Solutions
Attached.

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

MATH 209/2 all sections except EC: - Fundamental Mathematics II
Midterm - Sunday, November 1, 2015 (1h30min)

Only approved calculators are permitted.
Justify all your answers.

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1. (a) [*7 marks*] Find
$$\lim_{x\to 3} \frac{x^2+4x+4}{x^2-x-6}.$$
   (b) [*8 marks*] Give an example of a function $g(x)$ and a function $h(x)$ with the following properties:
$$(i)\ \lim_{x\to 5} g(x)=0 \qquad (ii)\ \lim_{x\to 5} h(x)=0 \qquad (iii)\ \lim_{x\to 5} \frac{[h(x)]^2}{g(x)}=2$$

2. [*7 marks*] Let $h(x)=x^3-5$. Work out the following in detail:
$$\lim_{s\to 0} \frac{h(x+s)-h(x)}{s}.$$

3. [*12 marks*]
   (a) If $f(x)=3x^{4/3}-x^{-7}$, find $f'(1)$. You don't have to simplify the answer.
   (b) If $g(x)=[3x^4+5][2-\ln(x^2)]$, find $g'(2)$. You don't have to simplify the answer.
   (c) Find $h'(x)$ if $h(x)=\dfrac{e^x-x^3}{x^3+\frac{1}{x}}$. You don't have to simplify the answer.
   (d) Find the value of dy if $y=\ln(x+1)$, $x=2$, and the change in x is 0.3.

4. [*6 marks*] A stock grew from \$35 to \$120,000 in 34 years. Assuming continuous compounding, what is the associated annual rate of growth?

5. [*10 marks*] The total profit (in dollars) from the sale of x charcoal grills is
$$P(x)=20x-0.02x^2-320 \qquad 0\le x\le 1{,}000$$
   (a) Find the average profit per grill if 40 grills are produced.
   (b) Find the marginal average profit at a production level of 40 grills and interpret.
   (c) Use the results from parts (a) and (b) to estimate the average profit per grill if 41 grills are produced.

PLEASE TURN OVER

6. [10 *marks*] Find x' for $x = x(t)$ defined implicitly by

$$t \ln x = xe^t - 1$$

and evaluate x' at $(t, x) = (0, 1)$.

7. [10 *marks*] The price p (in dollars) and demand x for a product are related by

$$2x^2 + 5xp + 50p^2 = 80,000.$$

If the price is increasing at a rate of \$2 per month, find the rate of change of the demand when the price is \$30.

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7 1a) $\lim_{x\to 3} \frac{x^2+4x+4}{x^2-x-6} = \frac{3^2+4(3)+4}{3^2-3-6} = \frac{25}{0}$ UNDEFINED

8 b) Let $g(x) = x-5$, $h(x) = \frac{1}{5}(x^2-25)$ $\lim_{x\to 5} \frac{\frac{1}{5}(x^2-25)}{(x-5)} = \lim_{x\to 5} \frac{\frac{1}{5}(\cancel{x-5})(x+5)}{(\cancel{x-5})} = \frac{1}{5}(5+5)=2$

2. $\lim_{s\to 0} \frac{h(x+s)-h(x)}{s}$ | $h(x) = x^3-5$

$h(x+s) = (x+s)^3 - 5$

7 $= \lim_{s\to 0} \frac{(x+s)^3-5-[x^3-5]}{s}$

$= \lim_{s\to 0} \frac{\cancel{x^3}+3x^2s+3s^2x+s^3-\cancel{5}-\cancel{x^3}+\cancel{5}}{s}$

$= \lim_{s\to 0} \frac{\cancel{s}(3x^2+3sx+s^2)}{\cancel{s}} = 3x^2+0+0 = 3x^2$

3 a) $f(x) = 3x^{4/3} - x^{-7} \Rightarrow f'(x) = 3 * \frac{4}{3}x^{\frac{1}{3}} - (-7)x^{-8} = 4x^{1/3} + 7x^{-8}$

$\Rightarrow f'(1) = 4(1) + 7 * \frac{1}{1} = 4+7 = 11$

b) $g(x) = [3x^4+5][2-\underbrace{\ln x^2}_{=2\ln x}] \Rightarrow g'(x) = [3x^4+5][0-2*\frac{1}{x}] + [2-\ln x^2][12x^3+0]$

$\Rightarrow g'(2) = [3(2)^4+5][-\frac{2}{2}] + [2-\ln 2^2][12(2)^3]$

4@3 c) $h(x) = \frac{e^x - x^3}{x^3+\frac{1}{x}} \Rightarrow h'(x) = \frac{(x^3+x^{-1})(e^x-3x^2)-(e^x-x^3)(3x^2-x^{-2})}{(x^3+x^{-1})^2}$

d) $y = \ln(x+1) \Rightarrow \frac{dy}{dx} = \frac{1}{x+1}(1) \Rightarrow dy = \frac{1}{x+1}dx$

If $x=2$ and change in x is $.3$ $(dx = .3)$

$dy = \frac{1}{2+1}(.3) \Rightarrow dy = .1$

4. $A = A_0 e^{rt}$

6 $120000 = 35e^{r(34)} \Rightarrow \frac{120\,000}{35} = e^{34r} \Rightarrow \log_e \frac{120\,000}{35} = 34r$

$\Rightarrow \frac{\ln \frac{120000}{35}}{34} = r \Rightarrow r = .239 \Rightarrow r = 23.9\%$

5. a) $\bar{P} = \frac{P}{x} = \frac{20x - .02x^2 - 320}{x} = 20 - .02x - \frac{320}{x}$

$\bar{P}(40) = 20 - .02(40) - \frac{320}{40} = 11.20\$$

b) $\bar{P}'(x) = \frac{d}{dx}(20 - .02x - 320x^{-1}) = 0 - .02 - 320(-1)x^{-2}$
$= .02 + \frac{320}{x^2}$

10 $\bar{P}'(40) = .02 + \frac{320}{40^2} = 0.22\$/\text{Art.}$

Price per Article (Average price) is increasing at Rate $.22\$/\text{Art}$ when we have sold 40 Articles

c) $\bar{P}(41) = \bar{P}(40) + \bar{P}'(40) = \$11.20 + \$.22 = 11.42\$$

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6. we are asked to find $\frac{dx}{dt}$. (Note: when $y = f(x)$ we can also write $y = y(x)$ and y' means $\frac{d}{dx}y$. So if $x = x(t)$ x' means $\frac{d}{dt}x$)

$\frac{d}{dt}(t\ln x) = \frac{d}{dt}xe^t - \frac{d}{dt}1$

$t\frac{d}{dt}\ln x + \ln x\frac{d}{dt}t = x\frac{d}{dt}e^t + e^t\frac{d}{dt}x - 0$

10 $t\frac{1}{x}\frac{d}{dt}x + \ln x(1) = xe^t\underbrace{\frac{d}{dt}t}_{=1} + e^t\frac{dx}{dt}$

$\frac{dx}{dt}\left[\frac{t}{x} - e^t\right] = xe^t - \ln x$

$\frac{dx}{dt} = \frac{xe^t - \ln x}{\frac{t}{x} - e^t}$

$\Rightarrow \left.\frac{dx}{dt}\right|_{\substack{t=0\\x=1}} = \frac{1e^0 - \ln 1}{\frac{0}{1} - e^0} = \frac{1-0}{0-1} = \frac{1}{-1} = -1$

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7. Step 1 List Rates $\frac{dp}{dt} = 2\$/\text{month}$

$\frac{dx}{dt} = ?$ Articles/month

Step 2 Equation with p, x is given

Step 3 do $\frac{d}{dt}$ of step 2 eq.

10 $\frac{d}{dt}(2x^2 + 5xp + 50p^2) = \frac{d}{dt}(80{,}000)$

$4x\frac{d}{dt}x + 5\left[x\frac{dp}{dt} + p\frac{dx}{dt}\right] + 50(2)p\frac{dp}{dt} = 0$

$\frac{dx}{dt}[4x + 5p] = -100p\frac{dp}{dt} - 5x\frac{dp}{dt}$

$\frac{dx}{dt} = \frac{\frac{dp}{dt}[-100p - 5x]}{4x + 5p}$

$\left.\frac{dx}{dt}\right|_{\substack{x=100\\p=30\\\frac{dp}{dt}=2}} = \frac{2[-100(30) - 5(100)]}{4(100) + 5(30)}$

$= -12.73$ Articles/month

when p 30 Find x

$2x^2 + 5xp + 50p^2 = 80000$

$2x^2 + 5x(30) + 50(30)^2 = 80000$

$2x^2 + 150x + 45000 - 80000 = 0$

$2x^2 + 150x - 35000 = 0$

$x^2 + 75x - 17500 = 0$

$(x-100)(x+175) = 0$

$x = 100$ | $x = -175$