NAME:________________________ **Student No.**_______________

DEPARTMENT OF MECHANICAL & INDUSTRIAL ENGINEERING
CONCORDIA UNIVERSITY
MECH 375/1-CC: Mechanical Vibrations

CLASS TEST — 29 July 2013
DURATION: 90 Minutes — **8 PAGES** — **Max. Mark: 100**

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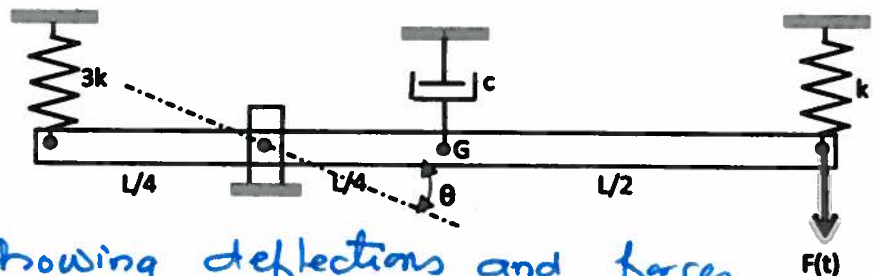
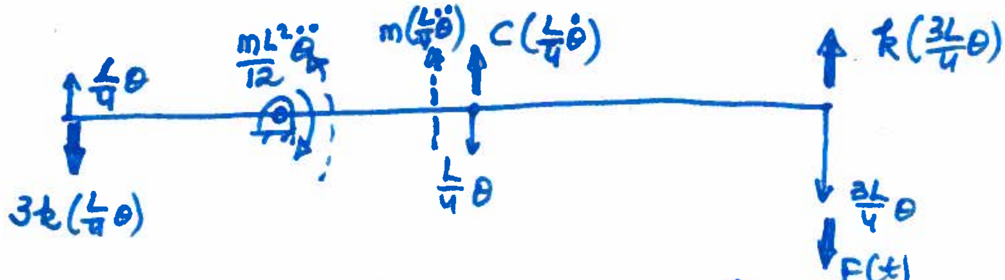
QUESTION # 1 (25)

For the uniform rigid bar of mass m and $I_G = mL^2/12$, compute the following, given that m=5 kg; k=6220 N/m; L=0.6 m; c=12Ns/m; F(t)= 120 cos(20πt) N:

a. Equivalent stiffness and inertia with generalized coordinate being bar rotation θ about the pivot.
b. Equation of motion of the system
c. Critical damping constant
d. Natural frequency and damping ratio of the system.



FBD showing deflections and forces



Σ moments = Σ moments due to ext. force

$$3k\left(\frac{L}{4}\theta\right)\left(\frac{L}{4}\right) + k\left(\frac{3L}{4}\theta\right)\left(\frac{3L}{4}\right) + c\left(\frac{L}{4}\dot\theta\right)\left(\frac{L}{4}\right) + m\left(\frac{L}{4}\ddot\theta\right)\left(\frac{L}{4}\right) + \frac{mL^2}{12}\ddot\theta = F(t)\cdot\left(\frac{3L}{4}\right)$$

$$\frac{3}{4}kL^2\theta + c\frac{L^2}{16}\dot\theta + \left(\frac{mL^2}{16} + \frac{mL^2}{12}\right)\ddot\theta = \frac{3L}{4}F(t)$$

$$\frac{7}{48}mL^2\ddot\theta + c\frac{L^2}{16}\dot\theta + \frac{3}{4}kL^2\theta = \frac{3L}{4}F(t)$$

a) $k_{eq} = \frac{3}{4} kL^2 = \frac{3}{4}(6220)(\cdot 6)^2 = 1679.4 \text{ Nm/rad}$

$m_{eq} = \frac{7}{48} mL^2 = \frac{7}{48}(5)(\cdot 6)^2 = 0.2625 \text{ kg m}^2$

b) Eqn. of motion $\qquad cL^2/16 = 12 \times \frac{\cdot 6^2}{16} = 0.27$

$$\cdot 2625\,\ddot{\theta} + 0.27\,\dot{\theta} + 1679.4\,\theta = 54 \cos(20\pi t)$$

c) $C_c = 2\sqrt{k_{eq} m_{eq}} = 42 \text{ Nms/rad}$

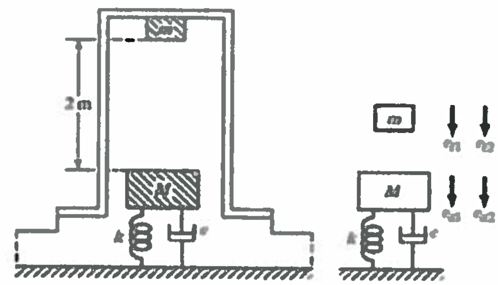
d) $\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{1679.4}{\cdot 2625}} = 79.99 \text{ rad/s}$

$\zeta = \frac{C_{eq}}{c_c} = \frac{0.27}{42} = 0.0064.$

QUESTION # 2 (25)

The anvil of the forging hammer, shown below, is supported on an elastic support with vertical stiffness of 500,000 N/m and critical damping constant of 20,000 Ns/m. The log decrement of the support was measured as 0.75. During the forging operation, the hammer of mass m is made to drop on the anvil from a height of 2 m. The hammer, however, bounces back immediately after the impact with the anvil mass M and it causes a velocity of the anvil mass M of 2.5 m/s.

a. Determine the vibration response of M
b. Maximum displacement of M



Eqn. of motion

$$M\ddot{x} + c\dot{x} + kx = 0$$

$$\dot{x}(0) = 2.5 \text{ m/s} \qquad x(0) = 0$$

Given $\quad \delta = 0.75 \approx 2\pi\zeta \longrightarrow \zeta = 0.1194$

Alternatively $\quad \delta = \dfrac{2\pi\zeta}{\sqrt{1-\zeta^2}} \; ; \quad \zeta = \sqrt{\dfrac{\delta^2}{\delta^2 + 4\pi^2}} = 0.1185$

Let $\zeta = 0.119$

$C_c = 20{,}000$ Ns/m (Given)

$C = C_c * \zeta = 20{,}000 \times 0.119 = 2{,}380$ Ns/m

Also $\quad C_c = 2\sqrt{kM} \; ; \quad M = \dfrac{C_c^2}{4k} = \dfrac{20000^2}{4 \times 500{,}000}$

$M = 200$ kg

$\therefore \quad \omega_n = \sqrt{\dfrac{k}{M}} = \sqrt{\dfrac{500{,}000}{200}} = 50$ rad/s

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 50\sqrt{1-.119^2} = 49.645$ rad/s

a) Free Vibration Response

$$x(t) = e^{-\zeta\omega_n t}\left[A\cos\omega_d t + B\sin\omega_d t\right]$$

$x(0) = 0$ yields $A = 0$

$\dot{x}(0) = 2.5\ m/s$ yields $B = \dfrac{2.5}{\omega_d} = 0.05036$

$\zeta\omega_n = 0.119 \times 50 = 5.95$

$$\therefore\ x(t) = 0.05036\, e^{-5.95t}\sin 49.645t, \ m$$

b) Maximum displacement response occurs when

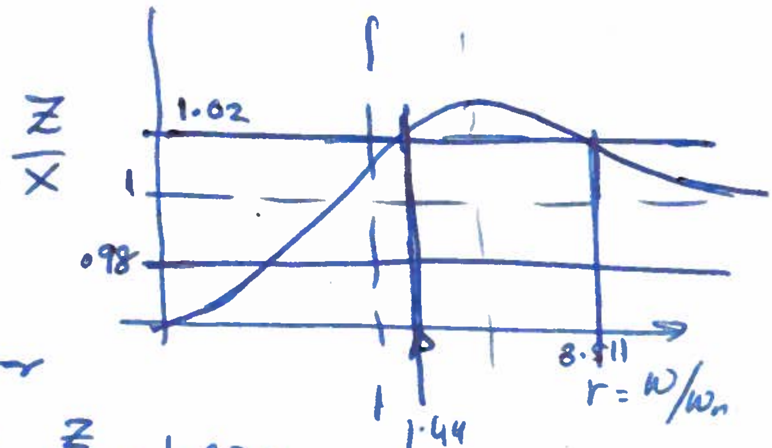
$$49.645t = \pi/2; \quad t = 0.0316$$

$$X_{max} = 0.05036\, e^{-5.95\times.0316} \underline{\underline{= 0.0417\ m}}$$

QUESTION # 3 (15)
The undamped natural frequency of a vibrometer is 10 Hz and its frequency of damped oscillations is reported as 8 Hz. Determine the lowest frequency so that it can measure the displacement of a structure with less than 2 percent error.

Given $f_n = 10\ Hz$; $f_d = 8 = f_n\sqrt{1-\zeta^2}$

$\therefore\ \zeta = 0.6$



It appears that the error would be corresponding to $\frac{Z}{X} = 1.02$.
Since $\zeta = 0.6$ is quite high, it is also possible that peak value may not approach 1.02 (Caution)

Let $$\left|\frac{Z}{X}\right| = 1.02 = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$\frac{r^4}{1.02^2} = 1 + r^4 - 2r^2 + 4\zeta^2 r^2$$

$$0.9611\,7\,r^4 = r^4 + 1 - .56\,r^2$$

$$.03883\,r^4 - .56\,r^2 + 1 = 0$$

or $$r^4 - 14.42\,r^2 + 25.75 = 0$$

$$\therefore\ r^2 = 7.21 \pm \sqrt{7.21^2 - 25.75} = 7.21 \pm 5.12$$

$$r^2 = 2.09,\ 12.33 \text{ or } r = 1.445,\ 3.511$$

from the $\frac{Z}{X}$ plot

select root $r = 3.511 = \frac{f}{f_n}$

$f \geq 3.511 \times f_n$

$f \geq 35.11$ Hz will ensure error within 2%.

QUESTION # 4 (35)

An air compressor (M=100kg) is supported on an elastic support with spring constant *k* N/m and damping constant *c* Ns/m. The compressor has an unbalanced mass of 4 kg at a radial distance of 0.01 m from the axis of rotation. The damping ratio of the elastic support is measured as 0.12.

a. It is desired that the magnitude of force transmitted to the foundation must not exceed 150 N, when operating at a speed of 1200 rpm. Determine the spring rate of the support to limit the magnitude of force transmitted to this target value.

b. The steady-state vibration response of the compressor mass.

Given $M = 100$ kg , $me = 4 \times .01 = .04$ kgm ; $\zeta = 0.12$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1200}{60} = 40\pi \text{ rad/s}$$

Input force magnitude, $F_o = me\omega^2 = .04 \times (40\pi)^2$

$$F_o = 631.65 \text{ N}$$

a)

$$\therefore \frac{F_T}{F_T} \le \frac{150}{631.65} = 0.2374 = \sqrt{\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}}$$

$$1 + r^4 - 2r^2 + (.24r)^2 = \frac{1}{(.2374)^2}\left[1 + (.24r)^2\right]$$

$$r^4 - 1.9424 r^2 + 1 = 17.74 + 1.022 r^2$$

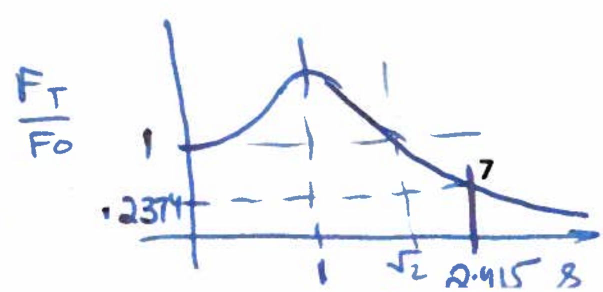
$$r^4 - 2.9644 r^2 - 16.74 = 0$$

$$r^2 = 1.4822 \pm \sqrt{1.4822^2 + 16.74} = 1.4822 \pm 4.35$$

only feasible root is $r^2 = 5.83$ or $r = 2.415$

$$r = 2.415 = \frac{\omega}{\omega_n} ; \quad \omega_n = \frac{\omega}{2.415}$$

$$\omega_n = \frac{40\pi}{2.415} = 52.03 \text{ rad/s}$$



$$\omega_n = \sqrt{\frac{k}{M}} \; ; \quad k = M\omega_n^2 = 100(52.03)^2$$

$$\underline{k = 270,786 \text{ N/m}}$$

b) Steady state response

$$x(t) = X \sin(\omega t + \varphi)$$

$$\frac{MX}{me} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \; ; \quad \zeta = 0.12 ; \quad r = 2.415$$

$$\frac{MX}{me} = \frac{2.415^2}{\sqrt{(1-2.415^2)^2 + (.24 \times 2.415)^2}} = 1.198$$

$$X = \frac{me}{M}(1.198) = \frac{0.04}{100}(1.198) = \underline{4.8 \times 10^{-4} \text{ m}}$$

$$\varphi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) = \tan^{-1}\left(\frac{0.58}{-4.83}\right) = -0.12 \text{ ra}$$

$$\varphi = \pi - .12 = 3.022 \text{ rad}$$

$$\underline{x(t) = 4.8 \times 10^{-4} \cos(40\pi t - 3.022) \text{ m}}$$