Assignment 3

Your submission **must be created using Microsoft Word, Google Docs, or LaTeX**.

Your submission **must be saved as a "pdf"** document and **have the name "a3.lastname.firstname.pdf"**

Do not compress your submission into a "zip" file.

Late assignments will not be accepted and will receive a mark of 0.

Submissions **written by hand**, **compressed into an archive**, or submitted in the **wrong format** (i.e., are not "pdf" documents) **will receive a mark of 0.**

The due date for this assignment is June 1st, 2017, by 11:30pm.

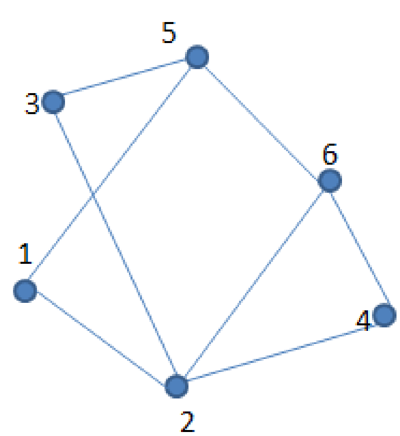
1. Determine which of the following graphs is simple. If the graph is simple, draw an example and if not then explain (in English) what the problem is.

   a. G has five vertices with the following degrees deg(v1) = 1; deg(v2) = 5; deg(v3) = 2; deg(v4) = 4; deg(v5) = 2.

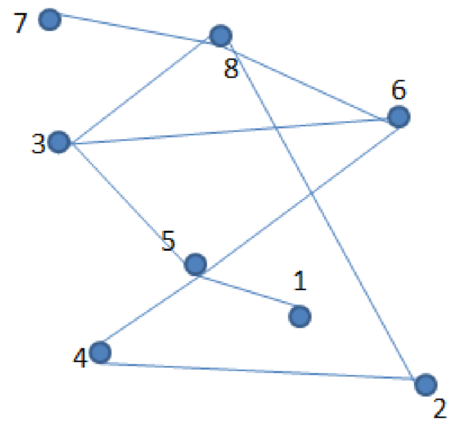
   Solution: The graph is not a simple graph because deg(v2) = 5. The graph has only five vertices and therefore, the maximum degree of a vertex is 4.

   b. G has six vertices with the following degrees deg(v1) = 2; deg(v2) = 4; deg(v3) = 2; deg(v4) = 2; deg(v5) = 3; deg(v6) = 3.

   Solution: Yes the graph is simple. Example of possible solution:



For questions 2-7 all refer to the undirected graph specified below:



2. If the graph above is denoted G where G = (V, E), define V and E.

   V = {1, 2, 3, 4, 5, 6, 7, 8}
   E = { {1, 5}, {2, 4}, {2, 8}, {3, 5}, {3, 6}, {3, 8}, {4, 5}, {5, 6}, {6, 8}, {7, 8} }

3. Provide an adjacency matrix representation of this graph.

	1	2	3	4	5	6	7	8
1	0	0	0	0	1	0	0	0
2	0	0	0	1	0	0	0	1
3	0	0	0	0	1	1	0	1
4	0	1	0	0	1	0	0	0
5	1	0	1	1	0	1	0	0
6	0	0	1	0	1	0	0	1
7	0	0	0	0	0	0	0	1
8	0	1	1	0	0	1	1	0

4. Provide an adjacency list representation of this graph.

   1: **5**
   2: **4, 8**
   3: **5, 6, 8**
   4: **2, 5**
   5: **1, 3, 4, 6**
   6: **3, 5, 8**
   7: **8**
   8: **2, 3, 6, 7**

5. Transform this undirected graph into a directed graph G' by changing each undirected edge into a directed edge that moves away from vertex 1. Provide the values of V' and E' such that G' = (V', E') and please note that there is more than one correct solution to this question.

   V' = {1, 2, 3, 4, 5, 6, 7, 8}
   E' = { (1, 5), (5, 3), (5, 4), (5, 6), (3, 6), (3, 8), (4, 2), (6, 8), (2, 8), (8, 7) }

6. Provide a subgraph G'' of the graph above that is an element of the set $\mathbf{K_n}$. Specify this graph by listing the elements of both V'' and E'' such that G'' = (V'', E'') and then provide an adjacency matrix representation of this graph as well.
   Any of the following would constitute a correct answer!

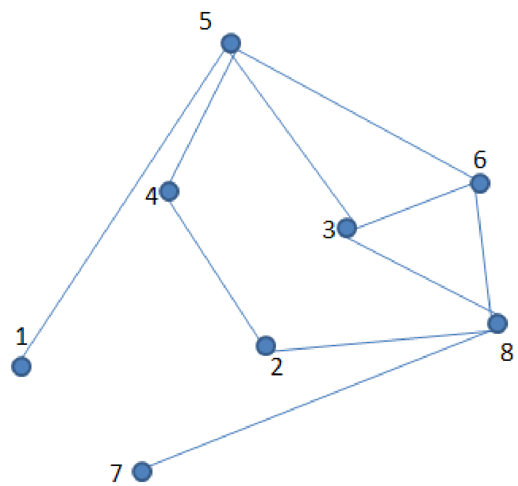
K_3 @	V'' = {3, 5, 6}	E'' = {{3, 5}, {5, 6}, {3, 6}}
K_3 @	V'' = {3, 6, 8}	E'' = {{3, 6}, {6, 8}, {3, 8}}
K_2 @	V'' = {1, 5}	E'' = {{3, 6}, {6, 8}, {3, 8}}
K_2 @	V'' = {1, 5}	E'' = { {1, 5} }
K_2 @	V'' = {2, 4}	E'' = { {2, 4} }
K_2 @	V'' = {2, 8}	E'' = { {2, 8} }
K_2 @	V'' = {3, 5}	E'' = { {3, 5} }
K_2 @	V'' = {3, 6}	E'' = { {3, 6} }
K_2 @	V'' = {3, 8}	E'' = { {3, 8} }
K_2 @	V'' = {4, 5}	E'' = { {4, 5} }
K_2 @	V'' = {5, 6}	E'' = { {5, 6} }
K_2 @	V'' = {6, 8}	E'' = { {6, 8} }
K_2 @	V'' = {7, 8}	E'' = { {7, 8} }
K_1 @	V'' = {1}	E'' = $\varnothing$
K_1 @	V'' = {2}	E'' = $\varnothing$
K_1 @	V'' = {3}	E'' = $\varnothing$
K_1 @	V'' = {4}	E'' = $\varnothing$
K_1 @	V'' = {5}	E'' = $\varnothing$
K_1 @	V'' = {6}	E'' = $\varnothing$
K_1 @	V'' = {7}	E'' = $\varnothing$
K_1 @	V'' = {8}	E'' = $\varnothing$

7. From the original graph, consider the smallest set of edges E''' that needs to be added such that the graph (V, E ∪ E''') contains W_6. Provide an exhaustive list of the elements of E''' and specify the cardinality of E'''.

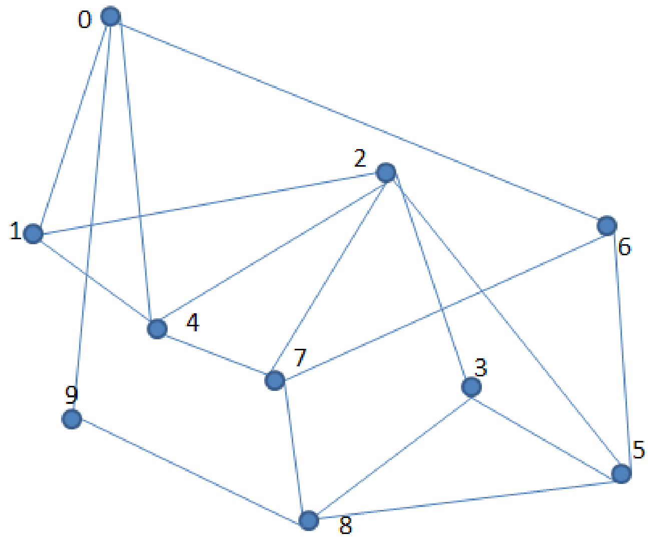
Solution:
E''' = {{3, 4}, {2, 3}}
|E'''| = 2

Another possible solution:
E''' = {{6, 4}, {2, 6}}
|E'''| = 2

Isomorphic graph that shows which two edges are missing: (NOT necessary to get full marks, here to illustrate the solution)



Questions 8-9 all refer to the undirected graph specified below:



8. Given the graph G = (V, E) in the figure above, compute the breadth-first search starting from the vertex that is labelled with the third-to-last digit in your student number. To clarify, if your student number is 100123456, start your breadth-first search tree from vertex 4.
   Provide the result of your search on the graph above (superimposed). Then, treat your search result as a new, **un**directed subgraph. Provide the adjacency **list** for this new subgraph. Whenever you have a "choice" of which adjacent vertex to consider, you must consider them in numerical order from least to greatest.

   (Graph not included here – will vary from student to student)
   Example of a solution: Assuming we start at 4.
   0: 4, 6, 9
   1: 4
   2: 3, 4, 5
   3: 2
   4: 0, 1, 2, 7
   5: 2
   6: 0
   7: 4, 8
   8: 7
   9: 0

9. Given the graph G = (V, E) in the figure above, compute the depth-first search tree starting from the vertex that is labelled with the fourth-to-last digit in your student number. To clarify, if your student number is 100123456, start your depth-first search tree from vertex 3.

Provide the result of your search on the graph above (superimposed). Then, treat your search result as a new, **un**directed subgraph. Provide the adjacency **matrix** for this new subgraph. Whenever you have a "choice" of which adjacent vertex to consider, you must consider them in numerical order from least to greatest.

(Graph not included here – will vary from student to student)
Example of a solution: Assuming we start at 3.

$$\begin{array}{c|cccccccccc|}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
7 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
8 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\hline
\end{array}$$