Concordia University

EMAT 213 - Final Exam

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Course Examiner: M. Bertola

Date: April 2006.

Time allowed: 3 hours.

[30 pts] Problem 1

Solve the following *first* order ODEs by finding the general solution.

(**a**) $y\,\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x+1}{y+3}$

(**b**) $\cos(x)\,y' + \sin(x)\,y = 1$ $\quad\left[\textbf{Note:} \int \tan(s)\mathrm{d}s = \ln|\cos(s)| + C\right]$

(**c**) $(x+y)^2\mathrm{d}x + (2xy + x^2 - 1)\mathrm{d}y = 0$

[20 pts] Problem 2

Solve the following linear ODEs by finding the general solution (complementary plus particular)

(**a**) $y'''' + 2y'' + y = x^2$ **[Const. Coeffs + Undet. Coeffs.]**

(**b**) $x^2y'' - 3xy' + 3y = 2x^4\mathrm{e}^x$ **[Cauchy–Euler + variation of parms]**
NOTE: $\int x^2\mathrm{e}^x\mathrm{d}x$ **by parts twice**

[10 pts] Problem 3

A thermometer is taken from an inside room to the outside, where the air temperature is -10^oC. After one minute the thermometer reads 14^oC and after three minutes it reads 4^oC. What was the temperature of the room?

[15 pts] Problem 4

Find the general solution (complementary plus particular) of the following system of ODEs

$$\begin{cases} x' = 4x + 2y + 3\mathrm{e}^t \\ y' = 2x + y + \mathrm{e}^t \end{cases}$$

[10 pts] Problem 5

Find two power-series solutions centered at $x = 0$ and up to order 4 of the given ODE

$$(x^2+1)y'' - 6y = 0$$

[15 pts] Problem 6

A force of 32 N. stretches a spring by $2m$. A mass of 1Kg is attached to the spring and let come to **rest in the equilibrium position**. Starting at $t = 0$ a force equal to

$$f(t) = 8\sin(4t)$$

is applied to the system. Find the equation of motion if the surrounding medium offers a **damping** force numerically equal to 8 times the instantaneous velocity.