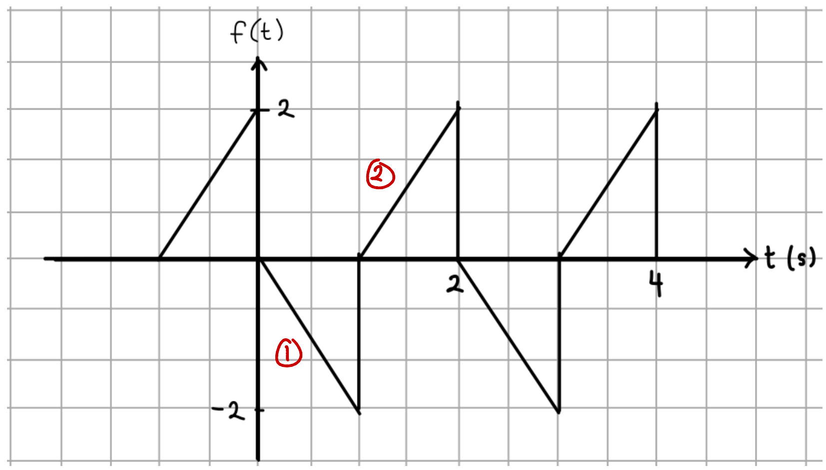
ELEC 2501 Quiz 1 - Alpha

Name: Solution Student No: ________________

1) Given the periodic function *f(t)*, find the average and root mean square (rms) value:



Step 1: Find equation that describes the function

Eqn ①: $m_1 = \frac{-2-0}{1-0} = -2$

$f_1(t) = -2t + b$

at $(0,0)$:

$f_1(0) = -2(0) + b = 0 \Rightarrow b = 0$

$\Rightarrow f_1(t) = -2t$

Eqn ②: $m_2 = \frac{2-0}{2-1} = 2$

$f_2(t) = 2t + b$

at $(1,0)$:

$f_2(1) = 2(1) + b = 0 \Rightarrow b = -2$

$\Rightarrow f_2(t) = 2t - 2$

Step 2: Find the average value

$$\text{Average} = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{2}\left(\int_0^1 -2t\,dt + \int_1^2 (2t-2)\,dt\right) = \frac{1}{2}\left(-t^2\Big|_0^1 + t^2 - 2t\Big|_1^2\right) = 0\,V$$

Step 3: Find the rms value

$$\text{RMS} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} = \sqrt{\frac{1}{2}\left[\int_0^1 (-2t)^2\,dt + \int_1^2 (2t-2)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{2}\left[\int_0^1 4t^2\,dt + \int_1^2 (4t^2 - 8t + 4)\right]} = \sqrt{\frac{1}{2}\left(\frac{4t^3}{3}\Big|_0^1 + \left(\frac{4t^3}{3} - 4t^2 + 4t\right)\Big|_1^2\right.}$$

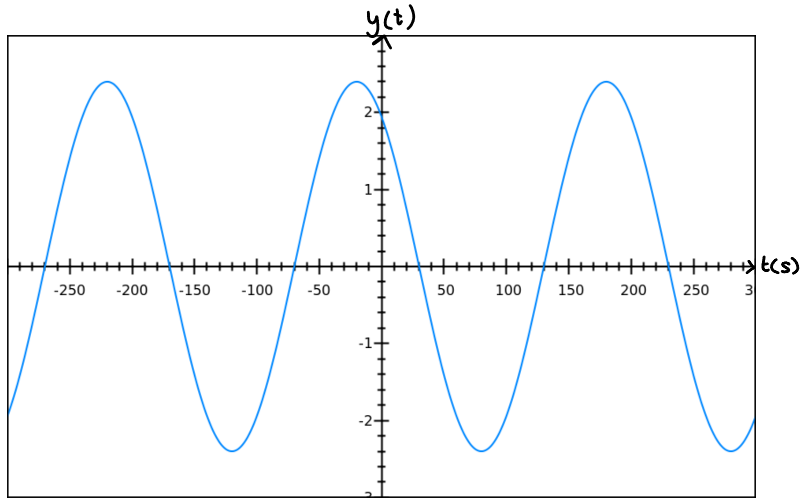
$$= \sqrt{\frac{1}{2}\left(\frac{4}{3} + \left(\frac{32}{3} - 16 + 8\right) - \left(\frac{4}{3} - 4 + 4\right)\right)} = 1.154\,V \text{ or } \sqrt{\frac{4}{3}}\,V \text{ or } \frac{2}{\sqrt{3}}\,V$$

2) Given the following periodic function, find:

a. the period, T

b. the angular frequency, ω_0

c. write the standard expression for the function in the form of $y(t) = A \cos(\omega_0 t + \theta)$



a) $T = 130 - (-70) = 200s$

b) frequency, $f = \frac{1}{T} = \frac{1}{200}$ Hz

$\omega_0 = 2\pi f = 2\pi\left(\frac{1}{200}\right) = \frac{\pi}{100}$ rad/s

c) $A = 2.4$

$y(t) = 2.4 \cos\left(\frac{\pi}{100}t + \theta\right)$

Substitute $t = 30$, $y(t) = 0$

$y(30) = 2.4 \cos\left(\frac{\pi}{100}(30) + \theta\right) = 0$

$\cos\left(\frac{3\pi}{10} + \theta\right) = 0 \Rightarrow \frac{3\pi}{10} + \theta = \cos^{-1} 0 = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

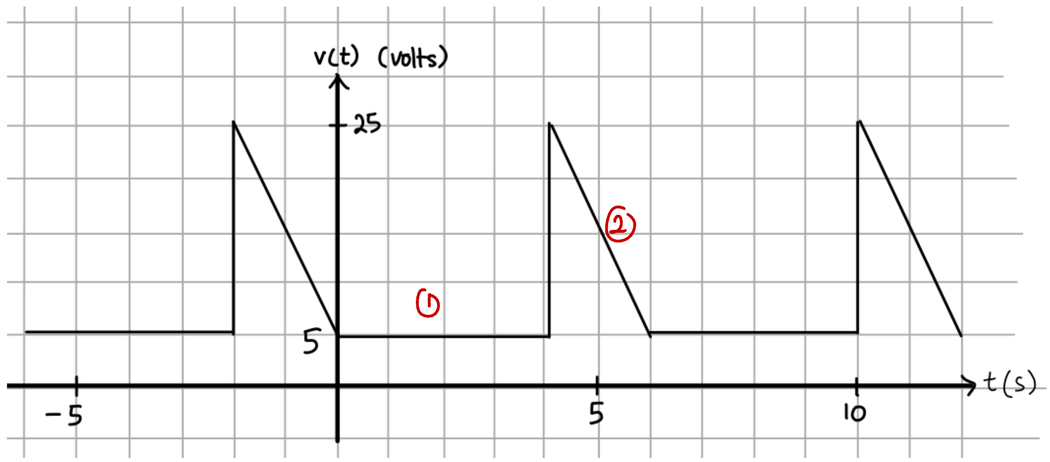
For $\cos^{-1} 0 = \frac{\pi}{2}$, $\theta = \frac{\pi}{2} - \frac{3\pi}{10} = \frac{\pi}{5}$

$\therefore y(t) = 2.4 \cos\left(\frac{\pi}{100}t + \frac{\pi}{5}\right)$

ELEC 2501 Quiz 1 - Bravo

Name: Solution Student No: ________________

1) Find the average and root mean square (rms) value of the following periodic function:



Eqn 1 : $v(t) = 5$

Eqn 2 : $m = \frac{5-25}{6-4} = -10$

$v_2(t) = -10t + b$

at $(4, 5)$:

$v_2(4) = -10(4) + b = 5$

$\Rightarrow b = 45$

$\Rightarrow V_2(t) = -10t + 45$

Average $= \frac{1}{T}\int_0^T f(t)\,dt$

$= \frac{1}{6}\left(\int_0^4 5\,dt + \int_4^6 -10t + 45\right)$

$= \frac{1}{6}\left(20 + (-10)\right)$

$= \frac{5}{3}V$ or $1.67\,V$

$$RMS = \sqrt{\frac{1}{6}\left[\int_0^4 5^2\,dt + \int_4^6 (-10t+45)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{6}\left[\int_0^4 25\,dt + \int_4^6 (100t^2 - 900t + 2025)\,dt\right]}$$

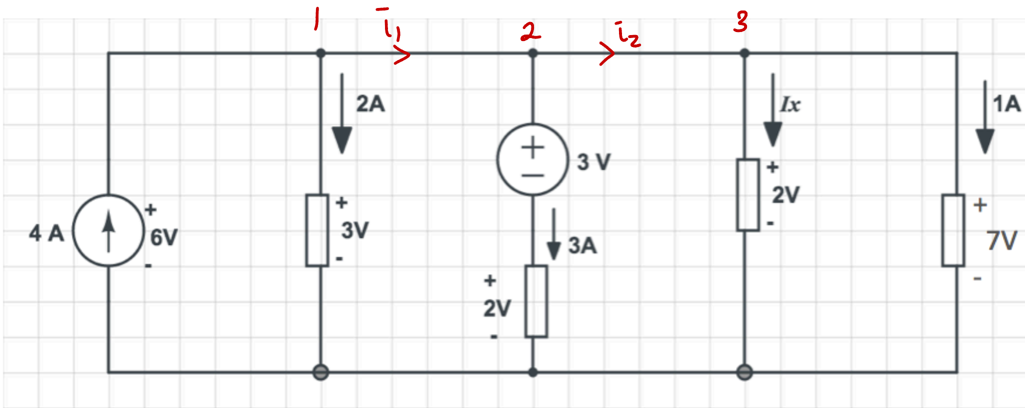
$$= \sqrt{\frac{1}{6}\left[25t\Big|_0^4 + \frac{100t^3}{3} - 450t^2 + 2025t\Big|_4^6\right]}$$

$$= \sqrt{\frac{1}{6}(100 + 116.67)}$$

$$= \sqrt{36.11}$$

$$= 6.01 \quad V$$

2) Find the value of I_x



Method 1: Using summation of power

$$-(4A\cdot 6V) + 2A\cdot 3V + 3A(3V+2V) + I_x\cdot 2V + 1A\cdot 7V = 0$$

$$2I_x + 4 = 0$$

$$I_x = \frac{-4}{2} = -2A$$

Method 2: Using KCL

at 1: $4A = 2A + \bar{i}_1$

$\bar{i}_1 = 2A$

at 2: $\bar{i}_1 = 3A + i_2$

$2A = 3A + \bar{i}_2$

$\bar{i}_2 = -1A$

at 3: $i_2 = I_x + 1A$

$-1A = I_x + 1A$

$I_x = -2A$