UNIVERSITY OF OTTAWA
**Principles of Physics I**
**PHY1321/31 Fall 2015**
***Dr. A. Czajkowski***

STUDENT'S# ________________

NAME:______________________

# Assignment 4: Thermal Physics IV: Maxwell Boltzmann Distribution Heat Engines, Second Law of Thermodynamics,

*Assigned: Oct 9* *Due: Oct 16 18:00 sharp!*

**1.** If we assume that the Earth's atmosphere has a uniform temperature of 20°C and uniform composition, with an effective molar mass of 28.9 g/mol. It is possible to derive the *law of atmospheres in the form below:*

$$n_V(y) = n_0 e^{-mgy/k_B T}$$

where $n_v(y)$ number density of molecules as function of the height $y$, $n_0$ is the number density at sea level, where $y = 0$. Commercial jetliners typically cruise at an altitude of 11.0 km. Find the ratio of the atmospheric density there, to the density at sea level.

**SOLUTION:**

(b)
$$\frac{n(y)}{n_0} = e^{-mgy/k_B T} = e^{-Mgy/N_A k_B T} = e^{-Mgy/RT}$$

$$= e^{-(28.9\times10^{-3}\text{ kg/mol})(9.8\text{ m/s}^2)(11\times10^3\text{ m})/(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})} = e^{-1.279} = \boxed{0.278}$$

2 Given is distribution of speeds of cars at 417 Highway as measured by OPP.

a) Is this a discrete or continuous distribution?

b) Find the Vmp, Vrms, Vavg

c) Find the probability that a randomly picked car will have speed larger than 120km/h.

d) Find the probability that a randomly picked car will have speed larger than 90km/h and less then 110km/h

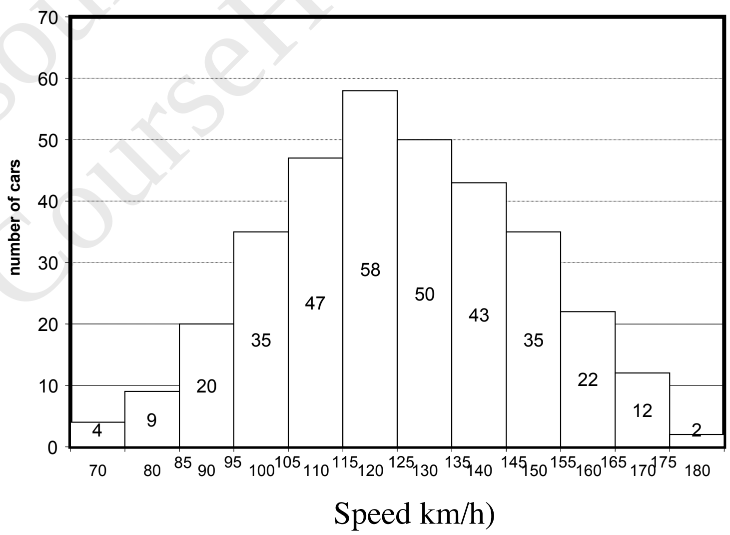
Answers:

a) discrete

b) $V_{mp}$=120km/h; $V_{rms}$ =127km/h; $V_{avg}$=125km/h

c) 0.49

d) 0.16




3 Given is 1mole of nitrogen molecules at atmospheric pressure and temperature of 20ºC.

a) Find the number of molecules having their speed between 200m/s and 201m/s $5.95\text{x}10^{20}$

$$Since: P(v)dv = 4\pi\left[\frac{1}{2\pi}\frac{m}{kT}\right]^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}}dv \quad and \quad P(v) = \frac{N}{N_A} \Rightarrow N_A = NP(v)$$

$$N = N_A 4\pi\left[\frac{1}{2\pi}\frac{M}{RT}\right]^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}}dv = N_A[3.11153\cdot10^{-8}]\cdot v^2 e^{-\frac{Mv^2}{2RT}}[1m/s]$$

b) Find the most probable speed: 402m/s

c) Find the number of molecules with the most probable speed (within 1 m/s from it) $11.99\text{ x}10^{20}$

d) Find the number of molecules with the speed of 1000m/s $0.60\text{ x}10^{20}$

e) Find the number of molecules with the speed of 2000m/s $7.69\text{x}10^{12}$

4. A particular heat engine has a useful power output of 5.00 kW and an efficiency of 25.0%. The engine expels 8 000 J of exhaust energy in each cycle. Find (a) the energy taken in during each cycle and (b) the time interval for each cycle.

(a) We have $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|} = 0.250$ with $|Q_c| = 8\,000$ J, we have $|Q_h| = \boxed{10.7 \text{ kJ}}$

(b) $W_{\text{eng}} = |Q_h| - |Q_c| = 2\,667$ J and from $P = \frac{W_{\text{eng}}}{\Delta t}$, we have $\Delta t = \frac{W_{\text{eng}}}{P} = \frac{2\,667 \text{ J}}{5\,000 \text{ J/s}} = \boxed{0.533 \text{ s}}$.

5. Heat engine *X* takes in four times more energy by heat from the hot reservoir than heat engine *Y*. Engine *X* delivers two times more work, and it rejects seven times more energy by heat to the cold reservoir than heat engine *Y*. Find the efficiency of (a) heat engine *X* and (b) heat engine *Y*.

We have $Q_{hx} = 4Q_{hy}$, $W_{\text{eng}\,x} = 2W_{\text{eng}\,y}$ and $Q_{cx} = 7Q_{cy}$. As well as $Q_{hx} = W_{\text{eng}\,x} + Q_{cx}$ and $Q_{hy} = W_{\text{eng}\,y} + Q_{cy}$

Substituting, $4Q_{hy} = 2W_{\text{eng}\,y} + 7Q_{cy}$

$$4Q_{hy} = 2W_{\text{eng}\,y} + 7Q_{hy} - 7W_{\text{eng}\,y}$$
$$5W_{\text{eng}\,y} = 3Q_{hy}$$

(b) $e_y = \frac{W_{\text{eng}\,y}}{Q_{hy}} = \frac{3}{5} = \boxed{60.0\%}$

(a) $e_x = \frac{W_{\text{eng}\,x}}{Q_{hx}} = \frac{2W_{\text{eng}\,y}}{4Q_{hy}} = \frac{2}{4}(0.600) = 0.300 = \boxed{30.0\%}$

6. Suppose a heat engine is connected to two energy reservoirs, one a pool of molten freezing 1.00 g of aluminum and melting 15.0 g of mercury during each cycle. The heat of fusion of aluminum is $3.97 \times 10^5$ J/kg; the heat of fusion of mercury is $1.18 \times 10^4$ J/kg. What is the efficiency of this engine?

The heat to melt 15.0 g of Hg is $|Q_c| = mL_f = (15 \times 10^{-3} \text{ kg})(1.18 \times 10^4 \text{ J/kg}) = 177$ J

The energy absorbed to freeze 1.00 g of aluminum is

$$|Q_h| = mL_f = (10^{-3} \text{ kg})(3.97 \times 10^5 \text{ J/kg}) = 397 \text{ J}$$

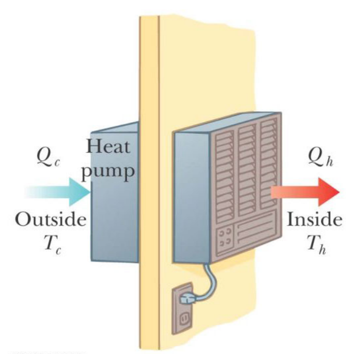
and the work output is $W_{\text{eng}} = |Q_h| - |Q_c| = 220$ J

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{220 \text{ J}}{397 \text{ J}} = 0.554 \text{, or } \boxed{55.4\%}$$

The theoretical (Carnot) efficiency is

$$\frac{T_h - T_c}{T_h} = \frac{933 \text{ K} - 243.1 \text{ K}}{933 \text{ K}} = 0.749 = 74.9\%$$

7 A heat engine operating between 200°C and 80.0°C achieves 20.0% of the maximum possible efficiency. What energy input will enable the engine to perform 10.0 kJ of work?




The Carnot efficiency of the engine is $e_c = \frac{\Delta T}{T_h} = \frac{120 \text{ K}}{473 \text{ K}} = 0.253$

At 20.0% of this maximum efficiency, $e = 0.200(0.253) = 0.050\,6$

From the definition of efficiency $W_{\text{eng}} = |Q_h|e$ and $|Q_h| = \frac{W_{\text{eng}}}{e} = \frac{10.0 \text{ kJ}}{0.050\,6} = \boxed{197 \text{ kJ}}$