

MATH 1300 A & B-MIDTERM # 1-2017.

Professors: Richard Blute & Termeh Kousha

Last NAME (In CAPITAL letters): _____ First Name: _____

Student ID# Solutions and Marking Scheme

INSTRUCTIONS: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 50 points.

Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.

By signing below, you acknowledge that you have ensured that you are complying with the above statement:

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Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages.

NO CALCULATORS. NO BOOKS. NO NOTES.

MULTIPLE CHOICE ANSWERS:

E

#1

D

#2

A

#3

A

#4

Multiple Choice Questions (1-4)

Question 1 Find all values of x for which the following function has a tangent line of slope 0 (i.e. $f'(x) = 0$).

$$f(x) = (x+3)^2 e^{-x}$$

- A) $x = -\frac{1}{2}$ B) $x = 1$ C) $x = -1$ D) $x = 1$ and $x = 3$ ~~E) $x = -1$ and $x = -3$~~

$$\begin{aligned} f'(x) &= 2(x+3) e^{-x} - e^{-x} (x+3)^2 \\ &= (x+3) e^{-x} (2 - (x+3)) = 0 \end{aligned}$$

$$\begin{aligned} x+3 &= 0 & x &= -3 \\ 2 - x - 3 &= 0 & -1 - x &= 0 & x &= -1 \end{aligned}$$

Question 2 Calculate

$$\lim_{x \rightarrow 1} \frac{x^2 - 5x + 4}{x^2 + x - 2}$$

- A) 2 B) $\frac{5}{2}$ C) The limit does not exist. ~~D) -1~~ E) -3

$$\lim_{x \rightarrow 1} \frac{(x-1)(x-4)}{(x+2)(x-1)} \stackrel{x \neq 1}{=} \lim_{x \rightarrow 1} \frac{x-4}{x+2} = \frac{1-4}{3} = \frac{-3}{3} = -1$$

$$D_f = \mathbb{R} - \{-2, 1\}$$

Question 3 Find the value of a and b , for which the following function is continuous everywhere:

$$f(x) = \begin{cases} |x-1| + a & \text{if } x < -3 \\ 9-x & \text{if } -3 \leq x < -2 \\ |-x-1| + b & \text{if } x \geq -2 \end{cases}$$

- ☒ A) $a = 8$ and $b = 10$
☐ B) $a = 9$ and $b = 12$
☐ C) $a = 12$ and $b = 8$
☐ D) $a = 6$ and $b = 7$
☐ E) $a = 8$ and $b = 6$

@ $x = -3$ $\lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^+} f(x)$

$$|-3-1| + a = 9 - (-3)$$

$$4 + a = 12 \quad a = 8$$

@ $x = -2$ $\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^+} f(x)$

$$9 - (-2) = |-(-2)-1| + b$$

$$11 = 1 + b \quad b = 10$$

Question 4 Solve the equation $\log_7(x-4) + \log_7(x+2) = 1$ for x .

- ☒ A) $x = 5$ ☐ B) $x = -5$ ☐ C) $x = 5$ and $x = -3$ ☐ D) $x = -2$ ☐ E) $x = -2$ and $x = -3$

$$\log_7 (x-4)(x+2) = 1$$

$$(x-4)(x+2) = 7 \quad x^2 - 2x - 8 = 7$$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

$$x = 5 \quad x = -3$$

$x = -3$
not in the domain

Long Answer Questions (5-7)

Question 5 (10 points)

Suppose a function is defined implicitly by the equation

$$x^2y + x^2 \ln(y) = 12 - 4y^3 - x^2$$

- Find $\frac{dy}{dx}$ at (2, 1).
- Find the equation of a tangent line to the curve at the point (2, 1).

5 pts

$$2xy + x^2y' + 2x \ln y + x^2 \frac{y'}{y} = -12y^2y' - 2x$$

@ (2, 1)

$$4 + 4y' + 4 \ln 1 + 4y' = -12y' - 4$$

3 pts

$$20y' = -8 \quad y' = -\frac{8}{20} \text{ or } -\frac{2}{5}$$

$$m = -\frac{2}{5}$$

@ (2, 1)

$$y = mx + b$$

$$1 = -\frac{2}{5}(2) + b \quad b = 1 + \frac{4}{5} = \frac{9}{5}$$

$$2 \text{ pts } y = -\frac{2}{5}x + \frac{9}{5}$$

If y' is wrong, but
line equation correct for
that y' , full 2 pts.

Question 6 (10 points)

Using only the definition of derivative as a limit, calculate $f'(x)$ where

$$f(x) = \sqrt{x+3}$$

$$(3 \text{ pts}) \quad f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h}$$

$$(3 \text{ pts}) = \lim_{h \rightarrow 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} \cdot \frac{\sqrt{x+h+3} + \sqrt{x+3}}{\sqrt{x+h+3} + \sqrt{x+3}}$$

$$(2 \text{ pt}) = \lim_{h \rightarrow 0} \frac{x+h+3 - (x+3)}{h(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$(2 \text{ pts}) = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+3} + \sqrt{x+3}} = \frac{1}{2\sqrt{x+3}}$$

If they don't write limit all the way through, -2.

Question 7 (10 points)

One thousand dollars is invested at a rate of 6%,

- 5 • How much money will be in the account in 3 years if interest is compounded every 6 months? (You do not need to simplify your answer.)
- 5 • How long will it take for the initial investment to double if interest is 6% and compounds continuously? (You do not need to simplify your answer.)

$$\bullet P_0 = 1000 \quad r = 0.06. \quad n = 2$$

$$A(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt}$$
$$A(3) = 1000 \left(1 + \frac{0.06}{2}\right)^{2(3)} \quad \text{or}$$
$$1000 (1 + 0.03)^6$$

$$\bullet P(t) = 2P_0 = P_0 e^{0.06t}$$

$$\Rightarrow 2 = e^{0.06t}$$

$$\ln 2 = 0.06t$$

$$t = \frac{\ln 2}{0.06}$$

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MULTIPLE CHOICE ANSWERS:

E

#1

A

#2

B

#3

B.

#4

Multiple Choice Questions (1-4)

Question 1 Find all values of x for which the following function has a tangent line of slope 0 (i.e. $f'(x) = 0$).

$$f(x) = (-x + 1)^2 e^{x-3}$$

- A) $x = -\frac{1}{2}$ B) $x = 1$ C) $x = -1$ D) $x = 1$ and $x = 3$ ☒ E) $x = -1$ and $x = 1$

$$f'(x) = -2(-x+1)e^{x-3} + e^{x-3}(-x+1)^2$$

$$= (-x+1)e^{x-3}(-2 + (-x+1)) = 0$$

$$-x+1=0$$

$$x=1$$

$$-2 - x + 1 = 0$$

$$-1 - x = 0$$

$$x = -1$$

Question 2 Calculate

$$\lim_{x \rightarrow -1} \frac{x^2 - 2x - 3}{x^2 - 1}$$

- ☒ A) $\frac{5}{2}$ B) $\frac{5}{2}$ C) The limit does not exist. D) 0 E) -3

$$D_f = \mathbb{R} - \{-1, +1\}$$

$$\lim_{x \rightarrow -1} \frac{(\cancel{x}+1)(x-3)}{(x-1)(\cancel{x}+1)} \stackrel{x \neq -1}{=} \lim_{x \rightarrow -1} \frac{x-3}{x-1}$$

$$= \frac{-4}{-2} = 2$$

Question 3 Find the value of a and b , for which the following function is continuous everywhere:

$$f(x) = \begin{cases} |x-1| + a & \text{if } x < -2 \\ 6-x & \text{if } -2 \leq x < -1 \\ |-x-1| + b & \text{if } x \geq -1 \end{cases}$$

- A) $a = 4$ and $b = 5$
~~B) $a = 5$ and $b = 7$~~
 C) $a = 5$ and $b = 8$
 D) $a = 4$ and $b = 7$
 E) $a = 5$ and $b = 6$

① $x = -2$ $\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^+} f(x)$

$$|-2-1| + a = 6 - (-2)$$

$$3 + a = 8 \quad a = 5$$

② $x = -1$ $\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^+} f(x)$

$$6 - (-1) = |-(-1)-1| + b$$

$$7 = 0 + b \quad b = 7$$

Question 4 Solve the equation $\log_6(x-3) + \log_6(x+2) = 1$ for x .

- A) $x = -4$ ~~B) $x = 4$~~ C) $x = 4$ and $x = -3$ D) $x = 2$ E) $x = -2$ and $x = 4$

$$\log_6 (x-3)(x+2) = 1 \quad (x-3)(x+2) = 6$$

$$x^2 - x - 6 = 6$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$x = 4$$

$$x = -3$$

not in the domain.

Long Answer Questions (5-7)

Question 5 (10 points)

Suppose a function is defined implicitly by the equation

$$x^2y + x \ln(y) = 14 - 2y^2 - x^3$$

- Find $\frac{dy}{dx}$ at $(2, 1)$.
- Find the equation of a tangent line to the curve at the point $(2, 1)$.

$$2xy + x^2y' + \ln y + x \frac{y'}{y} = -4yy' - 3x^2$$

$$4 + 4y' + \ln 1 + 2y' = -4y' - 12$$

$$10y' = -16 \quad y' = \frac{-16}{10} = -\frac{8}{5}$$

@ $(2, 1)$

$$m = -\frac{8}{5}$$

$$y = mx + b$$

$$1 = -\frac{8}{5}(2) + b$$

$$b = 1 + \frac{16}{5} = \frac{21}{5}$$

$$y = -\frac{8}{5}x + \frac{21}{5}$$

Question 6 (10 points)

Using only the definition of derivative as a limit, calculate $f'(x)$ where

$$f(x) = \sqrt{x-5}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{x+h-5} - \sqrt{x-5}}{h} \cdot \frac{\sqrt{x+h-5} + \sqrt{x-5}}{\sqrt{x+h-5} + \sqrt{x-5}}$$

$$= \lim_{h \rightarrow 0} \frac{x+h-5 - (x-5)}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$\lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h-5} + \sqrt{x-5}} = \frac{1}{2\sqrt{x-5}}$$

Question 7 (10 points)

One thousand dollars is invested at a rate of 4%,

- How much money will be in the account in 5 years if interest is compounded every 3 months? (You do not need to simplify your answer.)
- How long will it take for the initial investment to double if interest is 4% and compounds continuously? (You do not need to simplify your answer.)

② $P_0 = 1000$ $r = 0.04$ $n = 4$

$$A(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt}$$

$$A(5) = 1000 \left(1 + \frac{0.04}{4}\right)^{4(5)} \quad \text{or}$$

$$1000 (1 + 0.01)^{20}$$

③ $2P_0 = P_0 e^{0.04t}$

$$2 = e^{0.04t}$$

$$\ln 2 = 0.04t$$

$$t = \frac{\ln 2}{0.04}$$