Name:______________________ **I.D.**__________

DEPARTMENT OF MECHANICAL ENGINEERING
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
CONCORDIA UNIVERSITY

Mid-Term Class Test	
COURSE: MECH 443/2	**MAX. MARKS**: 20
DATE: 15 February, 2010	**TIME**: 15:15-16:05 HRS.

QUESTION # 1

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The stiffness and damping properties of a packaging for an equipment, weighing 50 kg, are measured through an impact test. The displacement-time history of the packaged mass revealed the following: period of oscillation: τ=0.1 s; and amplitude ratio of two consecutive oscillations: X_1/X_2=4.8.

The package and the equipment, represented by a mass-spring-damper system, is subject to base excitation during transportation. Assuming the base excitation as a harmonic vibration at 10 Hz with 2 cm amplitude, determine the magnitude of vibration of the equipment of the equipment mass.

$m = 50\ kg$

$\tau_d = 0.1\ s$ $\qquad \omega_d = \frac{2\pi}{0.1} = 20\pi\ rad/s$

$\frac{X_1}{X_2} = 4.8$ $\qquad \delta = \ln(4.8) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$ $\qquad \delta = \ln(4.8) = 2\pi\zeta$

$\delta = 1.568616$

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$(1-\zeta^2)\,\delta^2 = 4\pi^2\zeta^2$ $\qquad \zeta = 0.2496$

$\delta^2 = 4\pi^2\zeta^2 + \delta^2\zeta^2$

$= \zeta^2(4\pi^2 + \delta^2)$ $\qquad \zeta^2 = \frac{\delta^2}{4\pi^2+\delta^2} = .05867$

$\zeta = 0.2422$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{20\pi}{\sqrt{.94133}} = 64.76\ rad/s$

$\omega = 2\pi \times 10 = 20\pi\ rad/s = 62.83\ ;\quad \frac{\omega}{\omega_n} = 0.97 = r$

$Y = 0.02\ m$

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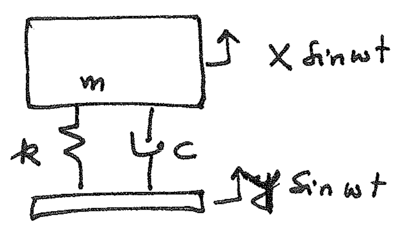
$\frac{X}{Y} = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}} = \sqrt{\frac{1+0.2208}{0.0035+.2208}} = \sqrt{5.443} = 2.333$

$X = 2.333 \times .02 = 0.0467\ m$



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QUESTION # 2

A uniform slender bar of mass m is supported in the manner shown. Determine expressions for natural frequency and critical damping coefficient for the arrangement. Assuming F_0=0 N and c<c_c, derive expressions for free vibration response in terms of angular motion of the bar about the pivot.

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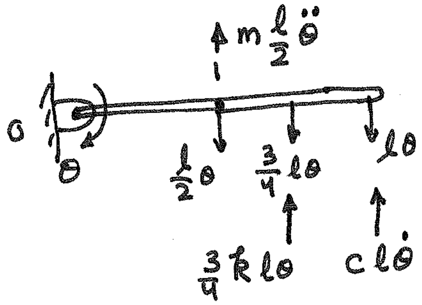

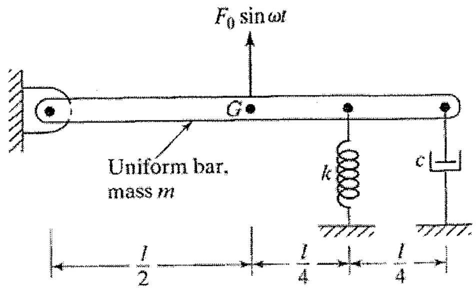

(a)

$$\Sigma M_0 = 0 \qquad m\left(\frac{l}{2}\right)^2\ddot{\theta} + cl^2\dot{\theta} + \frac{9}{16}kl^2\theta = 0 \quad — \quad 2$$

$$\ddot{\theta} + \frac{4c}{m}\dot{\theta} + \frac{9}{4}\frac{k}{m}\theta = 0$$

$$\omega_n^2 = \frac{9}{4}\frac{k}{m}; \quad \boxed{\omega_n = \frac{3}{2}\sqrt{\frac{k}{m}}} \longrightarrow 1$$

Characteristic eqn

$$s^2 + \frac{4c}{m}s + \omega_n^2 = 0$$

$$s = -\frac{4c}{2m} \pm \sqrt{\left(\frac{4c}{2m}\right)^2 - \omega_n^2}$$

$$\therefore \left(\frac{4c_c}{2m}\right)^2 - \omega_n^2 = 0 \qquad \left(\frac{2c_c}{m}\right)^2 = \omega_n^2 \qquad \cancel{c_c = \frac{m\omega_n^2}{2} = \frac{m}{2}\left(\frac{9}{4}\frac{k}{m}\right)} =$$

$$c_c = \frac{m}{2}\omega_n = \frac{m}{2}\frac{3}{2}\sqrt{\frac{k}{m}} = \boxed{\frac{3}{4}\sqrt{km} = c_c} \quad — \quad 2 \qquad \frac{m\omega_n}{2}$$

<u>Verify</u>

$$\frac{4c}{m} = \frac{4c}{m}\cdot\sqrt{\frac{k}{k}}\cdot\frac{3}{2}\cdot\frac{2}{3} = \left(\frac{4c}{\sqrt{km}}\cdot\frac{2}{3}\right)\frac{3}{2}\sqrt{\frac{k}{m}} = \frac{8c}{3\sqrt{km}}\cdot\omega_n$$

$$= \frac{4c}{3\sqrt{km}}\cdot 2\omega_n = 2\omega_n\cdot\frac{c}{c_c} = 2\zeta\omega_n$$

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$$\therefore \quad \ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0$$

$$s = -\zeta\omega_n \pm \sqrt{(\zeta\omega_n)^2 - \omega_n^2} = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2}; \quad \text{for } \zeta < 1$$

$$= -\zeta\omega_n \pm i\omega_d$$

$$\theta(t) = A e^{(-\zeta\omega_n + i\omega_d)t} + B e^{(-\zeta\omega_n - i\omega_d)t}$$

$$= e^{-\zeta\omega_n t}\left[C_1 e^{i\omega_d t} + C_2 e^{-i\omega_d t}\right]$$

$$\theta(t) = e^{-\zeta\omega_n t}\left[X_1\cos\omega_d t + X_2\sin\omega_d t\right]$$

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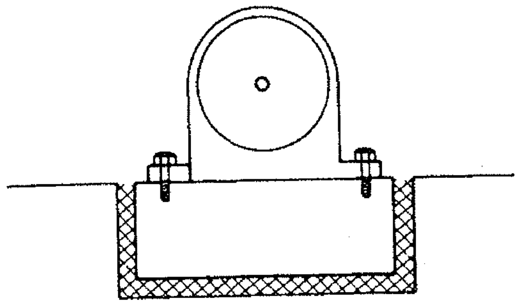
QUESTION # 3

An electric motor of mass 60 kg is mounted on an isolator block of mass 600 kg, as shown. The natural frequency of the total assembly is 60 Hz and its damping ratio is 0.1. The motor armature exhibits an unbalance of 0.01 kg-m. Determine the amplitude of vibration of the block when motor is operating at a speed of 900 rpm.



$m = 660$ kg

$\omega_n = 2\pi \times 60 = 120\pi$ rad/s

$\zeta = 0.1$

$me = 0.01$ kgm ; $\omega = \frac{2\pi N}{60} = 30\pi$ rad/s

$$\frac{MX}{me} = \frac{\omega^2/\omega_n^2}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}} \quad ; \quad \frac{\omega}{\omega_n} = \frac{30\pi}{120\pi} = 0.25$$

$$\frac{MX}{me} = \frac{.25^2}{\sqrt{(1-.25^2)^2 + (2\times.1\times.25)^2}} = 0.06657$$

$$X = \frac{.6657 \times me}{M} = \frac{.06657 \times .01}{660} = 1 \times 10^{-6} \text{ m}$$