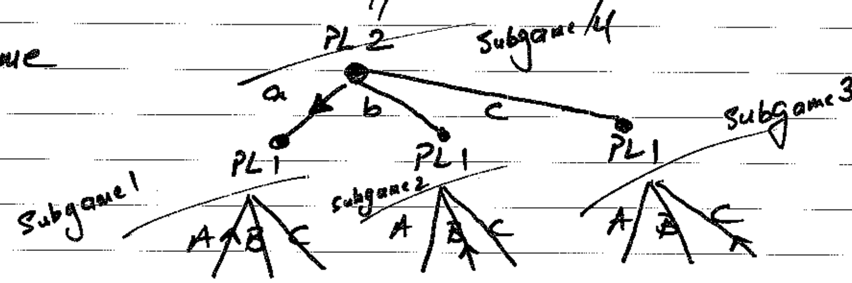
∴ (B,b) is a NE

* Similarly for (C,c).

(b) If both players talk before the game, they will choose the equilibrium that gives them the highest payoff.

⇒ They will choose (A,a) or (B,b) since both are better off as compared to (C,c)

(c) Sequential game



Payoffs: PL 1 6 4 0 5 7 -1 0 0 1
PL 2 7 5 0 4 6 0 0 -1 1

NE in sequential game is : {A, a)

Explanation:

In subgame 1 : Player 1 chooses A as $6 > 4 > 0$
& player 2 ~~will~~ gets $\boxed{7}$

in Subgame 2 : player 1 chooses B as $7 > 5 > -1$
& Player 2 gets $\boxed{6}$

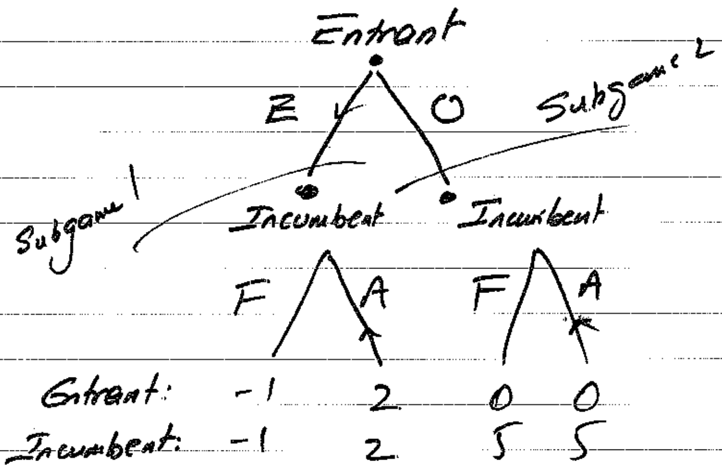
In Subgame 3: player 1 chooses C as $1 > 0$
& player 2 gets $\boxed{1}$

In subgame 4 (whole game), player 2 will choose to play "a" as $7 > 6 > 1$

$\therefore NE = \{A, a\}$.

Question (2)

a)



E ≡ Entrant
O ≡ Out
F ≡ fight
A ≡ Accomodate

b) No. In Subgame 1, the incumbent will accomodate and in subgame 2, he is indifferent.

if The Entrant plays E, the incumbent will accomodate & the entrants gets (2)

if the entrant plays O, the incumbent will accomodate & the entrants gets zero

$\therefore$ The equilibrium is therefore $\{E, A\}$ & ~~is~~ hence no entry deterrence.

(c) Yes. To do this, we need to make "fight" a dominant strategy that will be chosen by the incumbent in all subgames. Changing the incumbent's payoffs, for instance, in subgame 1 from −1 to 2 will make him choose "F" in both subgames

lead ⇒ the entrant will be forced to stay out in this case.

Problem #3

		Player 2	
		ℓ (q)	r ($1-q$)
player 1	t (p)	4, 3	1, 2
	b ($1-p$)	2, 1	3, 4

(a) NE in pure strategies:

$$NE = \{(t, \ell), (b, r)\} \quad + \text{Explain}$$

(b) NE in mixed strategies:

- Suppose PL1 plays "t" with a probability p and "b" with a probability $1-p$
- Suppose PL2 plays "ℓ" with a probability q and "r" with a probability $1-q$

The Expected Profit for PL1:

$$E(\pi_1) = 4pq + 1p(1-q) + 2(1-p)q + 3(1-p)(1-q)$$
$$= 4pq + p - pq + 2q - 2pq + 3(1-q-p+pq)$$
$$= pq + p + 2q + 3 - 3q - 3p + 3pq$$
$$= 4pq - 2p - q + 3$$

$$\frac{\partial E(\pi_1)}{\partial p} = 4q - 2 \quad \begin{cases} > 0 & \text{for } q > \frac{2}{4} = \frac{1}{2} \\ = 0 & \text{for } q = \frac{1}{2} \\ < 0 & \text{for } q < 1/2 \end{cases}$$

Best Response for PL 1:

(i) If $E(\pi_1)$ is increasing with p $\Rightarrow$ set p as high as possible, i.e., $p=1$

$\therefore$ for $q > 1/2 \Rightarrow p = 1$

(ii) if $E(\pi_1)$ is decreasing with p $\Rightarrow q < 1/2 \Rightarrow$ set $p = 0$ (as low as possible)

(iii) if $E(\pi_1)$ is constant with p $\Rightarrow q = \frac{1}{2}$

$\Rightarrow \therefore p \in (0, 1)$

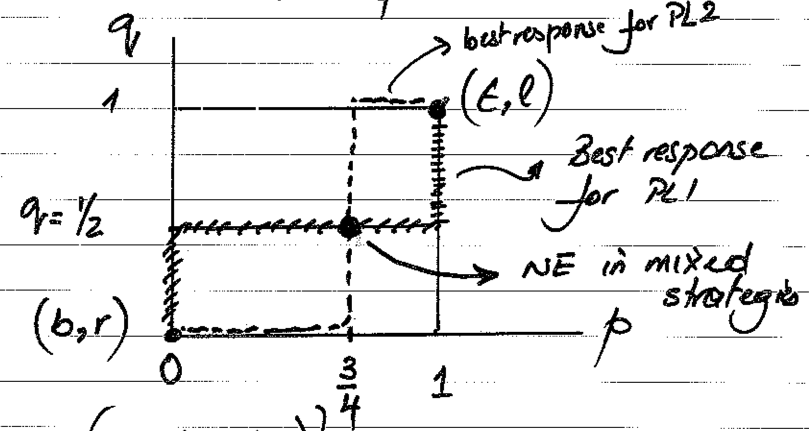
Similarly for Player 2

$$E(\pi_2) = 3pq + 2p(1-q) + 1(1-p)q + 4(1-p)(1-q)$$
$$= 3pq + 2p - 2pq + q - pq + 4(1 - q - p + pq)$$
$$= 2p + q + 4 - 4q - 4p + 4pq$$
$$= 4pq - 2p - 3q + 4$$

$$\frac{\partial E(\pi_2)}{\partial q} = 4p - 3 \quad \begin{cases} > 0 \Rightarrow p > \frac{3}{4} \Rightarrow \text{set } q = 1 \\ = 0 \Rightarrow p = \frac{3}{4} \Rightarrow \text{set } q \in (0,1) \\ < 0 \Rightarrow p < \frac{3}{4} \Rightarrow \text{set } q = 0 \end{cases}$$

$p^* = \frac{3}{4}$

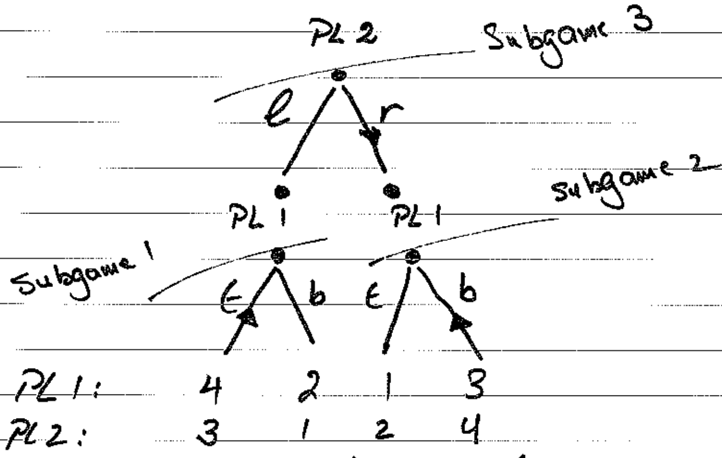
$q^* = \frac{1}{2}$



NE in mixed strategies:

$$= \left\{ \left(\tfrac{3}{4} t, \tfrac{1}{4} b\right), \left(\tfrac{1}{2} l, \tfrac{1}{2} r\right) \right\}$$

(b)



In Subgame 1, PL 1 will choose t as 4 > 2 and PL 2 gets 3

In Subgame 2, PL 1 will choose b as 3 > 1 and PL 2 gets 4

In Subgame 3, PL 2 will choose r

$\therefore$ NE = {r, b}

(c) In ~~pure~~ sequential strategies, player 1 receives $\underline{3}$

In mixed strategies, player 1 receives

$$E(\pi_1) = 4\left(\frac{3}{4}\right)\cdot\left(\frac{1}{2}\right) - 2\left(\frac{3}{4}\right) - \frac{1}{2} + 3 = \frac{3}{2} - \frac{3}{2} - \frac{1}{2} + 3 = \underline{2.5}$$

Since 3 > 2.5 $\therefore$ PL1 should choose the sequential game