Concordia University
ENCS
Engineering & Computer Science

ENGR 213

Applied Ordinary Differential Equations

Second Mid Term Exam

March 12th 2017

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ENGR 213 Section: W

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Q1	8
Q2	10
Q3	10
Q4	10
Q5	10
Total	48

- **Exam is closed book close notes.**
- **No use of any electronic devices.**
- **Use only the approved calculator.**
- **Write your answers in the provided space.**

Problem#1

$i = \sqrt{-1}$
$i^2 = -1$

Write the number $(3+6i) + (4\text{-}i)\ (3+5i) + \frac{1}{2-i}$ in the form $a+bi$.

Solution:

$(3+6i) + (4-i)(3+5i) + \frac{1}{2i}$

$(3+6i)+(12+20i-3i-5i^2)+\frac{1}{2i}$ $\frac{1}{2-i}$

$3+6i+12+17i+5+\frac{1}{2i}$

$20+23i+\left(\frac{1}{2i}\times\frac{2i}{2i}\right)$

$20+23i+\frac{2i}{4i^2}$

$20+23i-\frac{i}{2}$ α

$20+\frac{46i}{2}-\frac{i}{2}$

$20+\frac{45}{2}i$

8

Problem#2

Verify that $y_1 = e^{5x}$ and $y_2 = e^{-7x}$ are form a fundamental set of solutions of the differential equation $y''+2y'-35y=0$ on the interval $(-\infty,\infty)$. Write the form of the general solution.

Solution:

$$y''+2y'-35y=0$$

$$\downarrow$$

$$y=e^{mx} \quad m^2+2m-35=0$$

A: 1
B: 2
C: -35

$$\frac{-B\pm\sqrt{B^2-4AC}}{2A} = \frac{-2\pm\sqrt{2^2-4(1)(-35)}}{2}$$

$$= \frac{-2\pm\sqrt{4+140}}{2}$$

$$= \frac{-2\pm\sqrt{144}}{2}$$

$$= \frac{-2\pm 12}{2}$$

$$= -1\pm 6 \quad m_1=-7 \quad m_2=5$$

10

Hence, The General Solution is:

$$y = c_1e^{-7x}+c_2e^{5x} \quad \text{where} \quad x\in\mathbb{R}$$

By determining the root of the DE $y''+2y'-35y=0$, it is verified that $y_1=e^{5x}$ and $y_2=e^{-7x}$ form a fundemental set in the equation the interval $(-\infty,\infty)$

Problem#3

Solution:

(a) Write the characteristic equation and the general solution of the differential equation:

$y^{(6)} - 13y^{(5)} + 70y^{(4)} - 198y^{(3)} + 308y'' - 248y' + 80y = 0.$

$y^{(k)}$ is the k-th derivative of y. For Your convenience, the roots of the characteristic equation are: 1, 2, 2, 2, 3 + i, 3 − i. You do not have to check this.

$\alpha = 3 \quad \beta = 1$

(b) Find the general solution of the differential equation: $x^2 y'' - 7xy' + 16y = 0$, given one solution $y_1 = x^4$.

Solution:

3

a)

$$y = c_1 e^{x} + c_2 e^{2x} + c_3 x e^{2x} + c_4 x^2 e^{2x} + e^{3x}\left[c_5 \cos(x) + c_6 \sin(x)\right]$$

b)

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{(y_1)^2} dx$$

$$= x^4 \int \frac{e^{-\int -\frac{7}{x}dx}}{(x^4)^2} dx$$

$$= x^4 \int \frac{e^{7\ln x}}{x^8} dx$$

$$= x^4 \int \frac{x^7}{x^8} dx$$

$$= x^4 \int \frac{1}{x} dx$$

$$y_2 = x^4 \ln x$$

$$x^2 y'' - 7xy' + 16y = 0$$

$$\Downarrow$$

$$y'' - \frac{7}{x} y' + \frac{16}{x^2} y = 0$$

$$y = c_1 x^4 + c_2 x^4 \ln x$$

7

Problem#4

Using the method of undetermined coefficients, solve the initial value problem

$y''+y'-2y=(6x+2)\,e^{x};\ y(0)=0;\ y'(0)=0.$

Solution:

Homogeneous

$$y''+y'-2y=0$$

$y=e^{mx}$

$$m^2+m-2=0$$

A: 1
B: 1
C: −2

$$\frac{-1\pm\sqrt{1-4(-2)}}{2}=\frac{-1\pm 3}{2}\quad m_1=-2,\ m_2=1$$

$$y_h=c_1e^{-2x}+c_2e^{x}$$

* We must multiply y_p by x because Be^x is found in y_h (c_2e^x)

Particular

Assumed $y_p=(Ax+B)e^x$

New $y_p=(Ax^2+Bx)e^x$

$$y_p'=(2Ax+B)e^x+(Ax^2+Bx)e^x$$

$$y_p''=(2A)e^x+(2Ax+B)e^x+(2Ax+B)e^x+(Ax^2+Bx)e^x$$

$$=2Ae^x+2(2Ax+B)e^x+(Ax^2+Bx)e^x$$

$$=(2A)e^x+(4Ax+2B)e^x+(Ax^2+Bx)e^x$$

$$y''+y'-2y=(6x+2)e^x$$

$$(2A)e^x+(4Ax+2B)e^x+(Ax^2+Bx)e^x+(2Ax+B)e^x+(Ax^2+Bx)e^x-2\left[(Ax^2+Bx)e^x\right]=(6x+2)e^x$$

$$2A+4Ax+2B+\cancel{Ax^2}+Bx+2Ax+B+\cancel{Ax^2}+Bx-\cancel{2Ax^2}-2Bx=6x+2$$

$$(4Ax+Bx+2Ax+Bx-2Bx)+(2A+2B+B)=6x+2$$

$$(6Ax)+(2A+3B)=6x+2$$

Hence,

$$6Ax=6x\ \Rightarrow\ A=1$$

$$2A+3B=2\ \Rightarrow\ 2(1)+3B=2\ \Rightarrow\ 3B=0\ \Rightarrow\ B=0$$

VERSO

$$y = y_h + y_p$$

$$y = c_1 e^{-2x} + c_2 e^{x} + x^2 e^{x}$$

$y(0) = 0$

$y'(0) = 0$

- $y(0) = c_1 e^{-2(0)} + c_2 e^0 + 0^2 e^0$

  $0 = c_1 + c_2$

- $y'(x) = -2c_1 e^{-2x} + c_2 e^{x} + 2x e^{x} + x^2 e^{x}$

  $y'(0) = -2c_1 e^{-2(0)} + c_2 e^0 + 2(0) e^0 + (0)^2 e^0$

  $0 = -2c_1 + c_2$

- $c_1 = -c_2$

  $0 = -2(-c_2) + c_2$

  $0 = 2c_2 + c_2$

  $0 = 3c_2$

  $0 = c_2$

- if $c_2 = 0$, then

  $0 = c_1 + c_2$

  $0 = c_1 + 0$

  $0 = c_1$

Hence,

$$y(x) = x^2 e^{x}$$

Problem#5

Solve the following differential Equation $y'' + (3/x)\, y' + (5/x^2)\, y = 0$

Solution:

$$y'' + \frac{3}{x} y' + \frac{5}{x^2} y = 0$$

Cauchy-Euler

$y = x^m$

$y' = (m)x^{m-1}$

$y'' = (m-1)(m)x^{m-2}$

$$x^2 y'' + 3xy' + 5y = 0$$

$$x^2 (m-1)(m)x^{m-2} + 3x(m)x^{m-1} + 5x^m = 0$$

$$x^m(m-1)(m) + x^m 3(m) + x^m 5 = 0$$

$$x^m \left[(m-1)(m) + 3m + 5\right] = 0$$

$$(m-1)(m) + 3m + 5 = 0$$

$$m^2 - m + 3m + 5 = 0$$

$$m^2 + 2m + 5 = 0$$

A: 1
B: 2
C: 5

$$\frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2} = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm \sqrt{-1}\sqrt{16}}{2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$$

$m_1 = -1 - 2i$ $\quad m_2 = -1 + 2i$

$\alpha = -1$

$\beta = 2$

$$y = x^{-1}\left[c_1 \cos(2\ln(x)) + c_2 \sin(2\ln(x))\right]$$

10