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**Specification for Assignment 2 of 4**

*Your submission must be created using Microsoft Word, Google Docs, or LaTeX.*

*Your submission must be saved as a "pdf" document and have the name "a2.pdf"*

*Do not compress your submission into a "zip" file.*

*Late assignments will not be accepted and will receive a mark of 0.*

*Submissions written by hand, compressed into an archive, or submitted in the wrong format (i.e., are not "pdf" documents) will receive a mark of 0.*

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***The due date for this assignment is February 11<sup>th</sup>, 2017, by 11:30pm.***

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1. Determine whether or not the following arguments are valid. If they are valid, then state the rules of inference used to prove validity. If they are invalid, outline precisely why they are invalid.
  - a. If it is raining then I bring my umbrella to work. I bring my umbrella. Therefore it must be raining.
  - b. Everyone who has a PC plays computer games. Everyone student taking COMP1501 next semester plays computer games. Therefore every student taking COMP1501 next semester has a PC.
2. Prove that  $\sqrt{4} + \sqrt{5}$  is an irrational number.
3. Prove, by indirect proof, that if  $n$  is an integer and  $n^5+7$  is odd, then  $n$  is even. Show all your work.
4. Find the error in the following proof that every positive integer equals the next largest positive integer: "Proof: Let  $P(n)$  be the proposition ' $n=n+1$ '. Assume that  $P(n)$  is true, so that  $n=n+1$ . Add 1 to both sides of this equation to obtain  $n+1=n+2$ . Since this is the statement  $P(n+1)$ , it follows that  $P(n)$  is true for all positive integers  $n$ ."
5. For integer  $x$ , such that  $-2 \leq x \leq 2$ , prove that  $y < 0$ , where  $y = x^4 - 4x^2 - 9x - 36$ .
6. Prove by induction that  $1+3+5+\dots+(2n-1) = n^2$ , for all positive integers  $n$ .
7. Prove that any integer  $n \geq 24$  can be expressed as  $n = 5x + 7y$ , where  $x$  and  $y$  are nonnegative integers.

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8. What is the power set of  $\{0, 3, 9\}$  ?
9. List explicitly the members of the following sets.
- $\{i \mid 0 \leq i < 20 \text{ and either } i \text{ begins with either a vowel or the letter 't'}\}$
  - $\{j \mid j \text{ is the first name of a COMP1805 teaching assistant that starts with a vowel}\}$
10. Let  $S = \{1, 5, \{1,5\}, \{\text{apples, bananas}\}, \{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}\}$  and  $T = \{\{1, 5, 10\}, \{5\}, 1, \{\text{fruits}\}, \text{cats}, \{\text{dogs}\}\}$  and answer the following questions.
- Which set has the larger cardinality?
  - What is the intersection,  $S \cap T$ ?
  - What is the cardinality of the union  $S \cup T$ ?
11. Determine whether or not the following is valid. Justify your answer by using membership tables.

$$(A \cup B \cup C) - B - (C \cap A) = (A - B) \cup (B - A)$$

12. Draw the Venn Diagrams for the following set:

$$(A \cup B \cup C) - B - (C \cap A) = (A - B) \cup (B - A)$$

13. What is the intersection,  $A$ , of the set of all the digits that appear in your student number,  $B$ , and the set of all even numbers,  $C$ . Draw the Venn Diagram for sets  $A$ ,  $B$ , and  $C$ .