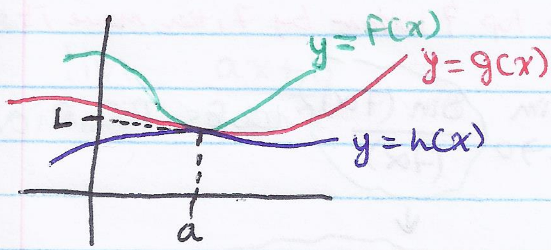
Chpt 2 Sep 14

* key limits of trig function
* lims at infinity
* Sadwich / Squeeze/pinch theorm

if $f(x) \leq g(x) \leq h(x)$ when $\underline{\underline{x}}$ near $\underline{\underline{a}}$
out in out

$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$ if f(x) and h(x) are equal then g(x) equals that

$\lim_{x \to a} g(x) = L$



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$\cos\theta < \frac{\sin\theta}{\theta} < 1 \rightarrow \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \rightarrow \lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta} = 0$

* if $1 \leq d(x) \leq x^2 + 2x + 2$

get $\lim_{x \to -1} d(x) = 1$

* $\lim_{x \to 0} \left(x^2 \sin\left(\frac{1}{x}\right)\right) \rightarrow \lim_{x \to 0} \sin\left(\frac{1}{x}\right) \nexists$

This can go to 0

$|\sin(\frac{1}{x})| \le 1 \rightarrow -1 \le \sin(\frac{1}{x}) \le 1$

$(x^2) \qquad (x^2)$

$\lim_{x \to 0} \quad -x^2 \le \sin(\frac{1}{x})x^2 \le x^2$

$0 = 0 = 0$

$\therefore \lim_{x \to 0} \left(x^2 \sin(\frac{1}{x})\right) = 0$

* $\lim_{\alpha \to 0} \dfrac{\sin(7\alpha)}{5\alpha}$ multiply top & bottom by 7, then move it out

$\frac{1}{5} \lim_{7\alpha \to 0} \dfrac{\sin(7\alpha)}{\alpha} \qquad \frac{7}{5} \lim_{7\alpha \to 0} \dfrac{\sin(7\alpha)}{(7\alpha)}$ looks familiar

$\dfrac{\sin\theta}{\theta} = 1$

$\boxed{= \frac{7}{5}}$

* $\lim_{3\alpha \to 0} \dfrac{\cos(3\alpha) - 1}{3\alpha} \rightarrow -3 \lim_{3\alpha \to 0} \dfrac{1 - \cos(3\alpha)}{3\alpha} \rightarrow \dfrac{1 - \cos\theta}{\theta} = 0$

$\boxed{= 0}$

* $\lim_{\boxed{\theta \to \pi}} \left(\dfrac{\sin\theta}{\theta - \pi}\right)$ set $\beta = \theta - \pi$ Then if $\theta \to \pi$ then $\beta \to 0$

$= \lim_{\beta \to 0} \dfrac{\sin(\beta + \pi)}{\beta}$ $\sin(\beta + \pi) = \sin\beta \cos\pi + \sin\pi \cos\beta$

$= \lim_{\beta \to 0} \dfrac{-\sin\beta}{\beta}$

$\boxed{= -1}$

$2\lim_{x\to\pi/2}$

$$\lim_{\boxed{x\to\pi/2}} \frac{2x-\pi}{\cos x} \longrightarrow \frac{x-\frac{\pi}{2}}{\cos x}$$

Set $\alpha = x - \pi/2$

as $x \to \pi/2$ $\alpha \to 0$

$$2\lim_{\alpha\to 0} \frac{\alpha}{\cos(\alpha+\pi/2)}$$

$\cos(\alpha+\pi/2) = \cos\alpha\cos\pi/2 - \sin\alpha\sin\pi/2$

$$2\lim_{\alpha\to 0}\frac{\alpha}{-\sin\alpha} \to -2\lim_{\alpha\to 0}\frac{\alpha}{\sin\alpha} \to = \boxed{-2}$$

$$\frac{1}{\frac{\sin\alpha}{\alpha}} \to 1 \quad \frac{1}{1} = 1$$

next lecture

get: a & b

$$\lim_{x\to\pi}\frac{ax+b}{2\sin x} = \pi/4$$