

Econ 301: Assignment 3

1. TRUE or FALSE. Provide full explanation for your answer. Credit will be given only for correct reasoning. (2 points each).

- a. If a good is a normal good, then the substitution effect and the income effect are of the same sign.

True. For a normal good, if we consider a price decrease, the substitution effect will be positive. Because of a decrease in price, purchasing power of income will rise, thus for a normal good, the income effect will also be positive. Thus both of them will have the same sign. The opposite is true for a price increase.

- b. If the consumer has Cobb-Douglas preferences, it is possible for one of the goods to be a Giffen good.

False. If a consumer has Cobb-Douglas preferences, her consumption demand for both goods rise proportionately to rise in income (homothetic preferences). Thus when income rises, it can't be that demand for one rises while the other falls.

- c. If a consumer has quasilinear preferences over two goods, then her consumption of neither good depends on her level of income.

False. When consumers have quasi-linear preferences, their Utility function is as follows: $U(x, y) = v(x) + y$. In this case, the demand for x does not depend upon income since her consumption of x can be derived from the condition: $v'(x) = \frac{p_x}{p_y}$, without using the budget constraint. However her demand for y needs to be derived by: $y = m - p_x \cdot x$, and thus it does depend upon income.

- d. In the case of two goods that are perfect complements, the entire effect of a price change is due to substitution effect.

False, when goods are perfect complements, it means that the substitution effect is zero. Thus the entire effect of price change is the income effect.

- e. The magnitude of Slutsky Substitution Effect and Hicks Substitution Effect will always be the same when preferences are strictly convex.

False. The magnitude to the Slutsky substitution effect and Hicks substitution effect depends on the curvature of the indifference curve and can be different.

2. (15 points) Consider the utility function $u(x, y) = 2 \ln x + \ln y$. Initially, the prices are $p_x = \$2/\text{unit}$ and $p_y = \$1/\text{unit}$. The consumer has an income of \$18. Illustrate your answers with graphs.

a. (3 points) Derive the consumer's optimal consumption bundle.

At the optimal :MRS = slope of the budget line.

$$MRS = -\frac{\frac{2}{x}}{\frac{1}{y}} = -\frac{2y}{x}$$

The slope of the budget line is: $-\frac{p_x}{p_y} = -\frac{2}{1} = -2$

Hence at the optimal: $-\frac{2y}{x} = -2; x = y$.

Plugging this into the budget line gives:

$$2x + y = 18;$$

$$2x + x = 18; \quad 2y + y = 18$$

$$x=6; \quad y=6.$$

b. (2 points) Now suppose the price of good x increases to $p'_x = \$3/\text{unit}$. What is the new optimal consumption bundle?

So now the slope of the budget line will become -3.

At the optimal: $-\frac{2y}{x} = -3$

$$2y = 3x$$

Plugging this into the budget line gives:

$$3x + y = 18$$

$$3x + \frac{3x}{2} = 18$$

$$x = 4; y = 6$$

c. (5 points) Calculate the substitution effect and the income effect. Illustrate it on a graph.

The income that corresponds to the pivoted budget line is: $m' = 3 * 6 + 1 * 6 = 24$. (We consider the new set of prices (3,1) but consume the original bundle (6,6).) **The budget equation that corresponds to the pivoted budget line is :** $3x + y = 24$.

Using the optimum condition: $-\frac{2y}{x} = -3$ **and plugging it into the pivoted budget line, we get:**

$$3x + y = 24$$

$$3x + \frac{3x}{2} = 24$$

$$x = \frac{16}{3}, y = 8$$

Thus the substitution effect is: $\frac{16}{3} - 6 = -\frac{2}{3}$

And the income effect is: $4 - \frac{16}{3} = -\frac{4}{3}$.

3. (10 points) Connie, as you may recall, always takes one cookies with one cup of coffee. Cookies cost \$2 per cup, and coffee costs \$1 per cup. Connie has \$12 to spend on cookies and coffee. Consider a price drop in cookies to \$1 per cup.

- a. (4 points) Illustrate Connie's optimal consumption bundles before and after the price change with a graph. Explain your reasoning.

Connie's utility function is: $\min\{x_1, x_2\}$ where x_1 represents cookie and x_2 represents coffee.

The budget line before the price drop is: $2x_1 + x_2 = 12$. At the consumption point, the following holds: $x_1 = x_2$. Plugging this in the budget constraint, we get:

$$2x_1 + x_2 = 12$$

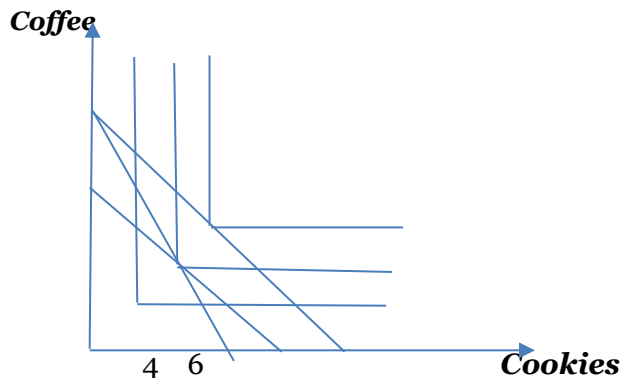
$$2x_1 + x_1 = 12$$

$$x_1 = 4; x_2 = 4$$

After the price drop, the budget line becomes $x_1 + x_2 = 12$

And so the optimal consumption becomes $x_1 = 6; x_2 = 6$

- b. (6 points) Decompose the change in Connie's consumption of cookies into the income effect and the substitution effect. Illustrate them in the graph.



Before price drop, 4 cookies were consumed. After price drop 6 cookies are consumed. Drawing the pivoted budget line through the original bundle but parallel to the new budget line, we see that still 4 cookies will be consumed. So substitution effect is zero. The entire effect is the income effect which is $(6-4)=2$ cookies.

