CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	203	All
Examination	**Date**	**Duration**
Midterm Test	23 October, 2016	1 h 30 min
Special Instructions:	Only approved calculators are allowed **Show your work for full marks**	

1. (12 marks): **(a)** Solve for x (find the *exact* values, do not approximate):

(i) $27 = 3^{5x}\, 9^{x^2}$ **(ii)** $\ln(\ln x) = 0$

(b) Let $f(x) = \sqrt{9-3x}$ and $g(x) = 3 - x^2$. Find the composite functions $f \circ g$ and $g \circ f$ and determine their domains.

(c) Let $f(x) = 2^{-x} - 1$. Find the inverse function f^{-1} and the domain and range of f^{-1}.

2. (8 marks) Find the limit or explain why the limit does not exist:

(a) $\lim\limits_{t\to 0} \left(\dfrac{1}{t\sqrt{1+t}} - \dfrac{1}{t} \right)$

(b) $\lim\limits_{x\to -5} \dfrac{x^2-25}{|\,x+5\,|}$

3. (6 marks) Find **(a)** all horizontal and **(b)** all vertical asymptotes of the graph

$$y = \frac{\sqrt{4x^6 + 2x^2 + 1}}{x^3 + x^2 - 6x}$$

4. (4 marks) Consider the following piecewise-defined function:

$$f(x) = \begin{cases} ax + 2b & \text{if} \quad x \le 0 \\ x^2 + 3a - b & \text{if} \quad 0 < x \le 2 \\ 3x - 5 & \text{if} \quad x > 2 \end{cases}$$

For what values of a and b is the function continuous at every x? Explain.

(continued on the other side)

5. (12 marks) Find the derivatives of the following functions. (you don't need to simplify the final answer, but you must show how you calculate it):

(**a**) $f(x) = x\sqrt{x}\,(x + x^{-3/2})$

(**b**) $f(x) = \dfrac{x3^x}{(x + 3^{-x})}$

(**c**) $f(x) = \cos\left(\sqrt[3]{x}\right) + \sqrt[3]{\tan x}$

(**d**) $f(x) = \sin[\, x\, \cos(x) + x^2 + \cos^2(x)\,]$

6. (8 marks) Given the function $f(x) = \sqrt{3x + 7}$,

(a) Calculate $f'(x)$ using its definition as a limit of difference quotient.

(b) Check that your calculation is correct using standard differentiation rules.

(c) Write equation of the tangent line to the curve $y = f(x)$ at the point $(3, 4)$.

Bonus Question (3 marks). Find a formula for the n-th derivative $f^{(n)}(x)$ if $f(x) = \dfrac{1}{1 - x}$ and n is any positive integer.