MATH 1009 - E TEST 1 Solutions

PART A: MULTIPLE CHOICE QUESTIONS.

2 marks are given for the correct answer, **0** - otherwise. (d, a, d, b, b, b, c, a.)

A1. The domain of the function $f(x) = \dfrac{1}{\sqrt{4-x}}$ is

(a) $[-2, 2]$, or, equivalently, $\{-2 \le x \le 2\}$.
(b) $(4, \infty)$, or, equivalently, $\{x > 4\}$.
(c) $[4, \infty)$, or, equivalently, $\{x \ge 4\}$.
(d) $(-\infty, 4)$, or, equivalently, $\{x < 4\}$.
(e) $(-\infty, 2)$, or, equivalently, $\{x < 2\}$.

Answer: (d)

A2. The domain of the function $f(x) = e^{-x}$ is

(a) $\mathbb{R}$, or, equivalently, $(-\infty, \infty)$.
(b) $(0, \infty)$, or, equivalently, $\{x > 0\}$.
(c) $[0, \infty)$, or, equivalently, $\{x \ge 0\}$.
(d) $(-\infty, 0)$, or, equivalently, $\{x < 0\}$.
(e) $(-\infty, 0]$, or, equivalently, $\{x \le 0\}$.

Answer: (a)

A3. The expression $(5^{2/3})^{-3/4} \cdot 5^{1/2}$ evaluates to

(a) $5^{-1/24}$. **(b)** $5^{1/4}$. **(c)** 5. **(d)** 1 . **(e)** None of the above.

Answer: (d)

A4. The domain of the function $f(x) = \log_2(x - 1)$ is

(a) $[1, \infty)$, or, equivalently, $\{x \ge 1\}$.
(b) $(1, \infty)$, or, equivalently, $\{x > 1\}$.
(c) $\mathbb{R}$, or, equivalently, $(-\infty, \infty)$.
(d) $(-1, \infty)$, or, equivalently, $\{x > -1\}$.
(e) $[-1, \infty)$, or, equivalently, $\{x \ge -1\}$.

Answer: (b)

A5. The expression $\dfrac{(a^2 \cdot a^{-1.4})^3}{a^{0.8}}$ simplifies to

(a) $a^{-0.6}$. (b) a. (c) a^2. (d) $a^{2.8}$. (e) none of the above.

Answer: (b)

A6. The expression $(16x^4y^6)^{1/2}$ simplifies to

(a) $4x^2y^4$. (b) $4x^2y^3$. (c) $\dfrac{16x^2y^4}{2}$. (d) $16x^2y^3$. (e) None of the above.

Answer: (b)

A7. The expression $\log_{\frac{1}{3}} 81$ evaluates to

(a) -27. (b) 27. (c) -4. (d) 4. (e) not defined.

Answer: (c)

A8. Write the expression $(\ln x - \ln 5 + 3\ln y)$ as the logarithm of a single quantity.

(a) $\ln\left(\dfrac{xy^3}{5}\right)$. (b) $\log_5\left(\dfrac{x}{y^3}\right)$. (c) $\ln(x - 5 + 3y)$. (d) $\ln\left(\dfrac{3xy}{5}\right)$.

(e) None of the above.

Answer: (a)

PART B: LONG STYLE QUESTIONS.

[**4 marks**] **B1**. The weekly demand and supply equations for a company are given by $p = -x^2 + 60$ and $p = 5x + 36$, respectively, where p is the price measured in dollars and x is measured in units **of a thousand**.

[**1**] **(a)** For the demand equation $p = -x^2 + 60$, determine the quantity demanded, when the price is set at 11 dollars.

$$11 = -x^2 + 60, \quad x^2 = 49, \quad x = \pm 7,$$

but we reject the negative root, so $x = 7$ thousand.

[**3**] **(b)** Find the equilibrium quantity and price.

At the equilibrium point, the supply is equal to the demand, and therefore

$$-x^2 + 60 = 5x + 36.$$

Solving this equation for x yields $x^2 + 5x - 24 = 0$, $x_{1,2} = \dfrac{-5 \pm 11}{2}$, $x_1 = 3$, $x_2 = -8$. We reject the negative root $x = -8$, since positive values of x demanded are meaningful. Thus, the equilibrium quantity is 3 thousand units, and the corresponding price is

$$p = 5 \cdot 3 + 36 = 51$$

dollars.

[12 marks] B2. Solve each of the following equations for x.

[4] **(a)** $e^{2x-1} = 7$ [4] **(b)** $\log_{24}(10 - x^2) = 0$ [4] **(c)** $\ln 5 - \ln x + 3\ln 2 = 1$

Solution:

NOTE: There is more than one way of solving each of the equations.

(a) Take the natural logarithm of both sides of the equation and use the laws of logarithms:

$$\ln(e^{2x-1}) = \ln 7 \quad \Rightarrow \quad 2x - 1 = \ln 7 \quad \Rightarrow \quad x = \frac{\ln 7 + 1}{2}.$$

(b) Since the logarithm equals zero, the argument must be equal to 1:

$$10 - x^2 = 1 \quad \Rightarrow x^2 = 9 \quad \Rightarrow x = \pm 3.$$

(c) Simplify the LHS of the equation and then exponentiate both sides of the relation:

$$\ln\left(\frac{5 \cdot 2^3}{x}\right) = 1 \quad \Rightarrow \quad \ln\left(\frac{40}{x}\right) = 1 \quad \Rightarrow \quad e^{\ln\left(\frac{40}{x}\right)} = e \quad \Rightarrow \quad \frac{40}{x} = e \quad \Rightarrow \quad x = \frac{40}{e}.$$

[8 marks] B3. The amount of \$20,000 is deposited in a bank that pays interest at the rate of 6% per year compounded **semiannually**. Using the compound interest formula

$$A(t) = P\left(1 + \frac{r}{m}\right)^{mt},$$

answer the following questions.

[2] **(a)** What is the accumulated amount on deposit in 5 years? (round to two decimals)

$$A(5) = 20,000\left(1 + \frac{0.06}{2}\right)^{2 \cdot 5} = 20,000(1.03)^{10} = 26,878.33$$

[1] **(b)** What is the interest earned in 5 years? (two decimals)

$$\text{Interest } I(5) = A(5) - P = 26,878.3 - 20,000 = 6,878.33$$

[5] **(c)** How long will it take to double the investment? (one decimal)

$$40,000 = 20,000\left(1 + \frac{0.06}{2}\right)^{2 \cdot t} \;\rightarrow\; 2 = (1.03)^{2t} \;\rightarrow\; \ln 2 = 2t\ln(1.03) \;\rightarrow\; t = \frac{\ln 2}{2\ln 1.03} = 11.7$$