

Mid-term Exam

ADM2303, Fall 2016
STATISTICS FOR MANAGEMENT I
Date: OCTOBER 15, 1.00-2.30

Duration: 1.5

INSTRUCTIONS

1. Books and notes **are not** permitted. **One** sheet of notes, 8.5"x11", is allowed as per instructions in course outline.
2. Calculators **are** permitted.
3. Use the space on the question paper for rough work.

GIVEN NAME: _____ **FAMILY NAME** _____ **SECTION** _____

STUDENT #: _____

ANSWER ALL QUESTIONS ON THE CODING SHEET

ALSO PUT YOUR ANSWER AND ALL INTERMEDIATE CALCULATION AND REASONING ON THE QUESTION PAPER.

Statement of Academic Integrity

The School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

Statement to be signed by the student:

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: _____

Note: an examination copy or booklet without that signed statement will not be graded and will receive a final exam grade of zero.

Questions 1-6

The company Innova manufactures small gasoline engines in three factories, M1, M2 and M3. All engines are tested before delivery and are declared to be acceptable, defective (minor defects that are repairable) or unacceptable (major issues that are not repairable). The following table provides the number of engines for each category of production for the three factory.

Categories	Factory			Total
	M1	M2	M3	
A: acceptable engine	450	315	390	1155
D: defective engine	60	40	60	160
U: unacceptable engine	30	25	30	85
Total	540	380	480	1400

- Q1.** What is the probability that an engine is defective or that it comes from factory M2?
- (a) 38.57%
 (b) 3.10%
 (c) 35.71%
 (d) 41.43%
 (e) None of the above
- Q2.** What is the probability that an engine is defective and unacceptable?
- (a) 0.00%
 (b) 17.50%
 (c) 0.69%
 (d) 5.36%
 (e) None of the above
- Q3.** Knowing that an engine was produced by factory M3, what is the probability that it is defective?
- (a) 41.43%
 (b) 12.50%
 (c) 37.50%
 (d) 4.29%
 (e) None of the above
- Q4.** Knowing that the events are disjoint, what is the probability that an engine is manufactured at M1 or M2?
- (a) 65.71%
 (b) 0.00%
 (c) 10.47%
 (d) 70.00%
 (e) None of the above
- Q5.** What is the probability that an engine is produced by factory M1 given that it is unacceptable?
- (a) 42.50%
 (b) 2.34%
 (c) 2.14%
 (d) 35.29%
 (e) None of the above

- Q6.** What is the probability that an engine is unacceptable and that it was manufactured either at factory M1 or M2?
- (a) 0.04%
 - (b) 3.93%
 - (c) 67.86%
 - (d) 71.79%
 - (e) None of the above

Questions 7-13

VillageNets is a microfinance NGO (non-governmental organization). They provide financing to small businesses in developing countries. Village Nets plans to assess the credit risk of loan applicants by assessing the applicant's internet-based social-network profile (to be categorized into one of three categories, A, B, C). Historical records of borrowers who proved to default show that the proportion falling into category A, B and C was 0.1, 0.2, 0.7, respectively. The same records show that the proportions among the non-defaulters (good loans) was 0.6, 0.3, and 0.1, respectively. Assuming that 10 percent of Village Nets loan applicants will ultimately default, answer the following (assuming that the above probabilities apply).

- Q7.** What proportion of all applicants will fall into Category C?
- (a) 0.07
 - (b) 0.16
 - (c) 0.7
 - (d) 0.4
 - (e) 0.09
- Q8.** What proportion will fall into either Category A or B?
- (a) 0.91
 - (b) 0.6
 - (c) 0.84
 - (d) 0.3
 - (e) 0.93
- Q9.** Given that potential customer is known to fall into category C, what is the probability that they would default on a loan (if loan given).
- (a) 0.7
 - (b) 0.07
 - (c) 0 because independent
 - (d) 0 because disjoint
 - (e) 0.4375
- Q10.** Consider a group of 5 Category C customers. What is the probability that none of them default?
- (a) 0.7
 - (b) 0.00243
 - (c) 0.0563135
 - (d) 0.0160284
 - (e) 0 because disjoint

- Q11.** Consider a group of 5 Category C customers. What is the probability that at least one of them defaults?
- (a) 0.83193
 - (b) 0.9839716
 - (c) 0.9436865
 - (d) Can't answer because not disjoint
 - (e) 0.99757

The context is different in the moderately developed world where the probability of default is 0.05, and the probability that a borrower would fall into category A is 0.8 (hint: these are marginal probabilities). Consider a randomly chosen customer from this moderately developed region, and determine the joint probability that they fall in Category A and will not default.

- Q12.** What would this joint probability be if these two events (namely (1) Category A and (2) being non-default) were disjoint?
- (a) 0.76
 - (b) 0.04
 - (c) 0
 - (d) 0.85
 - (e) 1

- Q13.** What would this joint probability be if these two events (namely (1) Category A and (2) being non-default) were independent?
- (a) 0.76
 - (b) 0.04
 - (c) 0
 - (d) 0.85
 - (e) 1

Questions 14-19

Karen is starting a career as a professional wildlife photographer and plans to photograph Canadian Geese at one of the staging grounds during their migration in North Eastern Manitoba. She booked a place in a hide at the edge of a lake and plans to photograph the geese as they land on the water. For the price of a room in the Hilton (\$200 per day), she gets a spot on a wooden bench shared by other photographers, a muddy floor, a bracket to mount her telephoto lens, a tent to sleep in and delicious meals of freshly caught fish. Last year, during a stay of 3 days in this hide, she got 2 shots worth \$5000 each. She regards this as typical for this time of year although good shots happen at random and each day is independent of the others. To establish her reputation Karen only sells \$5000 photographs. This year she has booked 4 days in the hide.

- Q14.** What is Karen's expected (mean) revenue from her first day in the hide.
- (a) \$2000
 - (b) \$3000
 - (c) \$3333
 - (d) \$5000
 - (e) \$6667
- Q15.** What is the probability of getting two or more \$5000 photos on the first day.
- (a) 0.114
 - (b) 0.144
 - (c) 0.189
 - (d) 0.213
 - (e) 0.856

- Q16.** What is the standard deviation (SD) of her revenue from one day.
- (a) \$950
 - (b) \$1178
 - (c) \$1546
 - (d) \$2441
 - (e) \$4082
- Q17.** What is the standard deviation (SD) of her revenue from the 4 days.
- (a) 25% of the answer to Q16
 - (b) 50% of the answer to Q16
 - (c) same as the answer to Q16
 - (d) 2 times the answer to Q16
 - (e) 4 times the answer to Q16
- Q18.** Use SD/mean as a measure of risk. How much more risky is 1 day than 4 days.
- (a) 1/4 as risky
 - (b) 1/2 as risky
 - (c) 2 times as risky
 - (d) 4 times as risky
 - (e) times as risky
- Q19.** The return charter flight from Winnipeg where Karen lives costs \$1,200. How much net income can she expect from the 4-day trip.
- (a) \$0
 - (b) \$1000
 - (c) \$11333
 - (d) \$12133
 - (e) \$13333

Questions 20-23

At a restaurant, patrons have the choice of sitting inside or on the outside patio. We are not concerned with the number of patrons in this exercise, only the number of tables taken inside versus outside. The restaurant has 25 tables. The manager knows from long-time experience that on a day like today, 75% of patrons choose the patio and 25% choose to sit inside. The manager was absent from the restaurant earlier in the day, and just arrives. A staff manager tells her that 7 tables are taken in total. In this exercise, the manager tries to guess how many tables are taken outside versus inside.

- Q20.** What is given in this exercise?
- (a) $p=.25$, $q=.75$, probability distribution
 - (b) $\lambda=.75$, $x=7$, poisson
 - (c) $n=25$, $p=.75$, binomial
 - (d) $n=7$, $p=.75$, binomial
 - (e) $n=5$, $p=0.25$, binomial

Q21. Without looking around, the manager calculates the expected value of how many of the tables are outside tables and says: Let me guess, this many are taken on the patio:

- (a) 1.75
- (b) 5.25
- (c) 6.25
- (d) 18.75
- (e) this can actually not be calculated so easily.

Q22. What is the standard deviation?

- (a) < 1.0
- (b) $1.0 < sd < 2.0$
- (c) $2.0 < sd < 4.0$
- (d) $4.0 < sd < 5.0$
- (e) we don't have enough information to calculate this.

Q23. What is the probability that exactly 3 tables outside and 4 tables inside are taken?

- (a) 0.001
- (b) 0.058
- (c) 0.428
- (d) 0.571
- (e) > 0.6

Questions 24-26

A worker takes a break from construction and watches bees on a patch of flowers. She notices that while there seem to be a lot of bees hovering around the flowers, the 'traffic' to and from the patch is not very heavy. During the 15min. morning break over a few days she observes a fairly steady pattern of 6 bees arriving per every 5min.

She wonders whether she could apply some statistics calculations to this.

Q24. Could she use one of the following approaches?

- (a) probability calculations
- (b) binomial distribution
- (c) Poisson distribution
- (d) (b) or (c)
- (e) none of the above

Q25. What is the probability of less than 2 bees arriving during 5 minutes of her morning break?

- a. 0.002
- b. 0.015
- c. 0.017
- d. 0.045
- e. 0.062

Q26. What is the probability that 4 or more bees arrive during only 2 minutes of the break?

- (a) 0.151
- (b) 0.779
- (c) 0.221
- (d) 0.849
- (e) cannot be calculated

Questions 27-29

Mr. Soandso is in the market to buy a car. He would like on that consumes under 7.5 litres/100km. The dealer has a car with a sticker that says the car consumes 7.2 litres/100km. Assume this is based on a normal distribution with a standard deviation of 2.3 litres/100km. Calculate the probability that on a test drive the car will consume more than 7.5 litres/100km.

Q27. Identify the given variables

- (a) $\lambda = 7.2 / 100\text{km}$
- (b) $\mu = 7.5, x=7.2$
- (c) $\mu = 7.2, x=7.5$
- (d) $\mu = 0.3, \sigma=2.3$
- (e) none of the above are correct

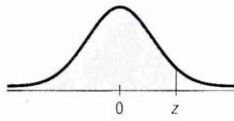
Q28. Can / should we calculate z?

- (a) yes, $z = (7.5-7.2)/2.3 = 0.13$
- (b) yes, $z = (7.2-7.5)/2.3 = -0.13$
- (c) yes, $z = (0.3-7.2)/2.3 = -3$
- (d) yes, $z = (7.2-0.3)/2.3 = 3$
- (e) no, this would not be appropriate

Q29. To not carry any errors forward, assume we can calculate z for this exercise and it is 0.95. Now calculate the probability that the car will consume more than 7.5 litres/100km on the test drive (values are approximate).

- (a) 5%
- (b) 17%
- (c) 83%
- (d) 95%
- (e) can't be calculated with this information.

Table Z (cont.)
Areas under the standard
Normal curve



z	Second decimal place in z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000 [†]									

[†] For $z \geq 3.90$, the areas are 1.0000 to four decimal places.

A-42 APPENDIX C

Table Z Areas under the standard Normal curve		Second decimal place in z									z	
		0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01		0.00
										0.0000 [†]	-3.9	
	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.8
	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.7
	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	-3.6
	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	-3.5
	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	-3.4
	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	-3.3
	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	-3.2
	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	-3.1
	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0012	0.0013	0.0013	0.0013	-3.0
	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0018	0.0019	-2.9
	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0024	0.0025	0.0026	-2.8
	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0034	0.0035	-2.7
	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0044	0.0045	0.0047	-2.6
	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0059	0.0060	0.0062	-2.5
	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0080	0.0082	-2.4
	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0104	0.0107	-2.3
	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0136	0.0139	-2.2
	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0174	0.0179	-2.1
	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0222	0.0228	-2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0281	0.0287	-1.9	
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0351	0.0359	-1.8	
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0436	0.0446	-1.7	
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0537	0.0548	-1.6	
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0655	0.0668	-1.5	
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0793	0.0808	-1.4	
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0951	0.0968	-1.3	
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1131	0.1151	-1.2	
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1335	0.1357	-1.1	
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1562	0.1587	-1.0	
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1814	0.1841	-0.9	
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2090	0.2119	-0.8	
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2389	0.2420	-0.7	
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2709	0.2743	-0.6	
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3050	0.3085	-0.5	
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3409	0.3446	-0.4	
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3783	0.3821	-0.3	
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4168	0.4207	-0.2	
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4562	0.4602	-0.1	
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.4960	0.5000	-0.0	

[†] For $z \leq -3.90$ the areas are 0.0000 to four decimal places.

ADM2303 formula sheet - Winter 2016

Probability theory

Rule of sum of probabilities:

$$P(S) = 1$$

Subtraction rule

(Let A^c be complement of A , i.e., Not A):

$$P(A) = 1 - P(A^c)$$

Addition rule for two mutually exclusive events (where \cup connotes "or" aka union):

$$P(A \cup B) = P(A) + P(B)$$

Addition rule for two not mutually exclusive events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Multiplication rule for two independent events (where \cap connotes "and" aka intersection):

$$P(A \cap B) = P(A) \times P(B)$$

Multiplication rule for n independent events:

$$P(A_1 \cap A_2 \cap \dots \cap A_n) = P(A_1) \times P(A_2) \times \dots \times P(A_n)$$

Multiplication rule for dependent events:

$$P(A \cap B) = P(B|A)P(A) = P(A|B)P(B)$$

Partition rule: for a partition B_1, B_2, \dots, B_k :

$$P(A) = \sum_{i=1}^k P(A \cap B_i) = \sum_{i=1}^k P(A|B_i)P(B_i)$$

Bayes' formula:

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A)} = \frac{P(A|B_i)P(B_i)}{\sum_{i=1}^k P(A|B_i)P(B_i)}$$

Events A and B are independent if:

$$P(A|B) = P(A) \text{ and } P(B|A) = P(B) \\ \text{or: } P(A \cap B) = P(A) \times P(B)$$

Random variables (RV)

Expected value of discrete RV X :

$$E(X) = \mu = \sum_{i=1}^n x_i P(X = x_i)$$

Variance of discrete RV X : $\text{Var}(X) = \sigma^2$

$$= \sum_{i=1}^n (x_i - \mu)^2 P(X = x_i) = \sum_{i=1}^n x_i^2 P(X = x_i) - \mu^2$$

Standard deviation of discrete RV X :

$$SD(X) = \sigma = \sqrt{\text{Var}(X)}$$

Coefficient of variation of discrete RV X :

$$CV(X) = \frac{SD(X)}{E(X)}$$

Correlation of two discrete RV X and Y : $\text{Corr}(X, Y)$

$$= \frac{\sum_{i=1}^n (x_i - \mu_x)(y_i - \mu_y)P(X = x_i \cap Y = y_i)}{\sigma_x \sigma_y}$$

Combining random variables

Adding a constant c to random variable X :

$$E(X \pm c) = E(X) \pm c \\ \text{Var}(X \pm c) = \text{Var}(X)$$

Multiplying random variable X by a constant a :

$$E(aX) = aE(X) \\ \text{Var}(aX) = a^2 \text{Var}(X)$$

Expected value of linear combination of RVs:

$$E(aX \pm bY \pm c) = aE(X) \pm bE(Y) \pm c$$

Variance of linear combination of **independent** RVs:

$$\text{Var}(aX \pm bY \pm c) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$$

Variance of linear combination of **dependent** RVs:

$$\text{Var}(aX \pm bY \pm c) \\ = a^2 \text{Var}(X) + b^2 \text{Var}(Y) \pm 2ab \text{Cov}(X, Y) \\ = a^2 \text{Var}(X) + b^2 \text{Var}(Y) \pm 2ab \text{Cov}(X, Y)$$

Discrete and continuous distributions

The Binomial probability distribution:

$$P(X = x) = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} \text{ for } x = 0, 1, \dots, n \\ E(X) = np, \text{Var}(X) = np(1-p)$$

The Poisson probability distribution

(if approx'n of binomial, $\lambda = np$):

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!} \text{ for } x = 0, 1, 2, \dots \\ E(X) = \lambda, \text{Var}(X) = \lambda, e = 2.718$$

The Geometric probability distribution:

$$P(X = x) = (1-p)^{x-1} p \text{ for } x = 1, 2, \dots \\ E(X) = \frac{1}{p}, \text{Var}(X) = \frac{1-p}{p^2}$$

The Normal distribution:

$$X \sim N(\mu, \sigma) \Rightarrow f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \\ Z = \frac{X-\mu}{\sigma} \sim N(0, 1) \Rightarrow P(Z < z) = \text{using normal table}$$

The Exponential distribution:

$$X \sim \text{Expo}(\lambda) \Rightarrow f(x) = \lambda e^{-\lambda x} \\ P(X \leq a) = 1 - e^{-a\lambda} \\ E(X) = \frac{1}{\lambda}, \text{Var}(X) = \left(\frac{1}{\lambda}\right)^2$$

The Uniform distribution:

$$X \sim \text{Uniform}(a, b) \Rightarrow f(x) = \frac{1}{b-a} \text{ for } a \leq x \leq b \\ P(x_1 < X < x_2) = \frac{x_2 - x_1}{b-a} \\ E(X) = \frac{a+b}{2}, \text{Var}(X) = \frac{(b-a)^2}{12}$$