

Solution:

MAT1320D Calculus 1

Midterm 01

Professor: Jose Malagon-Lopez

7 February

NAME: _____

STUDENT NUMBER: _____

- No calculators or other electronic aids allowed.
- No notes, books or other papers allowed.
- Answer all questions in the space provided. You must justify your answers and explain your reasoning.
- There are 4 pages. In all there are 7 questions worth a total of 70 marks.

1. Find the derivative of the following functions:

[3] a) $y = 3\sqrt{x} - \frac{4}{x^3} + e^2.$

Solution: $y' = \frac{3}{2\sqrt{x}} + \frac{12}{x^4}.$

[3] b) $y = e^x \sin(x).$

Solution: $y' = e^x \sin(x) + e^x \cos(x).$

[3] c) $y = \frac{\tan(x)}{\sqrt[5]{x}}.$

Solution: $y' = \frac{\sec^2(x)\sqrt[5]{x} - \tan(x)\frac{1}{5x^{4/5}}}{x^{2/5}}.$

[4] d) $y = \cos(x)x^2e^x.$

Solution: $y' = -\sin(x)x^2e^x + \cos(x)2xe^x + \cos(x)x^2e^x.$

2. An object is launched directly upward from a platform. If the height of the object with respect to the ground, in feet, after t seconds that it was launched is given by

$$h(t) = -2t^2 + 8t + 10.$$

- [4] a) Find the formulas that describe the velocity and acceleration of the object at time t .

Solution: velocity at time t is $v(t) = h'(t) = -4t + 8$, acceleration at time t is $a(t) = h''(t) = -4$.

- [2] b) Find the height and velocity of the object after 2 seconds. Give units.

Solution: $v(2) = 0$ feet/sec.

- [4] c) What is the speed of the object at the moment that hits the ground? Give the units.

Solution: The object hits the ground when $0 = h(t)$. Solving for t we get:

$$0 = -2t^2 + 8t + 10 = -2(t^2 - 4t - 5) = -2(t - 5)(t + 1)$$

Thus, the object hits the ground after 5 seconds of being launched. Hence, the speed of the object when it hits the ground is $|v(5)| = |-12| = 12$ feet/sec.

- [8] 3. Find an equation of the tangent line to $f(t) = \pi \sin(t)$ at $\left(\frac{2\pi}{3}, f\left(\frac{2\pi}{3}\right)\right)$.

Solution: We have that $f\left(\frac{2\pi}{3}\right) = \pi \sin\left(\frac{2\pi}{3}\right) = \frac{\pi\sqrt{3}}{2}$. Also, since $f'(t) = \pi \cos(t)$, we have that $f'\left(\frac{2\pi}{3}\right) = \pi \cos\left(\frac{2\pi}{3}\right) = \frac{\pi}{2}$.

Hence, the equation we want is

$$y = f'\left(\frac{2\pi}{3}\right)\left(x - \frac{2\pi}{3}\right) + f\left(\frac{2\pi}{3}\right) = \frac{\pi}{2}\left(x - \frac{2\pi}{3}\right) + \frac{\pi\sqrt{3}}{2}.$$

Please note that this exam from 2011. Our exam will be different. However, there are some similarities.

The best way to prepare for the midterm is to read the book and notes, and solve the suggested exercises given in the course outline (See Brightspace >> Course Outline)

4. Consider the function $f(x) = 4x^5 - 5x^4$.

- [3] a) Find the x -intercepts and y -intercept of f .
- [2] b) Find $f'(x)$ and $f''(x)$.
- [6] c) Determine the intervals where f is increasing and where is decreasing. Conclude giving the first entry of the local maximums and local minimums of f .
- [6] d) Determine the intervals where f is concave upward and where is concave downward. Conclude giving the first entry of the inflection points of f .

Solution:

This is not part of our midterm

**Our midterm will include all what
we covered so far except 3.10 Linear Approximation**

- [8] 5. Consider the function $f(x) = \sqrt{2x}$. Using limits, compute $f'(x)$. Also, determine the domain of $f'(x)$.

Solution:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{2x+2h} - \sqrt{2x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{2x+2h} - \sqrt{2x}}{h} \left(\frac{\sqrt{2x+2h} + \sqrt{2x}}{\sqrt{2x+2h} + \sqrt{2x}} \right) \\ &= \lim_{h \rightarrow 0} \frac{(\sqrt{2x+2h})^2 - (\sqrt{2x})^2}{h(\sqrt{2x+2h} + \sqrt{2x})} \\ &= \lim_{h \rightarrow 0} \frac{2h}{h(\sqrt{2x+2h} + \sqrt{2x})} \\ &= \frac{2}{\sqrt{2x} + \sqrt{2x}} \\ &= \frac{1}{\sqrt{2x}}. \end{aligned}$$

Domain: all $x > 0$.

- [7] 6. Find all the solutions of $\sin(2x) = 2 \sin(x)$ on $[0, 2\pi]$.

Solution: Since $\sin(2x) = 2 \sin(x) \cos(x)$, we have

$$\begin{aligned} \sin(2x) &= 2 \sin(x) \\ 2 \sin(x) \cos(x) &= 2 \sin(x) \\ 2 \sin(x) (\cos(x) - 1) &= 0 \end{aligned}$$

So we need to solve the equations $2 \sin(x) = 0$ and $\cos(x) = 0$.

The first equation gives $x = 0, \pi, 2\pi$. The second equation gives $x = 0, 2\pi$.

Hence, the solution to the equation is $x = 0, \pi, 2\pi$.

- [7] 7. Consider the function $f(x) = e^x$. Find all the points where the slope m of the tangent line to f is of the form $m = e^{x^2} e^{x-4}$.

Solution: Since $m = f'(x) = e^x$, we need to solve

$$e^x = e^{x^2} e^{x-4} = e^{x^2+x-4}.$$

Thus, we need to solve

$$\begin{aligned} x &= x^2 + x - 4 \\ 0 &= x^2 - 4 = (x-2)(x+2). \end{aligned}$$

Hence, the slope m of the tangent to f is $m = e^{x^2} e^{x-4}$ at $x = -2, 2$.