The domain of the function: $f(x) = \frac{\ln(3x+6)}{x-2}$ is:

Requirements : { • "argument of ln must be > 0"
• "Cannot divide by zero"

$(*)$ $\begin{cases} \bullet\ 3x+6>0 \Rightarrow x>-2 \Rightarrow x \in (-2,\infty) \\ \bullet\ x-2 \neq 0 \Rightarrow x \neq 2 \Rightarrow x \in \mathbb{R} \setminus \{2\} \end{cases}$

$\rightarrow$ Both conditions must be met :

$\therefore\ \text{dom}(f) = (-2,\infty) \setminus \{2\} = (-2,2) \cup (2,\infty)$

$\ldots = \{ x \in \mathbb{R} \mid x>-2,\ x \neq 2 \}$

Solve for x in $e^{\frac{1}{2}\ln(x^4)} + \ln(x) + \ln(x+2) - \ln(x^2+2x) = 9$

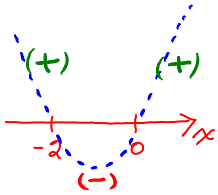
• With "ln" present in the equation, we need to determine the domain of the equation and verify the solutions.

Solve for x in $e^{\frac{1}{2}\ln(x^4)} + \ln(x) + \ln(x+2) - \ln(x^2+2x) = 9$ ...

$x>0 \quad x>0 \quad x>-2 \quad x^2+2x>0$

$x(x+2)>0$

$\begin{bmatrix} x>0 \\ x<-2 \end{bmatrix}$



$\Downarrow$

From this, the domain of the equation is $(0,\infty)$

And now we solve...

$e^{\frac{1}{2}\ln(x^4)} + \ln(x) + \ln(x+2) - \ln(x^2+2x) = 9$

$e^{\ln((x^4)^{1/2})} + \ln\left[\frac{\overbrace{x(x+2)}^{x^2+2x}}{x^2+2x}\right] = 9$

$x^2 + \ln 1 = 9$

$x^2 = 9$

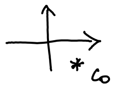
$x = \pm 3$

Based on the domain of $(0,\infty)$

we accept $x=3$ and reject $x=-3$

$\therefore$ SOLUTION : $\boxed{x=3}$

If $Cos(\theta) = \frac{2}{5}$ and $\underbrace{\frac{3\pi}{2} \leq \theta \leq 2\pi}_{\text{this is } Q4}$, then $Tan(\theta) =$



$\left.\begin{array}{l} * \cos > 0 \\ \sin < 0 \\ \tan < 0 \end{array}\right]$ IMPORTANT!

We can use identities to solve :

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{1-\cos^2\theta}}{\cos\theta}$$

$\boxed{\begin{array}{l} \sin^2+\cos^2=1 \\ \sin\theta = \pm\sqrt{1-\cos^2} \end{array}}$

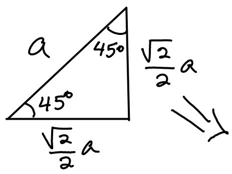
$$\tan\theta = -\frac{\sqrt{1-(2/5)^2}}{2/5}$$

$$= -\frac{\sqrt{25/25 - 4/25}}{2/5}$$

$$= -\frac{\sqrt{21}}{2}$$

$$\arcsin\left(\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

$\Big\} \Leftarrow$ one of the remarkable angles $(45°)$



$\Rightarrow$ $$\left(\frac{\sqrt{2}}{2}a\right)^2 + \left(\frac{\sqrt{2}}{2}a\right)^2 = a^2$$

$$\lim_{x\to\infty} \frac{\sqrt{x^2-4}}{2x+1} = \xrightarrow{\text{"D.S."}} \frac{\infty}{\infty}$$

One way to solve is to factor out x :

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2-4}}{2x+1}\right] = \lim_{x\to\infty}\left[\frac{\sqrt{x^2(1-4/x^2)}}{x(2+1/x)}\right] = \lim_{x\to\infty}\left[\frac{\sqrt{x^2}\cdot\sqrt{1-4/x^2}}{x(2+1/x)}\right] =$$

$$\ldots = \lim_{x\to\infty}\left[\frac{|x|\cdot\sqrt{1-4/x^2}}{\ldots}\right] = \lim_{x\to\infty}\left[\frac{\cancel{x}\cdot\sqrt{1-4/x^2}}{\ldots}\right] \xrightarrow{\text{"D.S."}}$$

$$\dots = \lim_{x\to\infty}\left[\frac{|x|\cdot\sqrt{1-4/x^2}}{x\,(2+1/x)}\right] = \lim_{x\to\infty}\left[\frac{\cancel{x}\cdot\sqrt{1-4/x^2}}{\cancel{x}\,(2+1/x)}\right] \xrightarrow{\text{"D.S."}}$$

$$\dots = \xrightarrow{\text{"D.S."}} \frac{\sqrt{1-4/\infty^2}}{2+1/\infty} = \frac{\sqrt{1-0}}{2+0} = \boxed{\frac{1}{2}}$$

$$\lim_{x\to\infty}\frac{\sin(x)}{\sqrt{3x-1}} = \xrightarrow{\text{"D.S."}} \frac{\sin\infty}{\sqrt{3(\infty)-1}} = \frac{?}{\infty}$$

- The problem is that $\lim_{x\to\infty} \sin x$ does not exist. It is not 1, not 0, not $\pm\infty$.

- However, "$\sin x$" is a bounded function: indeed: $-1 \le \sin x \le 1$

  $\to$ we can now use the Squeeze Theorem:

$$-1 \le \sin x \le 1$$

$$\frac{-1}{\sqrt{3x-1}} \le \frac{\sin x}{\sqrt{3x-1}} \le \frac{1}{\sqrt{3x-1}}$$

$$\lim_{x\to\infty}\left[\frac{-1}{\sqrt{3x-1}}\right] \le \lim_{x\to\infty}\left[\frac{\sin x}{\sqrt{3x-1}}\right] \le \lim_{x\to\infty}\left[\frac{1}{\sqrt{3x-1}}\right]$$

$$0 \le \lim_{x\to\infty}\left[\frac{\sin x}{\sqrt{3x-1}}\right] \le 0$$

... because $\frac{1}{\sqrt{3x-1}}$ $\left(\text{and } \frac{-1}{\sqrt{3x-1}}\right) \longrightarrow 0$ as $x\to\infty$

So, by the Squeeze Theorem:

$$\lim_{x\to\infty}\left[\frac{\sin x}{\sqrt{3x-1}}\right] = 0$$

$$\lim_{x\to 3}\frac{x^3-4x^2+9}{2x-6} = \xrightarrow{D.S.} \frac{3^3-4(3)^2+9}{2(3)-6} = \frac{27-36+9}{6-6} = \frac{0}{0}$$

we use L'Hospital's Rule (on $\frac{0}{0}$ or $\frac{\infty}{\infty}$ forms)

$$\lim_{x\to 3}\left[\frac{x^3-4x^2+9}{2x-6}\right] \xrightarrow{H.} \lim_{x\to 3}\left[\frac{3x^2-8x}{2}\right] \xrightarrow{D.S.}$$

$$\ldots \xrightarrow{D.S.} \frac{3(3)^2-8(3)}{2} = \boxed{\frac{3}{2}}$$

$$\lim_{x\to 0}\frac{1-\cos(2x)}{\sin(3x)} = \xrightarrow{D.S.} \frac{1-\cos(2(0))}{\sin(3(0))} = \frac{1-1}{0} = \frac{0}{0}$$

we use L'Hospital's Rule (or we can $\otimes$ by $\frac{1+\cos(3x)}{1+\cos(3x)}$)

this also works, but it is longer to do

$$\therefore \lim_{x\to 0}\left[\frac{1-\cos(2x)}{\sin(3x)}\right] \xrightarrow{H.} \lim_{x\to 0}\left[\frac{2\sin(2x)}{3\cos(3x)}\right] \xrightarrow{D.S.} \frac{0}{3} = \boxed{0}$$

$$\frac{d}{dx}3^{(x^2+1)} = (\ln 3)\cdot 3^{(x^2+1)}\cdot(2x)$$

Recall : $[f(g(x))]' = f'(g(x))\cdot g'(x)$ { CHAIN RULE

and : $[a^x]' = (\ln a)\cdot a^x$ { DERIVATIVE OF EXPONENTIAL FUNCTION

$$\lim_{x\to 1^+}(x-1)\ln(x-1) = \xrightarrow{D.S} (1-1)\cdot\ln(1-1) = 0\cdot\ln 0 = 0\cdot(-\infty)$$

We re-arrange to obtain an $\frac{\infty}{\infty}$ form, and then use L'Hospital's Rule :

$$\therefore \lim_{x\to 1^+}(x-1)\cdot\ln(x-1) = \lim_{x\to 1^+}\left[\frac{\ln(x-1)}{\frac{1}{x-1}}\right] \xrightarrow{D.S.} \frac{\ln(1-1)}{\frac{1}{1-1}} = \frac{-\infty}{\infty}$$

$$\therefore \lim_{x\to 1^+}\left[\frac{\ln(x-1)}{\frac{1}{x-1}}\right] \xrightarrow{H.} \lim_{x\to 1^+}\left[\frac{\frac{1}{x-1}}{\ldots}\right] = \lim_{x\to 1^+}\left(-(x-1)\right) \xrightarrow{D.S.} \boxed{0}$$

$x \to 1$ $\left[\frac{1}{x-1} \right]$ $\longrightarrow$ $\lim_{x \to 1^+} \left[\frac{-1}{(x-1)^2} \right]$ $= \lim_{x \to 1^+} (\ldots) \to \boxed{0}$

The absolute minimum value of $f(x) = x^3 - 6x + 1$ on $[-1, 3]$ is

We use the Closed Interval Method :

$f(a) = f(-1) = (-1)^3 - 6(-1) + 1 = \boxed{6}$

$f(b) = f(3) = (3)^3 - 6(3) + 1 = \boxed{10}$

Evaluate f at endpoints

we must now evaluate f at the critical points on the interval :

$f'(x) = 3x^2 - 6 \Rightarrow x = \pm\sqrt{2} \Rightarrow$ we ignore $x = -\sqrt{2} < -1$

(Recall : $\sqrt{2} \approx 1.4$)

$\therefore f(\sqrt{2}) = (\sqrt{2})^3 - 6(\sqrt{2}) + 1 = \sqrt{2}\left((\sqrt{2})^2 - 6\right) + 1$

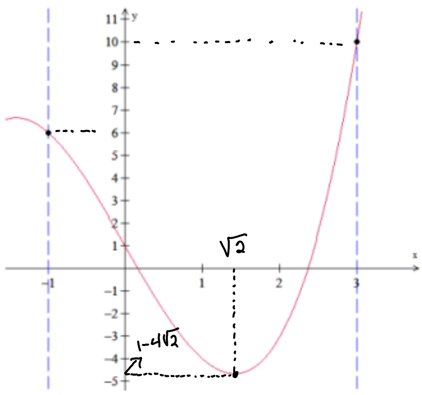
$\ldots = \sqrt{2}(2-6) + 1 = -4\sqrt{2} + 1$

$\ldots \approx -4(1.4) + 1 \approx -5.6 + 1 \approx \boxed{-4.6}$

$\therefore$ On this interval,

- the absolute min. is $\approx -4.6 = f(\sqrt{2})$
- the " max. is $10 = f(3)$

Here is the graph of the function:



The general antiderivative of $f(x) = 4e^{2x} + \sqrt{x} - \sin(x)$ is

$$\int f(x)\,dx = \int 4e^{2x} + \sqrt{x} - \sin(x)\,dx =$$

$$\ldots = 4\int e^{2x}\,dx + \int x^{1/2}\,dx - \int \sin(x)\,dx =$$

$$\ldots = 4\left(\frac{e^{2x}}{2}\right) + \frac{x^{3/2}}{3/2} + \cos x + C$$

$$\ldots = \boxed{2e^{2x} + \frac{2}{3}\sqrt{x^3} + \cos x + C}$$

$$\frac{d}{dx}\int_0^{x^2} e^t \sin(t)\,dt =$$

Let $f(t) = e^t \sin(t)$, and $F(t)$ its antiderivative.

Then: $$\frac{d}{dx}\left(\int_0^{x^2} e^t \sin(t)\,dt\right) = \frac{d}{dx}\left(\Big[F(t)\Big]_0^{x^2}\right)$$

$$\ldots = \frac{d}{dx}\Big(\underbrace{F(x^2)}_{\text{FUNCTION OF A FUNCTION}} - \underbrace{F(0)}_{\text{CONSTANT}}\Big) = \frac{d}{dx}\Big(F(x^2)\Big) - \cancel{\frac{d}{dx}\Big[F(0)\Big]}^{\,0}$$

$$\ldots = F'(x^2)\cdot\frac{d}{dx}(x^2) \qquad (\ldots \text{ by the Chain Rule})$$

$$\ldots = f(x^2)\cdot(2x) \qquad (\text{recall: } F' = f)$$

$$\ldots = \boxed{2x\,e^{x^2}\cdot\sin(x^2)}$$

The inverse of $f(x) = (x-1)^3 + 4$ is

3 steps:

- Start with $y = f(x)$
- Swap variables and work with $x = f(y)$
- Solve for $y = f^{-1}(x)$

OR

- Start with $y = f(x)$
- Solve for $x = f^{-1}(y)$
- Swap variables and obtain $y = f^{-1}(x)$

- $y = (x-1)^3 + 4 \;(= f(x))$
- SWAP:

$$x = (y-1)^3 + 4 \quad (= f(y))$$

$$x - 4 = (y-1)^3$$

$$y - 1 = \sqrt[3]{x-4}$$

$$\boxed{y = \sqrt[3]{x-4} + 1 \quad \left(= f^{-1}(x)\right)}$$

$$\int 3x - x^3 + e^{4x} + 3\sin(x)\, dx = 3\int x\, dx - \int x^3\, dx + \int e^{4x}\, dx + 3\int \sin x\, dx =$$

$$\ldots = 3\,\frac{x^2}{2} - \frac{x^4}{4} + \frac{1}{4}e^{4x} - 3\cos x + C_1$$

$$\ldots \boxed{= \frac{3}{2}x^2 - \frac{1}{4}x^4 + \frac{1}{4}e^{4x} - 3\cos x + C_1}$$
