

**PROBLEM 1 (50 MARKS)**

**NOTE THAT PROBLEM 1 CONSISTS OF 18 SEPARATE MULTIPLE-CHOICE QUESTIONS.**

1. **(2 MARKS)** LJP, Inc. has the following probability distribution of holding period returns on its stock.

State of Economy	Probability	HPR
Boom	.25	25%
Normal Growth	.45	15%
Recession	.30	5%

The expected return on LJP's stock is

- A) 12.4%.
  - B) 16.5%.
  - C) 14.5%.
  - D) 17.8%.
  - E) 11.6%.
2. **(2 MARKS)** According to the Capital Asset Pricing Model (CAPM), fairly priced securities
- A) have positive betas.
  - B) have zero alphas.
  - C) have negative betas.
  - D) have positive alphas.
  - E) none of the above.
3. **(2 MARKS)** Your personal opinion is that security X has an expected rate of return of 0.11. It has a beta of 1.5. The risk-free rate is 0.05 and the market expected rate of return is 0.09. According to the Capital Asset Pricing Model, this security is
- A) underpriced.
  - B) overpriced.
  - C) fairly priced.
  - D) cannot be determined from data provided.
  - E) none of the above.

### **PROBLEM 1 CONTINUED**

4. **(2 MARKS)** The security market line depicts
- A) A security's expected return as a function of its systematic risk
  - B) The market portfolio as the optimal portfolio of risky securities
  - C) The relationship between a security's return and the return on an index
  - D) The complete portfolio as a combination of the market portfolio and the risk-free asset
  - E) Investment opportunity set, that is the combination of available expected returns and corresponding standard deviations of security's return
5. **(3 MARKS)** Which statement is not true regarding the market portfolio?
- A) It includes all publicly traded financial assets.
  - B) It lies on the efficient frontier.
  - C) All securities in the market portfolio are held in proportion to their market values.
  - D) It is the tangency point between the capital market line and the indifference curve.
  - E) all of the above are true.
6. **(3 MARKS)** If a T-bill pays 5 percent, which of the following investments would **not** be chosen by a risk-averse investor?
- A) An asset that pays 10 percent with a probability of 0.60 or 2 percent with a probability of 0.40.
  - B) An asset that pays 10 percent with a probability of 0.40 or 2 percent with a probability of 0.60.
  - C) An asset that pays 10 percent with a probability of 0.20 or 3.75 percent with a probability of 0.80.
  - D) An asset that pays 10 percent with a probability of 0.30 or 3.75 percent with a probability of 0.70.
  - E) neither **a** nor **b** would be chosen.
7. **(3 MARKS)** Assume that the return of a risk-free asset is 3%, while the expected return and standard deviation of the risky asset are 12% and 20%, respectively. What percentages of your money must be invested in the risk-free asset and the risky asset, respectively, to form a portfolio with a standard deviation of 0.06?
- A) 30% and 70%
  - B) 50% and 50%
  - C) 60% and 40%
  - D) 70% and 30%
  - E) cannot be determined

### **PROBLEM 1 CONTINUED**

8. **(3 MARKS)** Which of the following assumptions are **not** used in CAPM?
- A) Each investor in the market is a price-taker
  - B) Investments are limited to publicly traded financial assets (stocks and bonds)
  - C) No taxes and transaction costs
  - D) Investors are rational mean-variance optimizers
  - E) **Investors may have different beliefs about future returns.**
9. **(3 MARKS)** Which of the following statements regarding risk-averse investors is **true**?
- A) They only care about the rate of return.
  - B) They accept investments that are fair games.
  - C) **They only accept risky investments that offer risk premiums over the risk-free rate.**
  - D) They are willing to accept lower returns and high risk.
  - E) **a and b.**
10. **(3 MARKS)** Elias is a risk-averse investor. David is a less risk-averse investor than Elias. Therefore,
- A) for the same risk, David requires a higher rate of return than Elias.
  - B) for the same return, Elias tolerates higher risk than David.
  - C) for the same risk, Elias requires a lower rate of return than David.
  - D) **for the same return, David tolerates higher risk than Elias.**
  - E) cannot be determined.
11. **(3 MARKS)** Assume that short selling is allowed. Which of the following statements is (are) **true** regarding the variance of a portfolio of two risky securities?
- A) The higher the coefficient of correlation between securities, the greater the reduction in the portfolio variance.
  - B) It is impossible to create risk-free portfolio if securities have perfect negative correlation
  - C) **An investor can make a risk-free portfolio from two risky securities even if they are perfectly positively correlated**
  - D) **a and c.**
  - E) None of the above

### **PROBLEM 1 CONTINUED**

12. **(3 MARKS)** Which statement is **not** true regarding the Capital Market Line (CML)?
- A) The CML is the line from the risk-free rate through the market portfolio.
  - B) The CML is the best attainable capital allocation line.
  - C) **The CML is also called the security market line.**
  - D) The CML always has a positive slope.
  - E) All of the above statements are true.
13. **(3 MARKS)** Assume that an investor can trade many risky securities and a risk-free asset. Assume also that an investor has found her efficiency frontier and tries to find her optimal risky portfolio comprised with only risky assets. What she will have to do?
- A) Find point of tangency of her indifference curve and the efficiency frontier
  - B) Use formula for  $y^*$
  - C) **Find CAL with the highest possible slope**
  - D) She cannot know because more information is required
  - E) Find an input list
14. **(3 MARKS)** When wealth is shifted from the risky portfolio to the risk-free asset, what happens to the relative proportions of the various risky assets within the risky portfolio?
- A) They all decrease.
  - B) Some increase and some decrease.
  - C) They all increase.
  - D) **They are not changed.**
  - E) The answer depends on the specific circumstances.
15. **(3 MARKS)** Efficient portfolios of  $N$  risky securities are portfolios that
- A) are formed with the securities that have the highest rates of return regardless of their standard deviations.
  - B) **have the highest rates of return for a given level of risk.**
  - C) are selected from those securities with the lowest standard deviations regardless of their returns.
  - D) have the highest risk and rates of return and the highest standard deviations.
  - E) have the lowest standard deviations and the lowest rates of return.

### PROBLEM 1 CONTINUED

16. **(3 MARKS)** Assume that a lot of investors become more risk averse in the result of the liquidity crises. What should happen with the equilibrium risk premium of the stock market and why?
- A) The premium will stay the same because it is independent from risk aversions of investors
  - B) It will increase because market will clear only at a smaller price of the stock market
  - C) It will increase because market will clear only at a higher price of the stock market
  - D) It will decrease because market will clear only at higher price of the stock market
  - E) None of the above
17. **(3 MARKS)** Consider a portfolio with two risky assets such that each asset has positive Sharpe ratio. Is it possible that an investor will short one of these two assets in this portfolio to take a leveraged position in the other? Why?
- A) No it is not possible because  $y^*$  is positive if the Sharpe ratio of an asset is positive
  - B) No it is not possible because an investor cannot short an asset in the market
  - C) No it is not possible because an investor will not short an asset which price is likely to increase in the future
  - D) Yes it is possible because the price of the shorted asset is likely to fall in the future
  - E) Yes it is possible if correlation between two assets is significant since shorting will improve the portfolio return-to-risk characteristics
18. **(3 MARKS)** Consider a risky portfolio, A, with an expected rate of return of 0.15 and a standard deviation of 0.20, that lies on a given indifference curve. Which one of the following portfolios might lie on the same indifference curve?
- A)  $E(r) = 0.15$ ; Standard deviation = 0.25
  - B)  $E(r) = 0.15$ ; Standard deviation = 0.10
  - C)  $E(r) = 0.10$ ; Standard deviation = 0.10
  - D)  $E(r) = 0.20$ ; Standard deviation = 0.15
  - E)  $E(r) = 0.10$ ; Standard deviation = 0.20

## **PROBLEM 2 (15 MARKS)**

Consider a market with a risk-free security and a risky asset. Assume that investor is not a price-taker so that her trading moves the expected return of a risky security P as following:

$$E(r_P) = .12 - .10y,$$

where  $y$  is a fraction of her complete portfolio (in decimals) invested in the risky security. (It follows that if an investor buys more of the risky security, its price increases and the expected return decreases.) Assume that risk-free rate,  $r_f$ , is 3%,  $\sigma_P$  is 20% and does not change when an investor trades, and the coefficient of risk aversion of an investor is 2.

a. **(8 MARKS)** Find the optimal fraction of the complete portfolio allocated to the risky asset P by an investor,  $y$ , the expected return and standard deviation of the complete portfolio. Also find the maximal utility of an investor. *Hint 1: you can follow the steps we did in the class in deriving  $y^*$ . Hint 2: If you cannot find  $y$ , then you can assume that it is equal to some number and then find expected return and standard deviation.*

$$y = .32$$
$$U_{max} = 0.044$$

b. **(4 MARKS)** Now assume that an investor is a price taker so that the expected return of a risky security P is fixed for her and

$$E(r_P) = .12$$

Assume that risk-free rate,  $r_f$ , is 3% and coefficient of the risk aversion of an investor is 2. Find the optimal fraction of the complete portfolio allocated to the risky asset by an investor, the expected return and standard deviation of the complete portfolio. Also find the maximal utility of an investor.

$$y^*=1.125$$

$$U_{max}=0.080625$$

c. **(3 MARKS)** Explain the economical intuition behind the difference between the results in questions **a** and **b**.

A “large” investor receives smaller returns than a “small” investor for the given level of risk. Therefore, the former buys less risky portfolio than the latter and obtains smaller utility.

### **PROBLEM 3 (15 MARKS)**

Investors can design a risky portfolio based on two stocks, K and L. Stock K has an expected rate of return of 18% and a standard deviation of return of 30%. Stock L has an expected rate of return of 10% and a standard deviation of return of 12%. The correlation coefficient between the two stocks is 0.5. The risk-free rate is 3%.

- a. **(4 MARKS)** What is the weight of stock K in the optimal risky portfolio? What are the expected return and the standard deviation of return on this portfolio?

$$W_K=0.2, W_L=0.8$$

$$E(r)=11.6\%$$

$$\sigma=13.63\%$$

- b. **(4 MARKS)** Investor Peter wants to have a complete portfolio (portfolio that includes T-bills and risky securities) with an expected return of 15%. What are the weights of each security in the complete portfolio? What are the expected return and the standard deviation of return on this portfolio?

$$Y_{T\text{-bill}}=-0.395, Y_K=0.279, Y_L=1.116$$

$$E(r)=14.997\%$$

$$\sigma=19.01\%$$

**PROBLEM 3 CONTINUED**

c. **(4 MARKS)** Investor Julien has a coefficient of risk aversion of 5. What are the weights of each security in her optimal complete portfolio? What are the expected return and the standard deviation of return on this portfolio?

$$Y_{T\text{-bill}}=0.074, \quad Y_k=0.185, \quad Y_L=0.741$$

$$E(r)=10.96\%$$

$$\sigma=12.62\%$$

d. **(3 MARKS)** Sketch the portfolio opportunity set for Julien. Show her risky portfolio and her complete portfolio.

See your class notes

**PROBLEM 4 (15 MARKS)**

Assume that there are only two stocks in an economy and no T-bills:

Security	<u>Expected Return</u>	<u>Standard deviation of return</u>
X	.05	.15
Y	.12	.20

The correlation coefficient between the returns of the two stocks is  $\rho = -1$ .

- a. **(3 MARKS)** Sketch the graph of the portfolio opportunity set for an investor

See you class notes

**PROBLEM 4 CONTINUED**

b. **(6 MARKS)** Find the weights of the two securities in the risk-free portfolio in this market. What is a return of this portfolio?

$$W_X=0.571, W_L=0.429$$

$$R=8.01\%$$

c. **(6 MARKS)** Consider an investor who has a coefficient of risk aversion equal to 2. What is the weight of the risk-free portfolio found in question **b** in his/her optimal portfolio in this economy? What is an expected return and standard deviation of return of this portfolio?

$$Y_F=0.501,$$

$$E(r)=10.00\%$$

$$\sigma=9.98\%$$