ECOR 1010
Lecture 4

Measurements, Units and Errors

SI Magnitude Prefixes

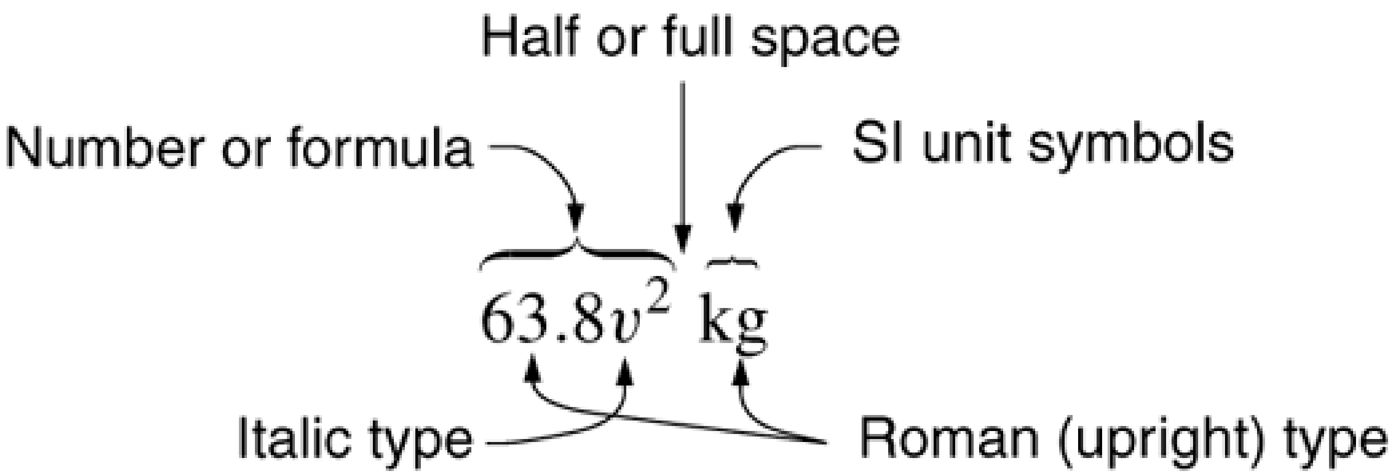
- The unit may be preceded by a magnitude prefix;
  - The prefix is regarded as an inseparable part of the symbol.

Table 10.3 Magnitude prefixes in the SI unit system by symbol, name, and numerical value.

Y	yotta	10^{24}	M	mega	10^{6}	f	femto	10^{-15}
Z	zetta	10^{21}	k	kilo	10^{3}	a	atto	10^{-18}
E	exa	10^{18}	m	milli	10^{-3}	z	zepto	10^{-21}
P	peta	10^{15}	μ	micro	10^{-6}	y	yocto	10^{-24}
T	tera	10^{12}	n	nano	10^{-9}			
G	giga	10^{9}	p	pico	10^{-12}			

Writing Quantities With Units

- Standardized rules for correctly representing quantities with units:



A physical quantity is written as a number or a formula representing a number, followed be unit symbols.

Rules for Writing With Units - 1

- Always a space between numeric value and unit:
  - 23.67 kg   not 23.67kg
- Unit symbols are treated like algebraic factors that can be multiplied or divided with numbers or other unit symbols:
  - Multiplication between units is indicated by a centred dot or a half-space,
  - Division by a slash or a negative superscript:
    - 27 kg·m/s^2 or 27 kg m/s^2 or 27 kg·m·s^{-2}

Rules for Writing With Units - 2

- Unit symbols are not modified with subscripts:
  - V_{max} = 200 V   not V = 200 V_{max}.
- Relation between numerical quantity and unit symbols must be clear:
  - 3.7 km × 2.8 km for an area; not 3.7 × 2.8 km
  - 87.2 g ± 0.4 g or (87.2 ± 0.4) g; but not 87.2 ± 0.4 g

Rules for Writing With Units - 3

- Unit symbols not mixed with unit names in sentences:
  - Use kg/m^2 or kilogram per square metre, but not $kilogram/m^2$.
- Long numbers, written in groups of 3, the groups should be separated by a non-breaking half space rather than a comma or period:
  - 93 000 000 miles   not 93,000,000 miles
  - 21 298.046 83 m   not 21298.04683 m

Common Units

	SI	FPS
Force	newton (N) = 1 kg·m/s^2	pound (lb)
Length	metre (m)	foot (ft)
Mass	kilogram (kg)	slug = 1 lb·sec^2/ft
Time	s	sec
Pressure	pascal (Pa) = 1 N/m^2	psi (lbs/in^2) or lbs/ft^2
Work & Energy	joule (J) = N·m	ft·lb
Power	watt (W) = N·m/s	horsepower HP = 550 ft·lb/sec
Temperature	K T(K) = T(°C)+273.15	°F T(°F) = 9/5 T(°C)+32

Fixed, Scientific and Engineering Notation

- Fixed notation
  - Ordinary decimal notation
    - e.g. 30140.0,
- Scientific notation
    - 3.014×10^4,
- Engineering notation
  - Exponent of 10 is a multiple of 3 to correspond to SI prefixes
    - e.g. 30.14×10^3.

Unit Algebra - 1

- Algebra can be performed on units,
  - with restriction that only quantities with same units can be equated, added, or subtracted.
- Consider conversion from one unit to another
  – 20 000 ft·lb/min = ? HP

$$20\,000\,\frac{\text{ft}\cdot\text{lb}}{\text{min}}\times\frac{1\,\text{min}}{60\,\text{s}}\times\frac{1\ \text{HP}}{550\ \text{ft}\cdot\text{lb/s}}=0.606\ \text{HP}$$

Unit Algebra - 2

- Consider the equation $s = vt + (1/2)at^2$, where
  - v is the initial velocity in m/s,
  - t is time in s,
  - a is acceleration in m/s^2.
- Then s has units (m/s)s + (m/s^2)s^2 = m + m
  - Since we can add quantities with same units,
    - s has units m.
    - s is a measure of length.

Dimensions

- Dimensions differ slightly from units
  - e.g., Speed
    - Dimensions = distance / time.
    - Units may be km/hr, or mm/s, etc.
- Fundamental dimensions
  - Distance, time, mass etc.

Derived Dimensions

- Products or ratios of basic dimensions

Acceleration: $$a = \frac{[l]}{[t]^2}$$

Force: $$f = ma = \frac{[m][l]}{[t]^2}$$

Dimensional Homogeneity: The dimensions on either side of an equation must be the same

→ Dimensions of f = dimensions of m times a

Derived Dimensions with Units (Pressure)

$$\text{Pressure} = \text{Force per area} \;\rightarrow\; \frac{[f]}{[l^2]}$$

Units:

$$\frac{\text{N}}{\text{m}^2} = \text{Pascal (Pa) in the SI system}$$

$$\frac{\text{lb}}{\text{in}^2} = \text{psi in the FPS system}$$

Derived Dimensions with Units (Work)

$$\text{Work} = \text{ force 'dot' displacement} = f \bullet d$$

$$\rightarrow [f][l]$$

Units:

$$\text{N} \cdot \text{m} = \text{Joules (J)}$$

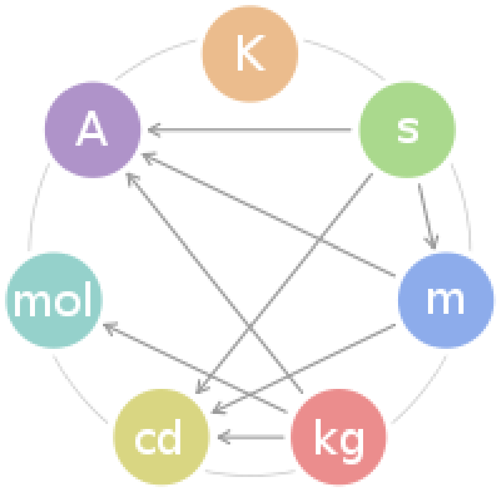
Derived Dimensions with Units (Power)

$$\text{Power} = \frac{\text{work}}{\text{time}} \rightarrow \frac{[f][l]}{[t]}$$

Units:

$$\frac{\text{N} \cdot \text{m}}{s} = \text{J/s} = \text{Watts (W)}$$

Derived Dimensions with Units (Charge)



$$Q = \int I \ dt$$

$$Units : \text{Coulomb} = \text{Ampere} \bullet \text{second}$$

6.241506×10^{18} electrons

Non-Dimensional Numbers

- Ratios where the numerator has the same dimension as the denominator

$$\text{Aspect Ratio } (\text{AR}) \text{ of an aircraft wing } = \frac{(\text{Wing Span})^2}{\text{Wing Area}}$$

$$AR = \frac{[l]^2}{[l]^2} = [1]$$

Unit Consistency (Dimensional Analysis)

- When performing calculations, numbers should always be written with correct units
- Units should be carried throughout the problem
- Very useful check!

The Gimli Glider

"If a Boeing 767 runs out of fuel at 41,000 feet, what do you have?   A 132 ton glider …"

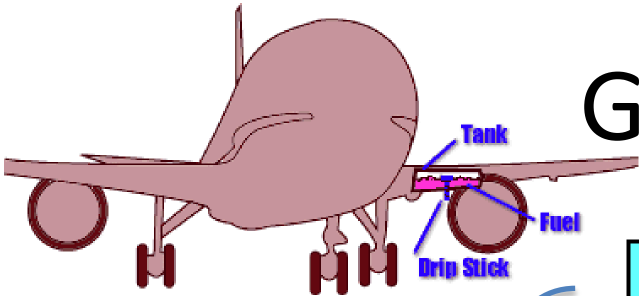
- Wade H. Nelson (Soaring Magazine, 1997)

Air Canada Flight 143: The Gimli Glider

The Scenario

- There was a problem with the fuel gauges, so they calculated the volume of fuel they needed *by hand*
- Previously there was a flight engineer on board, but not on these new planes.
- Among other things, the specific gravity of fuel is needed to make the proper calculations
  - They used 1.77 lb/L as the constant
  - They should have used 0.8 kg/L for this all-metric 767 aircraft!

Recall 2.2 lb =1 kg



Gimli Glider: What Happened

What they did

7,682 L x 1.77 = 13,597 kg of fuel remaining on board

22,300 kg needed – 13,597 kg on board = 8,703 kg to be added

8,703 kg ÷ 1.77 = 4,916 L of fuel to be added

If they had kept track of the units and verified that the units canceled properly, they could have calculated:

What they should have done

$$7{,}682\ \text{L} \times \frac{0.803\ \text{kg}}{\text{L}} = 6{,}169\ \text{kg remaining on board}$$

22,300 kg needed – 6,169 kg on board = 16,131 kg to be added

$$16{,}131\ \text{kg} \times \frac{\text{L}}{0.803\ \text{kg}} = 20{,}163\ \text{L of fuel to be added}$$

Sideslipping a 767

Aircraft too high. Speed slowed to 180 knots

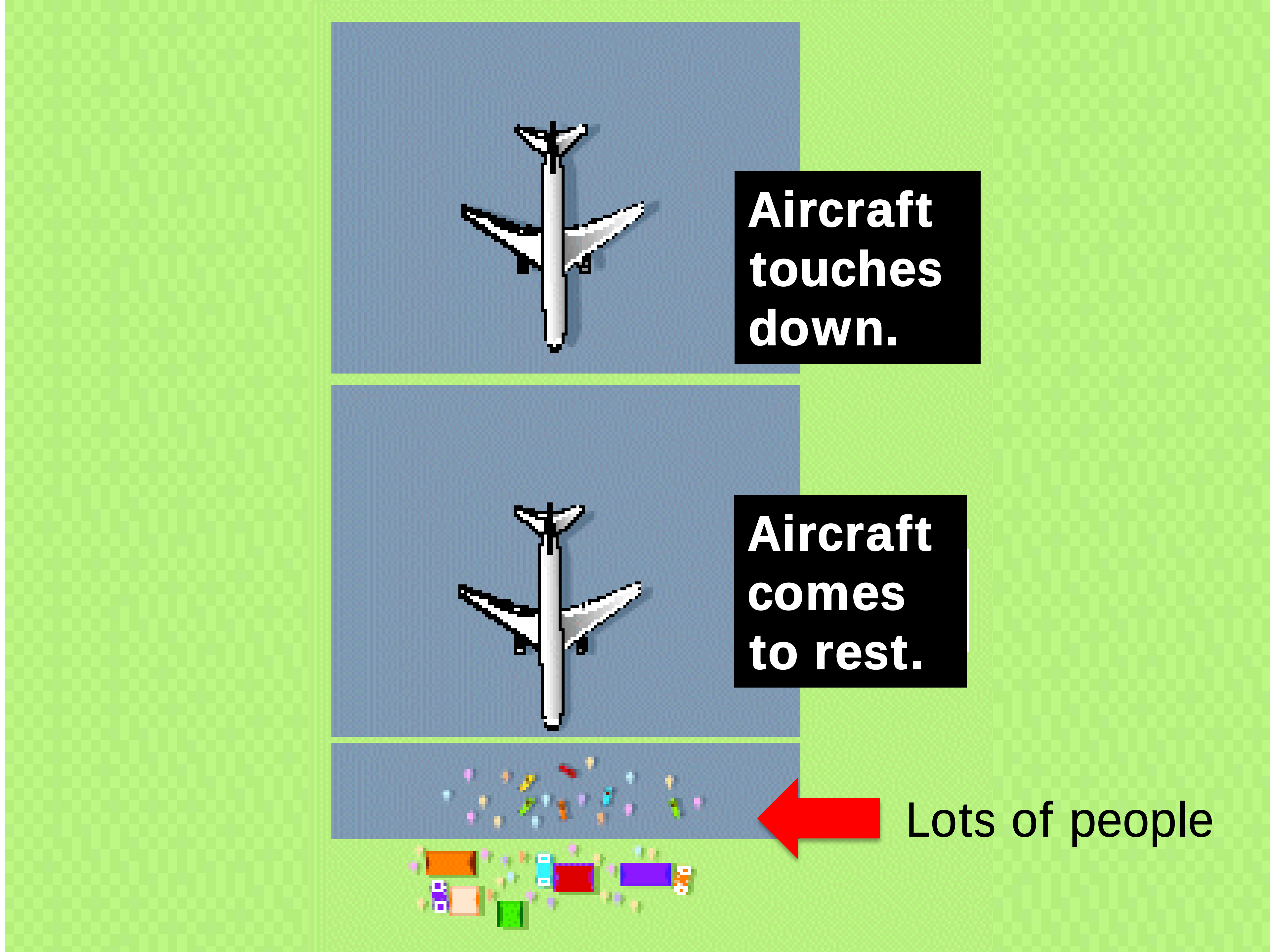
Pilot initiates sideslip: left rudder pedal pushed while he turns the yoke to the right. Aircraft manoeuvred into a steep angle. Aircraft rapidly loses altitude.

Sideslipping a 767

Aircraft too high. Speed slowed to 180 knots

Pilot initiates sideslip: left rudder pedal pushed while he turns the yoke to the right. Aircraft manoeuvred into a steep angle. Aircraft rapidly loses altitude.

Aircraft straightens out at an altitude of 40 feet.


Aircraft touches down.
Aircraft comes to rest.
Lots of people

Gimli Glider

Measurements

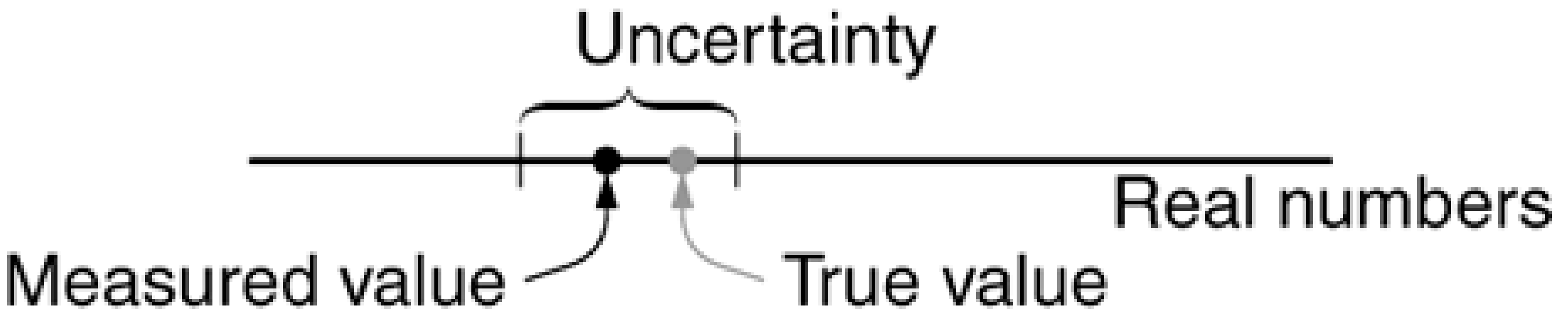
- Engineering students should know
  - Definition of traceable measurement;
  - Classification of uncertainties into systematic and random effects;
  - Correct use of words *accuracy, precision, bias;*
  - How to write inexact quantities using engineering and other notation;
  - The correct use of significant digits.

Measurement

- Measurement:
  - Property of a physical object that can be represented using a real number.
    - As opposed to counting, measurement is not exact.
  - Quantities usually measured with an instrument.
    - Calipers, scale, measuring cup etc.

Uncertainty

- Measuring instruments are not perfect;
  - Measurements typically yield
    - Estimate of true value, and
    - An interval around estimate in which true value lies.
    - This is called the uncertainty interval.



Illustrating a true (exact) value, which is a point on the line of real numbers, and its measured estimate, together with an interval of uncertainty.

True value – measured value = measurement error.

Systematic Errors

- Consistent deviation from the true value;
  - also called a *bias* or an *offset.*
- Error has same magnitude and sign when repeated measurements are made under same conditions.
- Can be detected by
  - Calibration,
  - Comparison with results obtained with an independent method

Three Types of Systematic Errors

- Natural Error
  - From environmental effects;
    - e.g. temperature changes affect electronic components and measuring instruments.
  - Correction factor can be applied to compensate if possible.
- Instrument Error
  - Also called *offset*.
  - Caused by imperfections in adjustment or construction of instrument.

Types of Systematic Errors

- Personal Error
  - Result from habits of the observer.
  - Can be reduced by proper training.

Random Errors

- Result of small variations in measurements
  - e.g. in repeated measurements, an observer may
    - exert slightly different pressures on a micrometer
    - connect voltmeter leads in slightly different locations.
- Random errors do not bias measurement,
  - They produce both positive and negative errors with zero mean value.
- If several measurements are made, their mean is normally a better estimate of true value than any single measurement.

Precision and Accuracy - 1

- Precision of a measurement
  - Repeatability.
    - Precise measurements have small random error
    - Discrepancies between repeated measurements taken under the same conditions are small.
- Accuracy
  - Accurate measurements are close to true value
    - Some authors use term to describe total error
    - Others to describe systematic error.
      - Accurate measurement has little bias.

Precision and Accuracy - 2

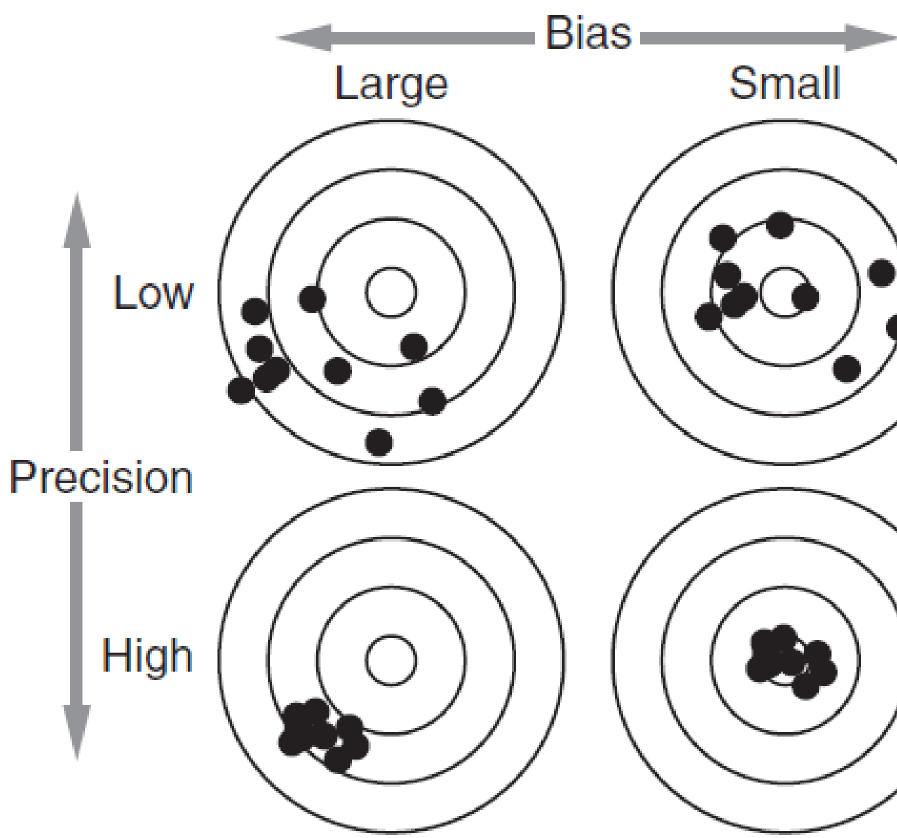


Figure 11.3 Precision, bias, and accuracy: The two right patterns have small bias (small systematic error); the two bottom patterns have high precision (small random error). The lower right pattern is accurate; ~~less commonly~~ the two right patterns would be said to be accurate.

Estimating Measurement Error - 1

- When systematic errors are quantified, the measurement can be corrected to remove bias.
  - The offset is subtracted from measurement, or
  - The instrument is recalibrated.
- Random error remains
  - Must be estimated and quoted with measurement.

Estimating Measurement Error - 2

- Source of random errors
  - Technical specifications of instrument
  - Reading errors.
    - e.g., Estimating position of needle of a gauge or meter requires interpolation between scale markings,
      - Precision of measurement is limited by ability of observer to resolve small distances.
      - Precision of the readings will be a fraction of the amount between scale markings, typically one-tenth.

How to Write Inexact Quantities

- Measurement is complete only when a statement about its uncertainty is included.
- Uncertainty is normally expressed as an estimated range;
  - Can be stated explicitly or implicitly.

Explicit Uncertainty Notation

- Measurement may be expressed in fixed, scientific, or engineering notation, and
- Uncertainty may be given in either absolute or relative form.
  - Fixed notation with error in absolute form
    - v_s = 343.5 m/s ± 0.9 m/s = (343.5 ± 0.9) m/s
  - Fixed notation with error in relative form
    - v_s = 343.5(1 ± 0.26%) m/s.

Implicit Uncertainty Notation

- If a quantity is written without explicit uncertainty, uncertainty is taken to be ±5 in digit immediately to the right of least significant digit.
- Suppose measured speed is 94 km/h
  - Without more information, the error is ±0.5 km/h
- Indicated speed is 94 km/h
  - Error from speedometer graduations is ±1 km/h
  - >100 km/h error is ±2.5 km/h

Significant Digits

- Determine the precision of the number.
  - Should not use more significant digits than justified by the measurement.
- Significant digits are identified as follows:
  - All non-zeros are significant,
  - All zeros between significant digits are significant.
  - Leading zeros are not significant.
  - Trailing zeros are significant in fractional part of number.

Significant Digits

- Examples
  - 0.0350 oz has 3 significant digits
  - 90 000 000 miles has 1 significant digit, (ambiguous)
  - 92 900 000 miles has 3 significant digits, (ambiguous)
  - (92 900 000 ± 500) miles has 5 significant digits

Significant Digits

- In scientific notation, integer part is normally one non-zero digit,
  - Number of significant digits is one more than number of digits after decimal point.
    - 9×10^7 = 1 significant digit
    - 9.3×10^7 = 2 significant digit
    - 9.290×10^7 = 4 significant digits
- In engineering notation, all digits except leading zeros are significant
    - 0.09×10^9 miles = 1 significant digit
    - 93×10^6 miles = 2 significant digits
    - 92.90×10^6 miles = 4 significant digits

Rounding Numbers

- Rounding is used to reduce number to the appropriate significant digits.
  - When leftmost discarded digit is less than 5, then retained digits are unchanged;
    - 3.234 → 3.23 or 3.2
  - When leftmost discarded digit greater than 5, or is 5 with at least one nonzero digit to its right, increase rightmost retained digit by one;
    - 3.256 → 3.26 or 3.3
    - 9.747 → 9.75 or 9.7 *not 9.8*

Rounding Numbers

- When the leftmost discarded digit is 5 and all following digits are 0, then rightmost retained digit unchanged if it is even; otherwise it is increased by one.
  - 3.25 → 3.2
  - 3.250 → 3.2
  - 3.350 → 3.4
    - This rule avoids introducing a bias into the mean of a set of rounded numbers.

Effect of Algebraic Operations

- Computations performed using inexact measured quantities yield inexact results.
  - Final computed value should not have more significant digits than are justified;
    - Must be rounded to imply correct interval of uncertainty.

Absolutely NEVER give all the digits you get out of your calculator or a program such as Excel:
unless you can defend all of them!!

Addition and Subtraction of Inexact Quantities

- $(x \pm \Delta x) + (y \pm \Delta y)$
  - Max: $(x + \Delta x) + (y + \Delta y) = (x + y) + (\Delta x + \Delta y)$
  - Min: $(x - \Delta x) + (y - \Delta y) = (x + y) - (\Delta x + \Delta y)$
- Therefore

$$(x \pm \Delta x) + (y \pm \Delta y) = (x + y) \pm (\Delta x + \Delta y)$$

- Similarly

$$(x \pm \Delta x) - (y \pm \Delta y) = (x - y) \pm (\Delta x + \Delta y)$$

Addition and Subtraction of Inexact Quantities

- For addition or subtraction, we add absolute uncertainties.
  - (3.5 ± 0.4) m + (1.22 ± 0.03) m = (4.72 ± 0.43) m becomes (4.7 ± 0.4) m.
  - (3.93 ± 0.08) kg - (3 ± 0.1) kg = (0.93 ± 0.18) kg becomes (0.9 ± 0.2) kg.

Multiplication and Division Using Inexact Quantities

– For multiplication, we add relative uncertainties:

$$
\begin{aligned}
(x \pm \Delta x) \times (y \pm \Delta y) &= xy \pm y\Delta x \pm x\Delta y + \Delta x \Delta y \\
&= xy\left(1 \pm \frac{\Delta x}{x} \pm \frac{\Delta y}{y} + \frac{\Delta x}{x}\frac{\Delta y}{y}\right) \quad \frac{\Delta x}{x}\frac{\Delta y}{y} \approx 0 \\
&\approx xy\left(1 \pm \frac{\Delta x}{x} \pm \frac{\Delta y}{y}\right)
\end{aligned}
$$

– Similarly, we add relative uncertainties for division.

Multiplication and Division Using Inexact Quantities

(4.0 ± 0.5) kg $\times$ (2.0 ± 0.2) m/s^2 = $4(1\pm0.5/4.0)$ kg $\times$ $2(1\pm0.2/2.0)$ m/s^2

= $4(1\pm0.125)$ kg $\times$ $2(1\pm0.1)$ m/s^2

= $8(1\pm0.225)$ N + higher order terms

≈ (8 ± 1.8) N

≈ (8 ± 2) N

(4.0 ± 0.5) N / (2.0 ± 0.2) m^2 = $4(1\pm0.5/4.0)$ N / $2(1\pm0.2/2.0)$ m^2

= $4(1\pm0.125)$ N / $2(1\pm0.1)$ m^2

= $2(1\pm0.225)$ Pa + higher order terms

≈ (2 ± 0.45) Pa

≈ (2 ± 0.5) Pa

Multiplication and Division Using Inexact Quantities

Product of 1.1, 1.2, 1.3, 2.4 ? (Implied uncertainty is ±0.05)

Calculator product: 1.1×1.2×1.3×2.4 = 4.1184

Product rounded to two significant digits = 4.1

Product rounded with uncertainty is between 4.05 and 4.15

But, what if all errors are -0.05 ? Product = 3.5470

or, what if all errors are +0.05 ? Product = 4.7545

Worst case is much worse than 4.05 to 4.15 !

Should you give the answer as 4, with implied uncertainty of ±0.5?

Multiplication and Division Using Inexact Quantities

Product of 1.1, 1.2, 1.3, 2.4 ? (Implied uncertainty is ±0.05)

Consider propagation of errors from multiplying:

$(1.1\pm0.05)\times(1.2\pm0.05)\times(1.3\pm0.05)\times(2.4\pm0.05)$

$=1.1(1\pm0.05/1.1)\times1.2(1\pm0.05/1.2)\times1.3(1\pm0.05/1.3)\times2.4(1\pm0.05/2.4)$

$= 4.1184\ (1\pm0.05/1.1)\times(1\pm0.05/1.2)\times(1\pm0.05/1.3)\times(1\pm0.05/2.4)$

$= 4.1184\ (1\pm0.05/1.1\pm0.05/1.2\pm0.05/1.3\pm0.05/2.4)\ +\ \ldots$

$\approx 4.1184\ (1\pm0.146) = 4.1 \pm 0.6$

This is a worst case error estimate. Is this always justified?

Reading Assignment

- Read Chapter 11