LAB 3: Transient Response of a Series RC Circuit.

PURPOSE

To introduce and verify the theory of a series RC circuit response to square wave excitation. Lectures on this topic will follow at the end of the term.

EQUIPMENT

(i) Oscilloscope – Tektronix 3012.
(ii) Oscillator – Wavetek 182A.
(iii) Multimeter – Wavetek DM15XL.
(iv) Prototyping board.
(v) 10kΩ resistor.
(vi) 0.01μF capacitor.

PRELAB (_/2)

For this prelab, use the component values as given in Part 1, step 1.2.

1. (_/0.5) Figure 1 shows the basic series RC circuit. The input signal to the circuit is a square wave, as shown in Figure 2. For the theory of the series RC circuit response to square wave excitation, refer to Example 6 on transients at the back of the lab manual.

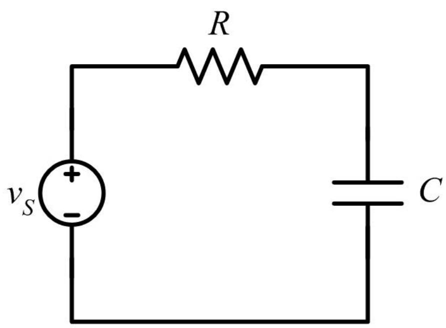


Figure 1. Basic series RC circuit.

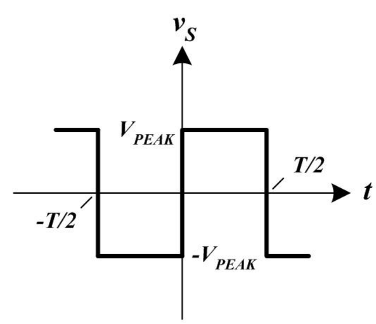


Figure 2. Squarewave input signal.

The voltage across the capacitor in the circuit shown in Figure 1 is given by the following equation:

$$v_C(t) = V_F + (V_I - V_F)e^{-\frac{t}{RC}} \tag{1}$$

where V_I is the initial value and V_F is the final value of the voltage on the capacitor as it is being charged. In this case, $V_F = +V_{PEAK}$ and, when the period of the input waveform is long enough to allow the capacitor to fully charge, $V_I = -V_{PEAK}$. Thus equation (1) becomes:

$$v_C(t) = V_{PEAK}(1 - 2e^{-\frac{t}{RC}}) \tag{2}$$

By KVL, the voltage across the resistor becomes:

$$v_R(t) = 2V_{PEAK}\, e^{-\frac{t}{RC}} \tag{3}$$

Using equations (2) and (3), calculate and tabulate the values for v_C and v_R for the positive half-period of a square wave with v_S = 1 V peak and a period T = 2000 μsec: divide the half-period of the waveform into 8 equal intervals, and for each time instant calculate the values of v_C and v_R. Complete the table below.

Interval (positive half-period)	**t (μs)**	**v_C (volts)**	**v_R (volts)**
1			
2			
3			
4			
5			
6			
7			
8			

2. (_/0.5) For the negative half-period, repeat question 1 using $V_I = +V_{PEAK}$ and $V_F = -V_{PEAK}$, and assuming the time origin is at $T/2$ by using the same values of t from question 1.

Interval (negative half-period)	**t (μs)**	**v_C (volts)**	**v_R (volts)**
1			
2			
3			
4			
5			
6			
7			
8			

3. (_/0.5) Plot accurately to scale on a sheet of graph paper one full period of the waveform v_C by combining the results obtained in questions 1 and 2. Make sure your plot matches the general shape of v_C as shown in Example 6 on transients at the back of the lab manual.

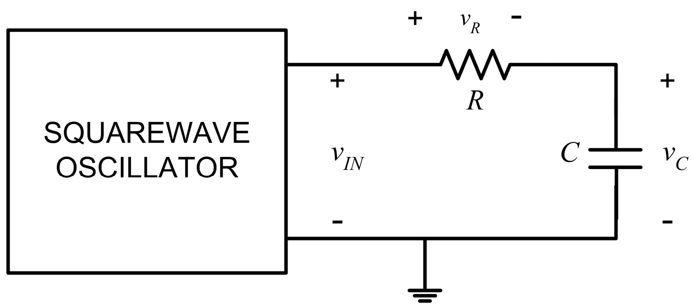
4. (_/0.5) Plot accurately to scale on a sheet of graph paper one full period of the waveform v_R by combining the results obtained in questions 1 and 2.

PROCEDURE (_/8)

Part 1. Series RC Circuit Response

1.1 Connect the oscilloscope across the output of the oscillator, and adjust the oscillator voltage to give a square wave signal with an amplitude of 2 V peak-to-peak and a period of T = 2000 μsec (f = 500 Hz). **Readjust the amplitude to 2 V peak-to-peak whenever you change the frequency of the input signal.**

1.2 Connect the circuit shown in Figure 3.



C = 0.01 μF R = 10 kΩ

Figure 3. Circuit configuration for step 1.2.

1.3 Connect channel 1 of the oscilloscope across the oscillator output, and channel 2 across the capacitor. Since the oscilloscope only has two channels, we cannot display v_{IN}, v_C, and v_R all at the same time. However, note that by KVL, $v_R = v_{IN} - v_C$, therefore we can use the 'MATH' menu of the oscilloscope to display v_R:

- Press the pink 'MATH' button on the oscilloscope panel.
- Set the operator to '–'.
- Set 1st source to channel 1 and 2nd source to channel 2

You should now be able to see three waveforms on the screen (v_{IN}, v_C, and v_R). Make sure the vertical scales of the three waveforms are the same (scales are displayed at the bottom left of the screen).

1.4 In this step, our goal is to obtain the v_C waveform and compare it to the plot produced in the prelab. In order to ensure that the rising edge of the v_C waveform starts at t = 0 like in the prelab, we need to set the trigger settings to an external trigger. To do this, perform the following steps:

- Make sure there is a cable connected between the 'SYNC OUT' output of the oscillator and the 'EXT TRIG' input of the oscilloscope.
- Press the 'menu' button at the bottom of the 'TRIGGER' area on the oscilloscope panel.
- Select 'EXT' from the list on the screen.
- Consider using 'Invert ON'.

Verify that the v_C waveform on the oscilloscope is similar to the waveform plotted in the prelab.

1.5 (_/1) Using the 'cursors' menu on the oscilloscope, record the values of v_C in the table at the of this handout for the same time instants used in the prelab. Plot accurately the waveform for v_C on the same plot that was produced in the prelab. Only one cycle of the waveforms is needed, and eight readings are required for each of the positive and the negative halves of the waveform.

1.6 (_/1) Repeat step 1.5 for the v_R waveform.

1.7 (_/1) Compare the experimental waveforms obtained in steps 1.5-1.6 to the waveforms plotted in the prelab.

1.8 (_/2) Change the oscillator to give an input signal with a period of T = 200 μsec (f = 5 KHz) and amplitude of 2 V peak-to-peak. Repeat the external trigger settings from step 1.4. Record and plot accurately the waveforms for v_C and v_R. Only one cycle of the waveforms is needed, and eight readings are required for each of the positive and the negative halves of the waveform. Observe that the v_C and v_R waveforms obtained here are actually the first 10% of the period of the waveforms obtained in steps 1.5-1.6.

1.9 (_/2) Repeat step 1.8 for T = 20 μsec (f = 50 KHz). Observe that the v_C and v_R waveforms obtained here are actually the first 10% of the period of the waveforms obtained in step 1.8, and the first 1% of the waveforms obtained in steps 1.5-1.6.

1.10 (_/1) Explain why the shapes of v_C and v_R waveforms change as the period of the input signal is varied.

Use the following tables to record your readings as required from steps 1.5 to 1.9

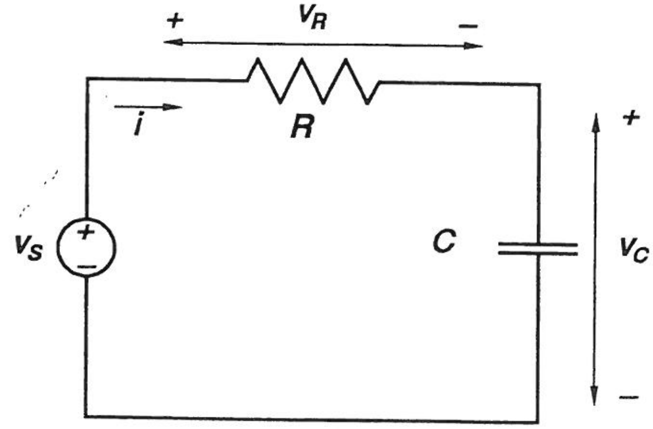
Waveform	Suggested Points (μs)	Measured t (μs)	T = 2000 μsec	
			v_C (volts)	v_R (volts)
Positive half-period	0			
	125			
	250			
	375			
	500			
	625			
	750			
	875			
Negative half- period	1000			
	1125			
	1250			
	1375			
	1500			
	1625			
	1750			
	1875			
	2000			

Waveform	Suggested Points (μs)	Measured t (μs)	T = 200 μsec	
			v_C (volts)	v_R (volts)
Positive half-period	0			
	12.5			
	25.0			
	37.5			
	50.0			
	62.5			
	75.0			
	87.5			
Negative half- period	100.0			
	112.5			
	125.0			
	137.5			
	150.0			
	162.5			
	175.0			
	187.5			
	200.0			

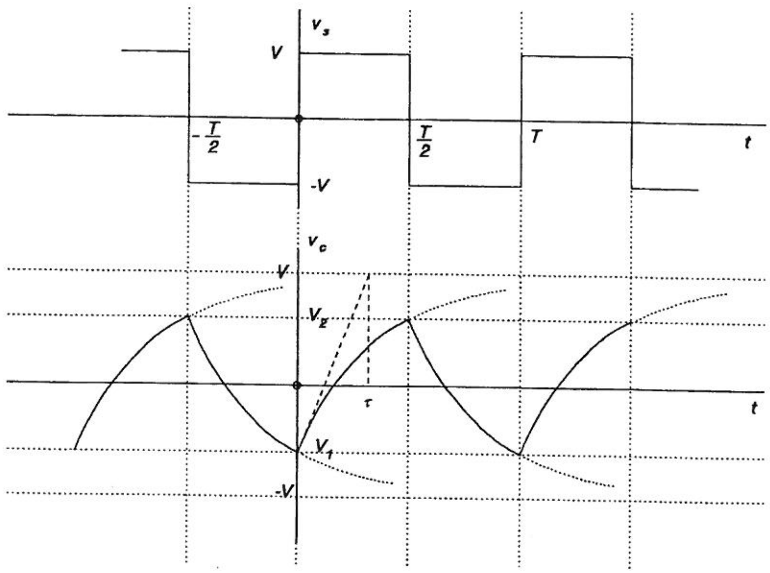
Waveform	Suggested Points (μs)	Measured t (μs)	T = 20 μsec	
			v_C (volts)	v_R (volts)
Positive half-period	0			
	1.25			
	2.50			
	3.75			
	5.00			
	6.25			
	7.50			
	8.75			
Negative half- period	10.00			
	11.25			
	12.50			
	13.75			
	15.00			
	16.25			
	17.50			
	18.75			
	20.00			

Example 6

v_s is a "square wave" (periodic) as shown below. Determine and sketch the waveform of the voltage v_c across the capacitor.



In general, since the capacitor voltage v_c cannot change instantaneously at the times where v_s suddenly changes (i.e. at $t = 0$, $t = T/2$ etc.), the waveform of v_c must be as illustrated below:



v_c increases exponentially from V_1 to V_2 while v_s is constant at the value V, and decreases exponentially from V_2 to V_1 while v_s is constant at the value $-V$. In each successive half-period, the capacitor charges towards either V or $-V$ but does not reach either value before v_s changes. If we assume that the circuit has been operating in the manner described for a long period so that its behaviour is cyclical, we can assign V_2 and V_1 as the maximum and minimum voltages respectively, on the capacitor. The voltages V_1 and V_2 are to be determined so that the waveform of v_c can be determined. It is clear that the values V_1 and V_2 must depend on the values of the time-constant, τ the period of the squarewave, T and the amplitude, v_s.

Since the squarewave is symmetrical i.e. $|V| = |-V|$, it follows that:

$$V_1 = -V_2 \qquad (1)$$

and the capacitor voltage changes by $2V_1$ (or $2V_2$) in each half period of v_s, reaching the same fraction of V or $-V$. In general, the waveform can be described as:

$$f(t) = (\text{Initial value}) + (\text{Final} - \text{Initial value})(1 - e^{-t/\tau}) \qquad (2)$$

The equation for the waveform of v_c for the period:

(a) $0 \leq t' \leq T/2$ is:

$$v_c = V_1 + [V - V_1][1 - e^{-t'/\tau}] \qquad (3)$$

Substituting $-V_2$ for V_1 from (1),

$$v_c = -V_2 + [V + V_2][1 - e^{-t'/\tau}] \qquad (4)$$

(b) $T/2 \leq t' \leq T$ (or $0 \leq t'' \leq T/2$) the equation of the waveform is:

$$v_c = V_2 + [(-V) - V_2][1 - e^{-t''/\tau}] \qquad (5)$$

This may be written as:

$$v_c = V_2 - [V + V_2][1 - e^{-t''/\tau}] \qquad (6)$$

Note

(i) The variable t' in (4) is equal to zero at the origin while the variable t'' in (6) is equal to zero at the instant when v_s switches from V to $-V$ i.e. at $t' = T/2$.

(ii) The voltage across the capacitor when when $t' = T/2$ hasto be the same as when $t'' = 0$.

From (4), when $t' = T/2$, $v_c = V_2$. We can therefore write:

$$V_2 = -V_2 + [V + V_2][1 - e^{-T/2\tau}] \qquad (7)$$

or

$$V_2 = \frac{V(1 - e^{-\frac{T}{2\tau}})}{(1 + e^{-\frac{T}{2\tau}})} \qquad (8)$$

The value of V_2 depends on the relative values of $T/2$ and τ, that is on the degree to which the capacitor can charge in the half-period $T/2$. If $T/2 >> \tau$

$$V_2 = V \tag{9}$$

and the charging of C to V or $-V$ is virtually completed in each half-period. If on the other hand $T/2 << \tau$, $V_2 << V$ and very little charging of C occurs in each half-period.