MATH 1009 TEST 1: Solutions

PART A: MULTIPLE CHOICE QUESTIONS.

A1. The domain of the function $f(x) = (3-x)^{3/5}$ is

(a) $(-\infty, 3)$, or, equivalently, $\{x < 3\}$.

(b) $(-\infty, 3]$, or, equivalently, $\{x \leq 3\}$.

(c) $(3, \infty)$, or, equivalently, $\{x > 3\}$.

(d) $[3, \infty)$, or, equivalently, $\{x \geq 3\}$.

(e) $\mathbb{R}$, or, equivalently, $(-\infty, \infty)$.

Answer: (e)

A2. If $f(x) = \dfrac{1}{x^2}$ and $g(x) = \dfrac{2}{x^3+1}$, then $f(g(x))$ is

(a) $\dfrac{2}{x^6+1}$. **(b)** $\dfrac{2x^2}{x^2+1}$. **(c)** $\dfrac{(x^3+1)^2}{2}$. **(d)** $\dfrac{(x^3+1)^2}{4}$.

(e) None of the above.

Answer: (d)

A3. The expression $(6^{-1/3} \cdot 6^3)^{3/8}$ evaluates to

(a) 6. **(b)** -6. **(c)** $\dfrac{1}{6}$. **(d)** $-\dfrac{1}{6}$. **(e)** None of the above.

Answer: (a)

A4. The domain of the function $f(x) = \log_{1/3}(x-4)$ is

(a) $[4, \infty)$, or, equivalently, $\{x \geq 4\}$.

(b) $(4, \infty)$, or, equivalently, $\{x > 4\}$.

(c) $[1/3, \infty)$, or, equivalently, $\{x \geq 1/3\}$.

(d) $(1/3, \infty)$, or, equivalently, $\{x > 1/3\}$.

(e) $\mathbb{R}$, or, equivalently, $(-\infty, \infty)$.

Answer: (b)

A5. The expression $\dfrac{e^{1.2} \cdot (e^{0.1})^{-2}}{e^{-5}}$ simplifies to

(a) $e^{4.3}$. (b) $e^{-5.7}$. (c) e^{4}. (d) e^{5}. (e) e^{6}.

Answer: (e)

A6. The expression $(27x^2y^9)^{1/3}$ is equivalent to

(a) $27x^{2/3}y^3$. (b) $27x^{7/3}y^{28/3}$. (c) $3x^{2/3}y^3$. (d) $3x^{7/3}y^{28/3}$.

(e) None of the above.

Answer: (c)

A7. The expression $\log_7 \dfrac{1}{49}$ evaluates to

(a) -2. (b) 2. (c) -1. (d) 1. (e) None of the above.

Answer: (a)

A8. Write the expression $(\ln 2 + \frac{1}{3}\ln x - 2\ln y)$ as the logarithm of a single quantity.

(a) $\ln\left(\dfrac{x}{3y}\right)$. (b) $\log_2\left(\dfrac{2x^{1/3}}{y^2}\right)$. (c) $\ln\left(2 + \dfrac{x}{3} - 2y\right)$. (d) $\ln\left(\dfrac{2x^{1/3}}{y^2}\right)$.

(e) None of the above.

Answer: (d)

PART B: LONG STYLE QUESTIONS.

[6 marks] B1. The weekly demand and supply equations for a company are given by $p = -2x^2 + 80$ and $p = 15x + 30$, respectively, where p is the price measured in dollars and x is measured in units **of a thousand**.

[2] **(a)** For the demand equation $p = -2x^2 + 80$, determine the quantity demanded, when the price is set at 8 dollars per thousand.

$$8 = -2x^2 + 80, \quad 2x^2 = 72, \quad x = \pm 6,$$

but we reject the negative root, so $x = 6$ thousand.

[4] **(b)** Find the equilibrium quantity and price.

At the equilibrium point, the supply is equal to the demand, and therefore

$$-2x^2 + 80 = 15x + 30.$$

Solving this equation for x yields $2x^2 + 15x - 50, \quad x_{1,2} = \dfrac{-15 \pm 25}{4}, \quad x_1 = 2.5, \ x_2 = -10.$ We reject the negative root $x = -10$, since positive values of x demanded are meaningful. Thus, the equilibrium quantity is 2.5 thousand units, and the corresponding price is

$$p = -2 \cdot (2.5)^2 + 80 = 67.5$$

dollars per thousand.

[11 marks] B2. Solve each of the following equations for x.

[4] **(a)** $3^{2x+1} = 5$ [3] **(b)** $\ln(x^2 - 4) = 1$ [4] **(c)** $\ln x + 3\ln 2 - \ln 4 = 5$

Solution:

NOTE: There is more than one way of solving each of the equations.

(a) Take the natural logarithm of both sides of the equation and use the laws of logarithms:

$$\ln(3^{2x+1}) = \ln 5 \;\Rightarrow\; (2x+1)\ln 3 = \ln 5 \;\Rightarrow\; 2x = \frac{\ln 5}{\ln 3} - 1, \; x = \frac{1}{2}\left(\frac{\ln 5}{\ln 3} - 1\right) = \frac{1}{2}(\log_3 5 - 1).$$

(b) Exponentiate both sides of the equation, simplify and solve for x:

$$e^{\ln x^2 - 4} = e^1 \quad \Rightarrow \quad x^2 - 4 = e \quad \Rightarrow \quad x^2 = 4 + e \quad \Rightarrow \quad x = \pm\sqrt{4 + e}.$$

(c) Simplify the LHS of the equation and then exponentiate both sides of the relation:

$$\ln\left(\frac{x \cdot 2^3}{4}\right) = 5 \quad \Rightarrow \quad \ln(2x) = 5 \quad \Rightarrow \quad e^{\ln(2x)} = e^5 \quad \Rightarrow \quad 2x = e^5 \quad \Rightarrow \quad x = \frac{e^5}{2}.$$

[7 marks] B3. The amount of \$10,000 is deposited in a bank that pays interest at the rate of 4% per year compounded **semiannually**. Using the compound interest formula

$$A(t) = P\left(1 + \frac{r}{m}\right)^{mt},$$

answer the following questions. (Round each answer to one decimal.)

[2] **(a)** What is the accumulated amount on deposit in 3 years?

$$A(3) = 10,000\left(1 + \frac{0.04}{2}\right)^{2\cdot 3} = 10,000(1.02)^6 = 11,261.6$$

[1] **(b)** What is the interest earned in 3 years?

$$\text{Interest } I(3) = A(3) - P = 11,261.6 - 10,000 = 1,261.6$$

[4] **(c)** How many years will it take to double the investment?

$$20,000 = 10,000\left(1 + \frac{0.04}{2}\right)^{2\cdot t} \;\rightarrow\; 2 = (1.02)^{2t} \;\rightarrow\; \ln 2 = 2t\ln(1.02) \;\rightarrow\; t = \frac{\ln 2}{2\ln 1.02} = 17.5$$