1. [10 marks] Solve the linear system using the Gauss-Jordan matrix elimination method

$$x - 2y + z = 5$$
$$-2x + 3y + z = 1$$
$$x + 3y + 2z = 2$$

[2] $\left[\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ -2 & 3 & 1 & 1 \\ 1 & 3 & 2 & 2 \end{array}\right]$

Row Reduction[6; **0.75 for each matrix]**

$$\sim\left[\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ 0 & -1 & 3 & 11 \\ 1 & 3 & 2 & 2 \end{array}\right] \begin{array}{c} R_1 = R_1 \\ R_2 = R_2 + 2R_1 \\ R_3 = R_3 \end{array} \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ 0 & -1 & 3 & 11 \\ 0 & 5 & 1 & -3 \end{array}\right] \begin{array}{c} R_1 = R_1 \\ R_2 = R_2 \\ R_3 = R_3 - R_1 \end{array} \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ 0 & 1 & -3 & -11 \\ 0 & 5 & 1 & -3 \end{array}\right] \begin{array}{c} R_1 = R_1 \\ R_2 = -R_2 \\ R_3 = R_3 \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ 0 & 1 & -3 & -11 \\ 0 & 0 & 16 & 52 \end{array}\right] \begin{array}{c} R_1 = R_1 \\ R_2 = R_2 \\ R_3 = R_3 - 5R_2 \end{array} \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ 0 & 1 & -3 & -11 \\ 0 & 0 & 1 & 13/4 \end{array}\right] \begin{array}{c} R_1 = R_1 \\ R_2 = R_2 \\ R_3 = R_3/16 \end{array} \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ 0 & 1 & 0 & -5/4 \\ 0 & 0 & 1 & 13/4 \end{array}\right] \begin{array}{c} R_1 = R_1 \\ R_2 = R_2 + 3R_3 \\ R_3 = R_3 \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 7/4 \\ 0 & 1 & 0 & -5/4 \\ 0 & 0 & 1 & 13/4 \end{array}\right] \begin{array}{c} R_1 = R_1 - R_3 \\ R_2 = R_2 \\ R_3 = R_3 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3/4 \\ 0 & 1 & 0 & -5/4 \\ 0 & 0 & 1 & 13/4 \end{array}\right] \begin{array}{c} R_1 = R_1 + 2R_2 \\ R_2 = R_2 \\ R_3 = R_3 \end{array}$$

[2]Therefore, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3/4 \\ -5/4 \\ 13/4 \end{bmatrix}$

2. Evaluate the following:

A. [4 marks] $(2 + 3j)^5$

First change $2 + 3i$ into the polar form. **[1]** $r = \sqrt{2^2 + 3^2} = \sqrt{13}$, **[1]** $\theta = tan^{-1}(3/2) = 56.3099$.

[1]Then $(2 + 3j)^5 = (\sqrt{13})^5 cis(5\times 56.3099) = (\sqrt{13})^5 cos(281.5495) + (\sqrt{13})^5 sin(281.5495) i$ = **[1]** $122 - 597i$

B. [5 marks] $\dfrac{7}{1-3i}$ =**[2]** $\dfrac{7}{1-3i}\dfrac{1+3i}{1+3i}$ =**[2]** $\dfrac{7+21i}{10}$ =**[1]** $\dfrac{7}{10} + \dfrac{21}{10}i$

3. [12 marks] Find all roots of $x^3 - 8 = 0$. The roots need to be in the form $a + bi$.
[1] $x^3 = 8$

First change 8 into the polar form. **[1]** $r = \sqrt{8^2} = 8$, **[1]** $\theta = tan^{-1}(0/8) = 0$.

First Root:**[3; 2for getting the root, 1 for changing it into the standard form]** $8^{\frac{1}{3}} cis(0) = 2cos0 + 2\, sin0\, i = 2$

Second Root:**[3; same as above]** $8^{\frac{1}{3}} cis\left(0 + \frac{2\pi}{3}\right) = 8^{\frac{1}{3}} cis\left(\frac{2\pi}{3}\right) = 2cos\left(\frac{2\pi}{3}\right) + 2\, sin\left(\frac{2\pi}{3}\right) = -1 + \sqrt{3}\, i$

Third Root:**[3; same as above]** $8^{\frac{1}{3}} cis\left(0 + \frac{4\pi}{3}\right) = 8^{\frac{1}{3}} cis\left(\frac{4\pi}{3}\right) = 2cos\left(\frac{4\pi}{3}\right) + 2\, sin\left(\frac{4\pi}{3}\right) = -1 - \sqrt{3}\, i$

4. [4marks; 1 each] Determine whether each of the following equations is linear or not. Explain why.
A. $x - 3y + 5 = -2w$. **[0.5]**Yes, **[0.5]**since it has the form of a linear equation, $\sum_{i=1}^{n} a_i x_i$
B. $x + 2sin\, y + 3z = 0$. **[0.5]**No, **[0.5]**since $sin\, y$ is not linear, and hence the equation is not linear.
C. $x + 2y - 3zw = 2$. **[0.5]**No, **[0.5]**because of zw.
D. $2x - y + z = 3y$. **[0.5]**Yes, **[0.5]**since it can be simplified to $\sum_{i=1}^{n} a_i x_i$.