Hint #1 - If we are dealing with independent samples, we look at the box-plot from each sample to determine normality. If we are dealing with dependent samples, we look at the box-plot of differences to determine normality.

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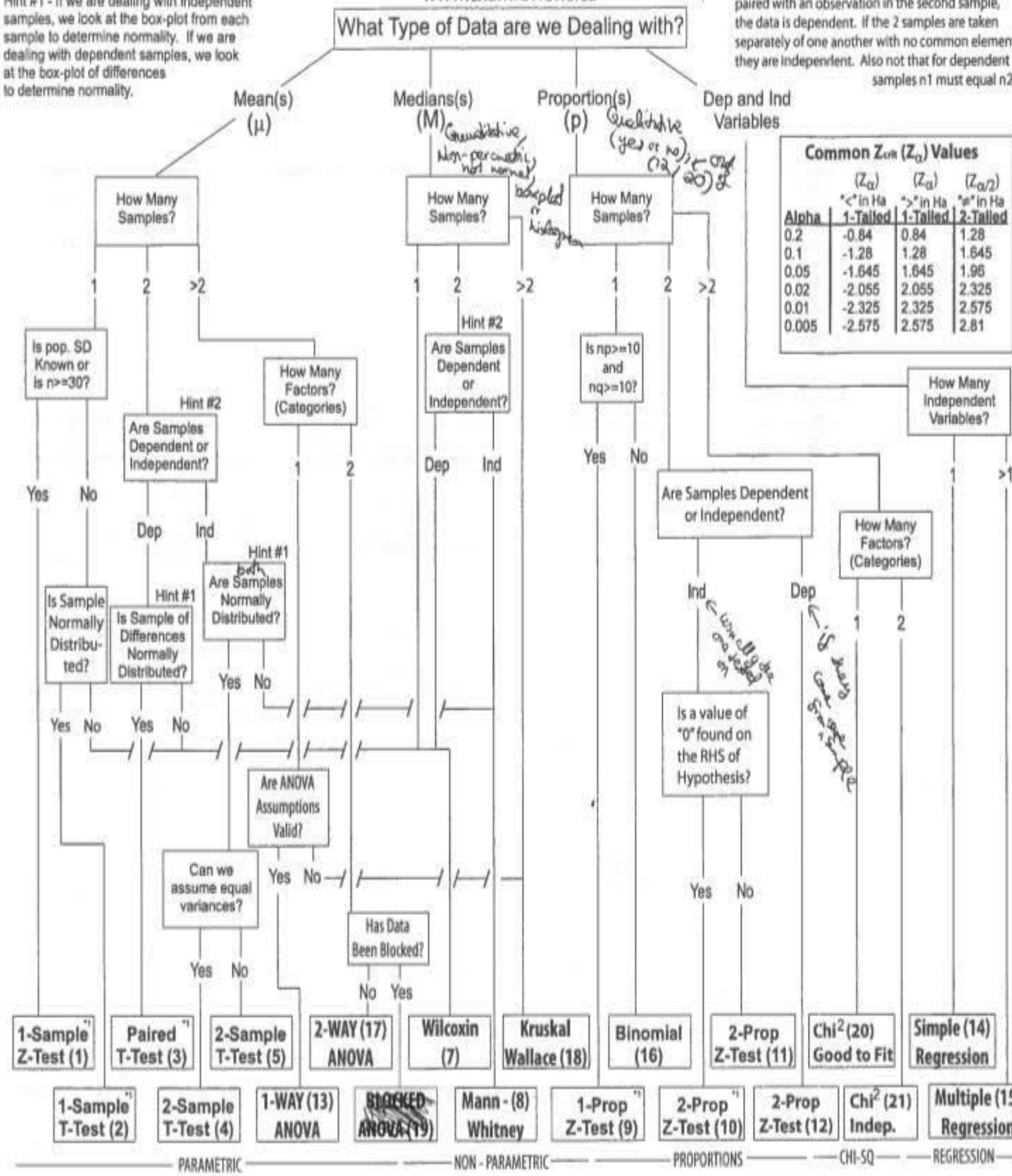
Hint #2 - If each observation from the first sample is paired with an observation in the second sample, the data is dependent. If the 2 samples are taken separately of one another with no common element, they are independent. Also not that for dependent samples n1 must equal n2.



Common Z_{crit} (Z_α) Values

Alpha	(Z_α) "<" in Ha 1-Tailed	(Z_α) ">" in Ha 1-Tailed	($Z_{\alpha/2}$) "≠" in Ha 2-Tailed
0.2	-0.84	0.84	1.28
0.1	-1.28	1.28	1.645
0.05	-1.645	1.645	1.96
0.02	-2.055	2.055	2.325
0.01	-2.325	2.325	2.575
0.005	-2.575	2.575	2.81

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Chi Sq-Goodness to Fit

Step 1 - Calculate the total of observed counts
Step 2 - Derive expected values as described in the question
Step 3 - Calculate Chi^2 for each row
Step 4 - Add the Chi^2 from each row to get your Chi^2_{stat}
Step 5 - Finish Hypothesis Test.

Ho: Data follows described dist.
Ha: Data follows some other dist.

$$Chi^2_{stat} = \sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i}$$ o=observed, e=expected, df=r-1

Reject Ho if $|Chi^2_{stat}| > |Chi^2_{crit}|$ or $p < \alpha$.

Table Headings for calculations:
| Group | Observed | Expected | Chi² |

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Chi Sq-Test of Independence

Step 1 - Calculate all row totals, column totals, grand total
Step 2 - Calculate expected value for each cell (e)
Step 3 - Calculate the Chi^2 value for each cell
Step 4 - Add the Chi^2 values from all cells to get Chi^2_{stat}
Step 5 - Finish Hypothesis Test.

Ho: Factor A and Factor B are independent
Ha: Factor A and Factor B are not independent

$$Chi^2_{stat} = \sum_{i=1}^{r}\sum_{j=1}^{c} \frac{(o_{ij} - e_{ij})^2}{e_{ij}} \qquad e_{ij} = \frac{(i \text{ row total})(j \text{ column total})}{\text{Total sample size}}$$

Reject Ho if $|Chi^2_{stat}| > |Chi^2_{crit}|$ or $p < \alpha$. o=observed, e=expected, df=(r-1)(c-1)

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Simple Regression (See box 15 for assumptions)

Manual Simple Regression Calculations:
Slope = $b_1 = r * (s_x / s_y)$
Intercept = $b_0 = \bar{y} - b_1\bar{x}$
Regression equation:
$\bar{y} = b_0 + b_1\bar{x}$

r = coefficient of correlation
s_x = standard dev. of x
s_y = standard dev. of y
$\bar{x}$ = mean of x values
y = mean of y values

Test #1-Significance of Model
Ho: $B_1 = 0$
Ha: $B_1 \neq 0$
$F_{stat} = MS_{reg}/MS_{err}$
Reject Ho if $|F_{stat}| > |F_{crit}|$ or $p < \alpha$.
$df_n = df_{reg}$ $df_d = df_{err}$

Test #2-Significance of Variable in Model
Ho: $B_n = 0$ $T_{stat} = \frac{b_n - B_n}{s_{bn}}$
Ha: $B_n \neq 0$
df=n-k-1
Reject Ho if $|T_{stat}| > |T_{crit}|$ or $p < \alpha$

All numbers come from minitab.

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Multiple Regression or Simple Regression

Assumptions
1) Normality
2) Equal Variance
3) Independence
(see other side under ANOVA)
4) Linear Model (See Below)

Test #1-Significance of Model
Ho: $B_1 = B_2 = B_3 ... = B_n = 0$
Ha: Atleast one $B_n \neq 0$
$F_{stat} = MS_{reg}/MS_{err}$
Reject Ho if $|F_{stat}| > |F_{crit}|$ or $p < \alpha$.
$df_n = df_{reg}$ $df_d = df_{err}$

Test #2-Significance of Variable in Model
Ho: $B_n = 0$ $T_{stat} = \frac{b_n - B_n}{s_{bn}}$ $df = df_e$
Ha: $B_n \neq 0$
Reject Ho if $|T_{stat}| > |T_{crit}|$ or $p < \alpha$
-All numbers come from minitab.
-This test can be repeated for each independent variable.
CI (coefficient) = $b_n +/- T_{\alpha/2} * s_{bn}$

CI = fit +/- $T_{\alpha/2}$ *SE_{fit} (Interval for average of all observations)
PI = fit +/- $T_{\alpha/2}$ *SQRT($SE_{fit}^2 + S^2$) (Interval for 1 observation)
df_{err} = n-k-1 (used for CI and PI)

-Regression Equation Given in Output as $\hat{y} = c + B_1x_1 + B_2x_2 + ... + B_nx_n$
-Residual= actual value - value predicted by model by subbing independent variables into the regression equation. (actual - fit)
-The best model will be the one with the highest R^2 or R^2_{adj}, the lowest S and the lowest number of independent variables.

Variance Inflation Factor (VIF)
VIF = $1/(1-R^2)$
Multicollinearity is a problem caused by highly correlated independent variables. It is a problem if VIF>10.

Linear Model Assumption (Residual vs Fit Graph)

IMPORTANT FORMULAS
$R^2 = 1-(SS_{err}/SS_{tot})$
$R^2_{adj} = 1-(MS_{err}/(SS_{tot}/df_{tot})$
s=SQRT(MSE) **MS = SS / df**

$df_{reg} = k$
$df_{err} = n-k-1$
$df_{tot} = n-1$
k=# independent variables

Correlation
-Measure the relationship between 2 variables
-Always between 1 and -1
-The closer to 1 or -1 the stronger the relationship

Ho:p=0 Ha:p≠0
Ho:p=0 Ha:p>0
$T_{stat} = \frac{r}{\sqrt{\frac{1-r^2}{n-1}}}$ df = n-2
- "r" is calculated by minitab
- Reject Ho if $|T_{stat}| > |T_{crit}|$ *see 1-tailed rules

[1]FPCF - If a sample(s) is more than 5% of the population we must apply the FPCF. Therefore, if n/N>0.05, we multiply the denominator by SQRT((N-n)/(N-1)).

*see bottom of page for 1 tailed

#	Test	1-Tailed Hyp	2-Tailed Hyp	Assumptions	Formulas	Decision (2-tailed)	CI (2-Sided)	CI (1-Sided Upper)	CI (1-Sided Lower)
1	**1 Sample Z-Test**	Ho:$\mu=5$ Ha:$\mu>5$	Ho:$\mu=3$ Ha:$\mu\neq3$	-Random Sample -Sample mean is normally distributed	$Z_{stat}=\frac{\bar{X}-\mu}{\sigma/\sqrt{(n)}}$	Reject Ho if $\|Z_{stat}\|>\|Z_{crit}\|$ or $p<\alpha$	$\bar{X}\pm Z_{\alpha/2}\,\sigma/\sqrt{(n)}$	if">"in Ha UB="infinite" LB= $\bar{X}-Z_{\alpha}\,\sigma/\sqrt{(n)}$	if"<"in Ha UB= $\bar{X}+Z_{\alpha}\,\sigma/\sqrt{(n)}$ LB="0" or "-infinite"
2	**1 Sample T-Test**	Ho:$\mu=6$ Ha:$\mu>6$	Ho:$\mu=8$ Ha:$\mu\neq8$	-Random Sample -Sample mean is normally distributed	$T_{stat}=\frac{\bar{X}-\mu}{s/\sqrt{(n)}}$ df = n -1	Reject Ho if $\|T_{stat}\|>\|T_{crit}\|$ or $p<\alpha$	$\bar{X}\pm T_{\alpha/2}\,s/\sqrt{(n)}$	if">"in Ha UB="infinite" LB= $\bar{X}-T_{\alpha}\,s/\sqrt{(n)}$	if"<"in Ha UB= $\bar{X}+T_{\alpha}\,s/\sqrt{(n)}$ LB="0" or "-infinite"
3	**Paired T-Test**	Ho:$\mu_d=0$ Ha:$\mu_d<0$	Ho:$\mu_d=0$ Ha:$\mu_d\neq0$	-Dependent Samples -Sample mean of differences is normally distributed	$T_{stat}=\frac{\bar{d}-\mu_d}{s_d/\sqrt{(n)}}$ df = n -1	Reject Ho if $\|T_{stat}\|>\|T_{crit}\|$ or $p<\alpha$	$\bar{d}\pm T_{\alpha/2}\,s_d/\sqrt{(n)}$	if">"in Ha UB="infinite" LB= $\bar{d}-T_{\alpha}\,s_d/\sqrt{(n)}$	if"<"in Ha UB= $\bar{d}+T_{\alpha}\,s_d/\sqrt{(n)}$ LB="-infinite"
4	**2 Sample T-Test**	Ho:$\mu_1-\mu_2=0$ Ha:$\mu_1-\mu_2<0$	Ho:$\mu_1-\mu_2=0$ Ha:$\mu_1-\mu_2\neq0$	-Independent Samples -Population Variances are Equal -Sample means are both normally distributed	$T_{stat}=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{s_p\sqrt{(1/n_1+1/n_2)}}$ $s_p=\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$ df $=n_1+n_2-2$	Reject Ho if $\|T_{stat}\|>\|T_{crit}\|$ or $p<\alpha$	$(\bar{X}_1-\bar{X}_2)\pm T_{\alpha/2}{}^{*}\,s_p\sqrt{(1/n_1+1/n_2)}$	if">"in Ha UB="infinite" LB= $(\bar{X}_1-\bar{X}_2)-T_{\alpha}{}^{*}\,s_p\sqrt{(1/n_1+1/n_2)}$	if"<"in Ha UB= $(\bar{X}_1-\bar{X}_2)+T_{\alpha}{}^{*}\,s_p\sqrt{(1/n_1+1/n_2)}$ LB="-infinite"
5	**2 Sample T-Test**	Ho:$\mu_1-\mu_2=0$ Ha:$\mu_1-\mu_2<0$	Ho:$\mu_1-\mu_2=0$ Ha:$\mu_1-\mu_2\neq0$	-Independent Samples -Population Variances are NOT Equal -Sample means are both normally distributed	$T_{stat}=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{(s_1^2/n_1+s_2^2/n_2)}}$ df $=\frac{(s_1^2/n_1+s_2^2/n_2)^2}{\left(\frac{(s_1^2/n_1)^2}{n_1-1}+\frac{(s_2^2/n_2)^2}{n_2-1}\right)}$	Reject Ho if $\|T_{stat}\|>\|T_{crit}\|$ or $p<\alpha$	$(\bar{X}_1-\bar{X}_2)\pm T_{\alpha/2}{}^{*}\,\sqrt{(s_1^2/n_1+s_2^2/n_2)}$	if">"in Ha UB="infinite" LB= $(\bar{X}_1-\bar{X}_2)-T_{\alpha}{}^{*}\,\sqrt{(s_1^2/n_1+s_2^2/n_2)}$	if"<"in Ha UB= $(\bar{X}_1-\bar{X}_2)+T_{\alpha}{}^{*}\,\sqrt{(s_1^2/n_1+s_2^2/n_2)}$ LB="-infinite"
7	**Wilcoxin**	Ho:$M_d=0$ Ha:$M_d<0$	Ho:$M_d=0$ Ha:$M_d\neq0$	-Dependent Samples -Sample of Diff is NOT Normally Distributed	p-value = from Minitab output	Reject Ho if $p<\alpha$	Minitab	Minitab	Minitab
8	**Mann-Whitney**	Ho:$M_1-M_2=0$ Ha:$M_1-M_2<0$	Ho:$M_1-M_2=0$ Ha:$M_1-M_2\neq0$	-Independent Samples -One or both of the samples is NOT normally distributed	p-value = from Minitab output	Reject Ho if $p<\alpha$	Minitab	Minitab	Minitab
9	**1 Proportion**	Ho:$p=0.3$ Ha:$p<0.3$	Ho:$p=0.4$ Ha:$p\neq0.4$	$-np\geq10$ $-nq\geq10$ Sample Proportion normally distributed	$Z_{stat}=\frac{\hat{p}-p}{\sqrt{(pq/n)}}$ $\hat{p}=x/n\quad\hat{q}=1-\hat{p}$	Reject Ho if $\|Z_{stat}\|>\|Z_{crit}\|$ or $p<\alpha$	$\hat{p}\pm Z_{\alpha/2}\sqrt{(\hat{p}\hat{q}/n)}$	if">"in Ha UB="1" LB= $\hat{p}-Z_{\alpha}\sqrt{(\hat{p}\hat{q}/n)}$	if"<"in Ha UB= $\hat{p}+Z_{\alpha}\sqrt{(\hat{p}\hat{q}/n)}$ LB="0"
10	**2 Proportions** (Independent Pooled)	Ho:$p_1-p_2=0.0$ Ha:$p_1-p_2<0.0$	Ho:$p_1-p_2=0.0$ Ha:$p_1-p_2\neq0.0$	Sample Proportions normally distributed	$Z_{stat}=\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\bar{p}(1-\bar{p})(1/n_1+1/n_2)}}$ $\bar{p}=(n_1\hat{p}_1+n_2\hat{p}_2)/(n_1+n_2)$ $\hat{p}_1=x_1/n_1\quad\hat{p}_2=x_2/n_2$	Reject Ho if $\|Z_{stat}\|>\|Z_{crit}\|$ or $p<\alpha$	$(\hat{p}_1-\hat{p}_2)\pm Z_{\alpha/2}{}^{*}\,\sqrt{(\hat{p}_1\hat{q}_1/n_1+\hat{p}_2\hat{q}_2/n_2)}$	if">"in Ha UB="1" LB= $(\hat{p}_1-\hat{p}_2)-Z_{\alpha}{}^{*}\,\sqrt{(\hat{p}_1\hat{q}_1/n_1+\hat{p}_2\hat{q}_2/n_2)}$	if"<"in Ha UB= $(\hat{p}_1-\hat{p}_2)+Z_{\alpha}{}^{*}\,\sqrt{(\hat{p}_1\hat{q}_1/n_1+\hat{p}_2\hat{q}_2/n_2)}$ LB="-1"
11	**2 Proportions** (Independent)	Ho:$p_1-p_2=0.1$ Ha:$p_1-p_2<0.1$	Ho:$p_1-p_2=0.1$ Ha:$p_1-p_2\neq0.1$	Sample Proportions normally distributed	$Z_{stat}=\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{(\hat{p}_1\hat{q}_1/n_1+\hat{p}_2\hat{q}_2/n_2)}}$ $\hat{p}_1=x_1/n_1\quad\hat{p}_2=x_2/n_2$	Reject Ho if $\|Z_{stat}\|>\|Z_{crit}\|$ or $p<\alpha$	$(\hat{p}_1-\hat{p}_2)\pm Z_{\alpha/2}{}^{*}\,\sqrt{(\hat{p}_1\hat{q}_1/n_1+\hat{p}_2\hat{q}_2/n_2)}$	if">"in Ha UB="1" LB= $(\hat{p}_1-\hat{p}_2)-Z_{\alpha}{}^{*}\,\sqrt{(\hat{p}_1\hat{q}_1/n_1+\hat{p}_2\hat{q}_2/n_2)}$	if"<"in Ha UB= $(\hat{p}_1-\hat{p}_2)+Z_{\alpha}{}^{*}\,\sqrt{(\hat{p}_1\hat{q}_1/n_1+\hat{p}_2\hat{q}_2/n_2)}$ LB="-1"
12	**2 Proportions** (Dependent, $\hat{p}_1$ and $\hat{p}_2$ from same sample)	Ho:$p_1-p_2=0.0$ Ha:$p_1-p_2<0.0$	Ho:$p_1-p_2=0.0$ Ha:$p_1-p_2\neq0.0$	Sample Proportions normally distributed	$Z_{stat}=\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{(\hat{p}_1\hat{q}_1/n+\hat{p}_2\hat{q}_2/n+2\hat{p}_{12}\hat{p}/n)}}$ $\hat{p}_1=x_1/n\quad\hat{p}_2=x_2/n$	Reject Ho if $\|Z_{stat}\|>\|Z_{crit}\|$ or $p<\alpha$	$(\hat{p}_1-\hat{p}_2)\pm Z_{\alpha/2}{}^{*}\,\sqrt{(\hat{p}_1\hat{q}_1/n+\hat{p}_2\hat{q}_2/n+2\hat{p}_{12}\hat{p}/n)}$	if">"in Ha UB="1" LB="do only subtraction" *Remember use 1 sided Z	if"<"in Ha UB="Do only addition" LB="-1"

Variables
E=margin of error
μ = pop. mean
$\bar{X}$ = sample mean
σ = pop. SD
s = sample SD
σ^2 = pop. variance
s^2 = sample variance
μ_d = pop. mean of differences
$\bar{d}$ = sample mean of differences
s_d = sample SD of differences
s_p = pooled SD
n = sample size
M_d = pop. median of differences
M = pop. median
p = pop. proportion
$\hat{p}$ = sample proportion
$\bar{p}$ = pooled proportion

Testing Steps
1) State Assumptions
2) Write Hypothesis
3) Calculate STAT Value
4) Calculate CRIT Value
5) Make Decision
6) Write Conclusion

1-Tailed Decision Rules
*If 1 tailed and you have a "<" sign in Ha, reject Ho if $Z_{stat}<Z_{crit}$ or $T_{stat}<T_{crit}$
*If 1 tailed and you have a ">" sign in Ha, reject Ho if $Z_{stat}>Z_{crit}$ or $T_{stat}>T_{crit}$
*ALWAYS Reject Ho if $p<\alpha$
Writing your conclusion
If you accept Ho -> "There is not sufficient evidence that ....(describe what Ha means)
If you reject Ho -> "There is sufficient evidence that ....(describe what Ha means)

Confidence Intervals
*Never input a negative $Z_{\alpha}(Z_{\alpha/2})$ or $T_{\alpha}(T_{\alpha/2})$ in your interval formulas.
*Decision Rule: Reject Ho if the C.I. does not contain the value (#) in your hypothesis.

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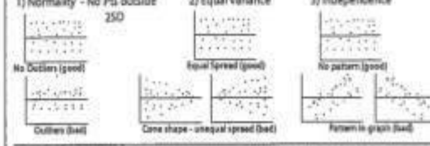
13 — 1-WAY ANOVA
Ho:$\mu_1=\mu_2=\mu_3=...\mu_k$
Ha:not all means equal
Assumptions
-Samples Normal
-Equal Variances
-Observations Ind
(see graphs below)
***Review 1 WAY ANOVA table in the book for detailed calculations.
$F_{stat}=MS_{tr}/MS_{err}$
Reject Ho if $|F_{stat}|>|F_{crit}|$ or $p<\alpha$.
Both F_{stat} and p come from minitab.
$df_{T}=k-1$
$df_{E}=N-k$
$df_{T}=N-1$
k=# rows

17 — 2-WAY ANOVA
**Review 2 WAY ANOVA table in the book for detailed calculations.
3 Possible Tests

Test #1-Interaction	Test #2-Factor A Means	Test #3-Factor B Means
Ho: Factors A and B do not interact Ha: Factors A and B interact	Ho:$\mu_{A_1}=\mu_{A_2}=...\mu_{A_k}$ Ha:not all means equal	Ho:$\mu_{B_1}=\mu_{B_2}=...\mu_{B_k}$ Ha:not all means equal
$F_{stat}=MS_{int}/MS_{err}$	$F_{stat}=MS_{facA}/MS_{err}$	$F_{stat}=MS_{facB}/MS_{err}$

For all 3 tests Reject Ho if $|F_{stat}|>|F_{crit}|$ or $p<\alpha$.
There will be 3 F-values and/or p-values in minitab.
$df_{facA}=r-1$ $df_{facB}=c-1$ $df_{int}=(r-1)*(c-1)$ $df_{err}=r*c*(n-1)$ $df_{tot}=N-1$

Assumptions - Residual vs FIT Graph



Important Formulas
$R^2=1-(SS_{err}/SS_{tot})$
$R^2_{adj}=1-\frac{MS_{err}}{(SS_{tot}/df_{tot})}$
s=SQRT(MSE)
SS_E = SUM[$(n_i-1)*s_i^2$]
$MS=\frac{SS}{df}$

Interaction Plot -Parallel lines suggest that there is not interaction present between the two factors
-Crossing/Non parallel lines suggest there is interaction between the two factors

Bonferroni used to identify where the actual differences between means are.
Step 1 - Determine number of rows (r), number of columns (c) and number of observations in each samples being compared (n)
Step 2 - Calculate "J" (the number of possible comparisons) J=[(rc)*(rc-1)]/2 s=SQRT(MSE)
Step 3 - Calculate alpha/2(J) and the df = df_e
Step 4 - Get T-value from table at a 1 tailed significance level of alpha/2(J) and df_e (pick closest value)
Step 5 - Interval = (x_bar₁ - x_bar₂) +/- $t_{\alpha/2J}$ * s *SQRT(1/n_1 + 1/n_2)
critical difference = margin of error = $t_{\alpha/2J}$ * s *SQRT(1/n_1 + 1/n_2)
Rule - If the interval contains "0", there is no difference between the means being compared
OR - If |actual diff| < |critical diff|, there is no difference between the means
-Used for 1-WAY ANOVA and 2-WAY ANOVA, do not use for BLOCKED ANOVA
-For 2-WAY ANOVA there are 3 types of possible comparisons, row comparisons, column comparisons and cell comparisons, all steps will be exactly the same except step 1. The values of "r", "c" "n" are determined based on the type of comparison.

19 — ANOVA-BLOCKED
Assumptions
-See 1-Way ANOVA
**Review BLOCKED ANOVA table in the book for detailed calculations.
2 Possible Tests

Test #1-Was Blocking Needed?	Test #2-Factor Means
Ho:$\mu_{b_1}=\mu_{b_2}=...\mu_{b_n}$ Ha:not all block means equal	Ho:$\mu_{t_1}=\mu_{t_2}=...\mu_{t_n}$ Ha:not all factor means equal
$F_{stat}=MS_{block}/MS_{err}$	$F_{stat}=MS_{fac}/MS_{err}$

***If in test #1 we DO NOT reject Ho we must run a 1-WAY ANOVA to complete test #2, otherwise, you can proceed with test #2 using the output.
For both tests Reject Ho if $|F_{stat}|>|F_{crit}|$ or $p<\alpha$.
There will be 2 F-values and/or p-values in minitab.
$df_{block}=b-1$ $df_{fac}=c-1$ $df_{err}=(b-1)*(c-1)$ $df_{tot}=N-1$ b = # of rows

18 — Kruskal-Wallace
Ho:$M_1=M_2=M_3...=M$
Ha:not all M equal
-Get p-value from minitab and compare to alpha.
Reject Ho $p<\alpha$

16 — Binomial
Ho:p=0.3 Ho:p=0.4
Ha:p<0.3 Ha:p≠0.4
$P(X=x)=nCx*p^xq^{n-x}$
n=# of trials (sample size)
p=probability of a success
q=probability of a failure
q=1-p
x=# of successes in "n" trials
Reject Ho if $p<\alpha$

Required Sample Size (always round up)
-proportion $n=\frac{Z_{\alpha/2}{}^2\hat{p}\hat{q}}{E^2}$
*$\hat{p}$,$\hat{q}$ and E are always entered as decimals
*If $\hat{p}$ and $\hat{q}$ are not given use values of 0.5
-means (σ known) $n=\frac{Z_{\alpha/2}{}^2\sigma^2}{E^2}$