Université d'Ottawa
Faculté de génie

University of Ottawa
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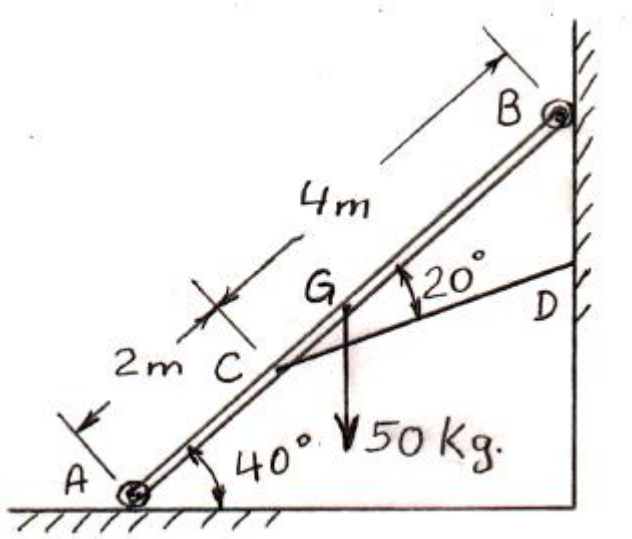
GNG 1105 B- Engineering Mechanics

Mid-Term Exam — 05 November 2015
Professor A. Skaff — Time: 80 min.

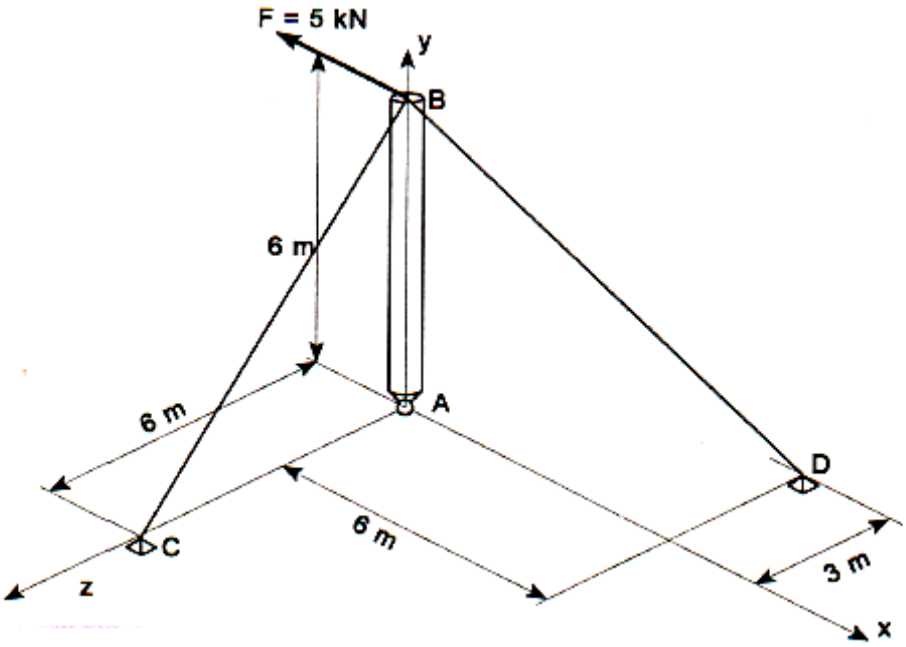
Closed Book. Non programmable calculators are allowed. Free-body diagrams must be drawn wherever appropriate.

1. (15 marks) Bar AB, 6m long, lies in a vertical plane and is supported by roller at A and B and by a cable at C as shown in the diagram. The mass of the bar is 50 kg and it acts at G (the mid-point of AB).
   a) Draw the FBD of bar AB.
   b) Determine the reactions at points A and B and the tension in cable CD.
   c) Reduce all of the forces acting on this bar into a force-couple system at point A.

   N.B. "g=9.81 m/S^2"



2. (15 marks) A mast AB is supported by a frictionless ball and socket joint at A and by two cables BC and BD. Determine the tensions in the cable BC and BD and the reactions at A.



Good luck,

GNG 1105 B — Nov. 5, 2015

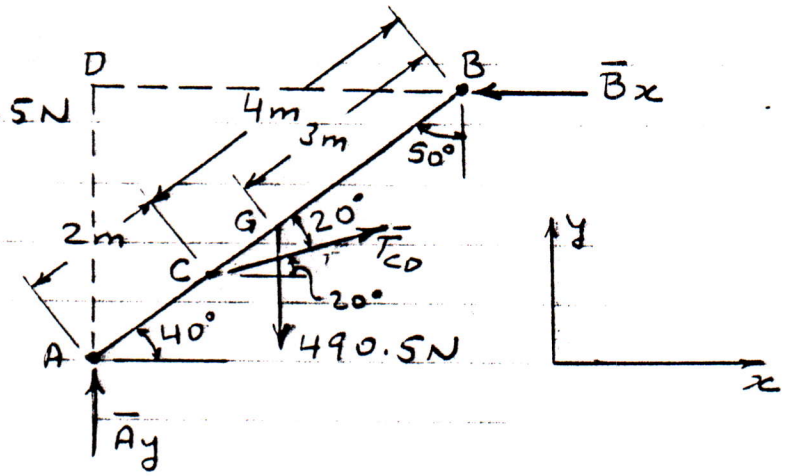
ENGINEERING MECHANICS

Mid-term Exam

SOLUTIONS

1.

a) FBD For Bar AB



$W = 50kg \times 9.81 = 492.5N$

b)

$+\circlearrowleft \Sigma M_D = 0;$

$T_{CD}\cos 20° \times 4m.\sin 40° + T_{CD}\sin 20° \times 2m.\cos 40° - 490.5 \times 3m\cos 40° = 0$

$2.416\,T_{CD} + 0.524\,T_{CD} - 1127.234 = 0$

$2.94\,T_{CD} = 1127.234.$ $\therefore T_{CD} = \underline{383.4N}$ ANS.

$\pm \rightarrow \Sigma F_x = 0;$

$383.4\cos 20° - Bx = 0;$ $\therefore Bx = \underline{360.3N}$ ANS.

$+\uparrow \Sigma F_y = 0;$

$Ay + 383.4\sin 20° - 490.5 = 0;$ $\therefore Ay = \underline{359.4N}$ ANS.

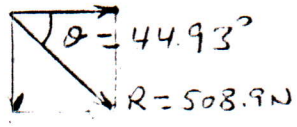
c) Reactions at A & B will be excluded.

$\Sigma F_x = Rx = T_{CD}\cdot\cos 20° = 383.4\cos 20 = 360.3N$

$\Sigma F_y = Ry = T_{CD}\cdot\sin 20° - 490.5N = -359.4N$

$\therefore R = \sqrt{(360.3)^2 + (-359.4)^2} = 508.9N$

$\angle\theta = \tan^{-1}\frac{-359.4}{360.3} = 44.93°$

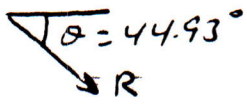


1. c) Cont'd.

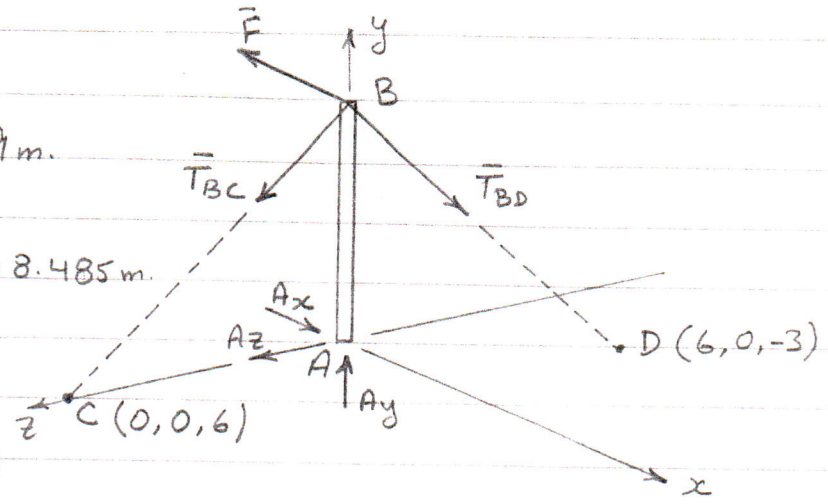
$$\Sigma M = T_{CD}\sin 20^\circ \times 2\cos 40^\circ - T_{CD}\cos 20^\circ \times 2\sin 40^\circ - 490.5 \times 3\cos 40^\circ$$
$$= 383.4\sin 20^\circ \times 2\cos 40^\circ - 383.4\cos 20^\circ \times 2\sin 40^\circ - 490.5 \times 3\cos 40^\circ$$
$$= 200.9 - 463.2 - 1127.2$$
$$= -1389.5\ \text{N.m}.$$

∴ The force acting at A is $R = \underline{\underline{508.9\ \text{N}}}$ $\theta = 44.93^\circ$ ANS.

and the couple $= 1389.5\ \text{N.m}\circlearrowright$ ANS.



2.



$\bar{BD} = +6\bar{i} - 6\bar{j} - 3\bar{k}$

$\therefore BD = \sqrt{(6)^2 + (-6)^2 + (-3)^2} = 9\,m.$

$\bar{BC} = 0\bar{i} - 6\bar{j} + 6\bar{k}$

$\therefore BC = \sqrt{(-6)^2 + (6)^2} = \sqrt{72} = 8.485\,m.$

$\bar{T}_{BD} = T_{BD}\,\bar{\lambda}_{BD} = T_{BD}\,\frac{\bar{BD}}{BD}$

$\bar{T}_{BD} = \frac{T_{BD}}{9}(6\bar{i} - 6\bar{j} - 3\bar{k})$

$\bar{T}_{BD} = T_{BD}\left(\frac{6}{9}\bar{i} - \frac{6}{9}\bar{j} - \frac{3}{9}\bar{k}\right)$

$\bar{T}_{BD} = T_{BD}\left(\frac{2}{3}\bar{i} - \frac{2}{3}\bar{j} - \frac{1}{3}\bar{k}\right)$

$\bar{T}_{BC} = T_{BC}\,\bar{\lambda}_{BC} = T_{BC}\,\frac{\bar{BC}}{BC}$

$\bar{T}_{BC} = \frac{T_{BC}}{8.485}(-6\bar{j} + 6\bar{k})$

$\bar{T}_{BC} = T_{BC}\left(\frac{-6}{8.485}\bar{j} + \frac{6}{8.485}\bar{k}\right) = T_{BC}(-0.707\bar{j} + 0.707\bar{k})$

$\bar{F} = -5\,kN\,\bar{i}$

$\Sigma M_A = 0 = \bar{AB} \times \bar{T}_{BD} + \bar{AB} \times \bar{T}_{BC} + \bar{AB} \times \bar{F}$ $\qquad \bar{AB} = 6\bar{j}$

$= 6\bar{j} \times T_{BD}\left(\frac{2}{3}\bar{i} - \frac{2}{3}\bar{j} - \frac{1}{3}\bar{k}\right) + 6\bar{j} \times T_{BC}(-0.707\bar{j} + 0.707\bar{k})$

$+ 6\bar{j} \times (-5\bar{i}) = 0$

$= -4T_{BD}\bar{k} - 2T_{BD}\bar{i} + 4.242\,T_{BC}\bar{i} + 30\bar{k} = 0$

Now, equate the coefficients of $\bar{i}$ & $\bar{k}$ to zero: $\bar{k}\ \bar{j}\ \bar{i}$

$(\bar{i})$: $-2T_{BD} + 4.242\,T_{BC} = 0$ — (1)

$(\bar{k})$: $-4T_{BD} + 30 = 0$; $\therefore T_{BD} = \frac{30}{4} = \underline{\underline{7.5\,KN}}$ ANS.

Insert in (1):

$-2 \times 7.5 + 4.242\,T_{BC} = 0$; $\therefore T_{BC} = \frac{15}{4.242} = \underline{\underline{3.54\,KN}}$ ANS.

$\Sigma F_x = 0$; $A_x - 5 + \frac{6}{9}T_{BD} = 0$

$A_x = 5 - \frac{6}{9} \times 7.5 = 0$; $\therefore A_x = \underline{\underline{0}}$

$\Sigma F_y = 0$, $A_y - \frac{6}{9}T_{BD} - \frac{6}{8.485} \times 3.54 = 0$

$A_y = \frac{6}{9} \times 7.5 + 2.5 = 7.5$; $\therefore A_y = \underline{\underline{7.5\,KN}}$

$\Sigma F_z = 0$; $A_z - \frac{3}{9}T_{BD} + \frac{6}{8.485}T_{BC} = 0$

$A_z = \frac{3}{9} \times 7.5 - \frac{6}{8.485} \times 3.54 = 0$; $\therefore A_z = \underline{\underline{0}}$

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