CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	206	AA
Examination	**Date**	**Pages**
Final	June 2016	3
Instructors		**Course Examiner**
L. Dube		D. San

Special Instructions
▷ **Only approved calculators are allowed.**

MARKS

[4] **1.** Simplify following expressions. Do not use a calculator.

(a) $5\sqrt{48} - 7\sqrt{27} + 2\sqrt{75}$ (b) $\frac{2}{3}\log_8 81 + \frac{1}{4}\log_2(5^2 - 9)$

[4] **2.** Rationalize the denominator:

(a) $\frac{\sqrt{3}}{5+2\sqrt{3}}$ (b) $\frac{4+\sqrt{5}}{\sqrt{2}-\sqrt{5}}$

[6] **3.** Simplify the expressions:

(a) $3x(x^4 + 2x^2 - 3x) + 4x(x^3 - 7^2 + 1)$ (b) $\frac{4x}{x^3 - 9x} - \frac{1}{x+3}$

[8] **4.** Factor the polynomials completely:

(a) $6x^2 + x - 15$ (b) $2x^4 - 54x$

[4] **5.** Perform the arithmetic operations and simplify:

$$\frac{-2x+3}{x^2+7x+12} - \frac{x+4}{x^2-16}$$

[9] **6.** Solve the equations:

(a) $\dfrac{x}{x-2}+\dfrac{2}{x+1}=\dfrac{7x+1}{x^2-x-2}$ (b) $\log_3(x+1)+\log_3(x+4)=2$

(c) $2(8)^x=256$

[8] **7.** Solve the inequalities, express your answer using bet notation or interval notation:

(a) $3x+4 \geq \dfrac{1}{3}(x-2)$ (b) $|1-2x|-4<-1$

[4] **8.** Solve the system of equations:

$$x^2+y^2=25$$
$$x^2-\tfrac{1}{2}y^2=19$$

[8] **9.** (a) The midpoint of the line segment P to Q is $(-1,4)$, if the coordinates of P is $(-3,6)$, what is the coordinates of Q?

(b) Write the equation of the circle: $x^2+y^2-x+2y+1=0$ in standard form; find the coordinates of the center and radius of the circle.

[6] **10.** Find the domain and range of the functions (do not graph):

(a) $f(x)=\dfrac{x+1}{x^2-9}$ (b) $g(x)=\sqrt{x+2}+2$

(c) $h(x)=|\,3x-4\,|+2$

[5] **11.** Sketch the graph of the function $f(x)=\ln(x+2)+3$ starting from the graph of $g(x)=\ln x$, and using proper transformations.

[8] **12.** Let $f(x)=\dfrac{3}{x-1}$, $g(x)=\frac{2}{x}$, Find:

(a) fg (b) $\dfrac{f}{g}$ (c) fog (d) gof

[8] **13.** (a) Find the inverse of the function: $f(x) = \dfrac{2x-3}{x+4}$

(b) Find vertical and horizontal asymptotes of both f and f^{-1}.

[5] **14.** Patrice can paint four rooms in 10 hours. If he hires John to help, they can do the job together in 6 hours. If he lets John work alone, how long it will take him to paint four rooms?

[5] **15.** A gardener has 46 feet to fencing to be used to enclose a rectangular garden that has a border 2 feet wide surrounding it. If the length of the garden is to be twice its width, what will be the dimensions of the garden?

[8] **16.** The number N of bacteria present in a culture at time t (in hours) obeys the law of uninhibited growth $N = 1000e^{0.01t}$:

(a) Determine the initial amount of bacteria.

(b) How long will it take the number of bacteria to reach 1700?

(c) What is the doubling time for the population?

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number		Section(s)
Mathematics	206/4		All
Examination	**Date**	**Time**	**Pages**
Final	April 2017	3 Hours	2
Instructors			**Course Examiner**
A. Butaev, I. Negrini, K. Lagota, U. Mgbemena			D. Sen

Special Instructions

▷ **Only approved calculators are allowed.**

MARKS

[4] **1.** Simplify the expressions below. Do not use a calculator.

(a) $5\sqrt{20} - \sqrt{45} + 2\sqrt{80}$ (b) $\frac{2}{3}\ln 8 - \ln(5^2 - 1)$

[4] **2.** Rationalize the denominator:

(a) $\frac{4-\sqrt{3}}{2-\sqrt{3}}$ (b) $\frac{1+\sqrt{7}}{2-\sqrt{5}}$

[6] **3.** Simplify the expressions:

(a) $5x^2(x^3 - x^2 - 4x + 5) - 4x(3x^4 - 2x^3 + 3x^2 + x)$ (b) $\frac{4x^2 + 8x}{12x + 24}$

[8] **4.** Factor the polynomials completely:

(a) $2x^2 - 9x + 4$ (b) $12x^2 + 7x - 10$

[4] **5.** Perform the arithmetic operations and simplify:

$$\frac{x}{x^2 - 7x + 6} - \frac{x}{x^2 - 2x - 24}$$

[9] **6.** Solve the equations:

(a) $\frac{2x+1}{9} - \frac{x+4}{6} = 1$ (b) $2\log_5 x = \log_5 9$

(c) $5^{x^2+8} = 125^{2x}$

[8] **7.** Solve the inequalities, express your answer using set notation or interval notation:

(a) $x(4x + 3) \leq (2x + 1)^2$ (b) $|1 - 4x| < 5$

[4] **8.** Solve the system of equations:

$$\begin{aligned} x^2 - y^2 &= 21 \\ x + y &= 7 \end{aligned}$$

[8] **9.** (a) Which of the points $A(2,4)$, $B(4,5)$ is closer to the point $C(1,5)$?

(b) Show that the equation $x^2 + y^2 + 4x - 6y + 12 = 0$ represents a circle. Find coordinates of the center and radius of the circle.

[6] **10.** Find the domain and range of the functions (do not graph):

(a) $f(x) = \dfrac{3x}{x^2 - 4}$ (b) $g(x) = \sqrt{3x - 12}$ (c) $h(x) = |x| + 4$

[5] **11.** Sketch the graph of the function $f(x) = \log(x-4) + 2$, starting from the graph of the function $g(x) = \log x$ and using appropriate transformations.

[8] **12.** Let $f(x) = \dfrac{1}{2x-1}$ and $g(x) = \dfrac{1}{x^2}$. Find:

(a) fg (b) $\dfrac{f}{g}$ (c) $f \circ g$ (d) $g \circ f$

[8] **13.** (a) Find the inverse of the function $f(x) = \dfrac{x+1}{x-2}$.

(b) Find the vertical and horizontal asymptotes of both f and f^{-1} above.

[5] **14.** A restaurant manager wants to purchase 200 sets of dishes. One design costs \$25 per set, while another costs \$45 per set. If she only has \$7400 to spend, how many of each design should be ordered??

[5] **15.** A movie theater charges \$9.00 for adults and \$7.00 for senior citizens. On a day when 325 people paid an admission, the total receipts were \$2495. How many who paid were adults? How many were seniors?

[8] **16.** The number N of bacteria present in a culture at time t (in hours) obeys the law of uninhibited growth

$$N(t) = 1000e^{0.01t}$$

(a) Determine the number of bacteria at $t = 0$ hours.

(b) What is the growth rate of the bacteria?

(c) When will the number of bacteria reach 1700?

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1. Simplify: (a) $-4\sqrt{50}-\sqrt{98}+3\sqrt{32}$

$= 4\sqrt{25\times 2}-\sqrt{49\times 2}+3\sqrt{16\times 2}$

$= -20\sqrt{2}-7\sqrt{2}+12\sqrt{2} = -15\sqrt{2}$

(b) $\log_2 20 - \log_2 30 + \log_2 12 = \log_2\left(\frac{20\times 12}{30}\right)$

$= \log_2 8 = \log_2 2^3 = 3\log_2 2 = 3.$

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2. Rationalize (a) $\frac{\sqrt{2}}{\sqrt{7}+2} = \frac{\sqrt{2}(\sqrt{7}-2)}{(\sqrt{7}+2)(\sqrt{7}-2)}$

$= \frac{\sqrt{14}-2\sqrt{2}}{7-4} = \frac{\sqrt{14}-2\sqrt{2}}{3}$

(b) $\frac{\sqrt{3}-1}{2\sqrt{3}+3}\times\frac{(2\sqrt{3}-3)}{(2\sqrt{3}-3)} = \frac{2\cdot 3-2\sqrt{3}-3\sqrt{3}+3}{12-9}$

$= \frac{9-5\sqrt{3}}{3}$

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3. Simplify:

(a) $8x^2(4x^3-3x^2-4)-4x(4x^4-4x^3+2x^2+7x)$

$= 32x^5-24x^4-32x^2-16x^5+16x^4-8x^3-4x^2$

$= 16x^5-8x^4-8x^3-36x^2$

(b) $\frac{x^2-x-2}{3x^2+5x+2} = \frac{(x-2)(x+1)}{(3x+2)(x+1)} = \frac{x-2}{3x+2}$

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4. Factor:

(a) $x^3+8x^2-20x = x(x^2+8x-20)$

$= x(x+10)(x-2)$

(b) $x^8-x^5 = x^5(x^3-1) = x^5(x^3-1^3)$

$= x^5(x-1)(x^2+x+1)$

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5. Simplify:

$\frac{1}{x}-\frac{2}{x(x-1)}+\frac{3}{x^2(x-1)} = \frac{x(x-1)-2x+3}{x^2(x-1)}$

$= \frac{x^2-3x+3}{x^2(x-1)}$

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6. Solve the eqns.

(a) $\frac{x}{x^2-1}-\frac{x+3}{x^2-x}+\frac{3}{x^2+x} = 0$

$\frac{x}{(x-1)(x+1)}-\frac{x+3}{x(x-1)}+\frac{3}{x(x+1)} = 0$

6a $\frac{x^2-(x+1)(x+3)+3(x-1)}{x(x-1)(x+1)} = 0$

$\frac{x^2-x^2-4x-3+3x-3}{x(x-1)(x+1)} = 0$

$\frac{-x-6}{x(x-1)(x+1)} = 0 \Rightarrow -x-6=0$

$x = -6.$

(b) $\log_8(x+6)+\log(x+4) = 1$

$\log_8(x+6)(x+4) = 1,$

$(x+6)(x+4) = 8^1$

$x^2+10x+24-8 = 0$

$x^2+10x+16 = 0$

$(x+8)(x+2) = 0 \Rightarrow x = -2, -8$

(c) $5^{x^2+8} = (5^3)^{2x} \Rightarrow 5^{x^2+8} = 5^{6x}$

$x^2+8 = 6x \Rightarrow x^2-6x+8 = 0$

$(x-2)(x-4) = 0 \Rightarrow x = 2, 4.$

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7 Solve:

(a) $-3 \le \frac{5-3x}{2} \le 6$

$-6 \le 5-3x \le 12$

$-5-6 \le -5+5-3x \le -5+12$

$-11 \le -3x \le 7$

$-3x \ge -11 \Rightarrow 3x \le 11 \Rightarrow x \le \frac{11}{3}$

$-3x \le 7 \Rightarrow 3x \ge -7 \Rightarrow x \ge -\frac{7}{3}$

$-\frac{7}{3} \le x \le \frac{11}{3},$

(b) $1-\left|\frac{2x-1}{2}\right| < -2$

$-\left|\frac{2x-1}{2}\right| < -3$

$\left|\frac{2x-1}{2}\right| > 3 \Rightarrow |2x-1| > 6$

$2x-1 > 6 \Rightarrow x > \frac{7}{2}$

$-(2x-1) > 6 \Rightarrow -2x+1 > 6$

$-2x > 5 \Rightarrow 2x < -5 \Rightarrow x < -\frac{5}{2}$

Ans: $x < -\frac{5}{2}$ or $x > \frac{7}{2}$

Solution: Math 206. DEC 2014 Exam.

#1: Solve: $x^2+y^2=13$, $x^2-y=7$

$y^2+y=6 \Rightarrow y^2+y-6=0$

$(y+3)(y-2)=0 \Rightarrow y=2,-3$

Now $x^2+y^2=13 \Rightarrow x^2+2^2=13 \Rightarrow x=\pm 3$

and $x^2+y^2=13 \Rightarrow x^2+(-3)^2=13 \Rightarrow x=\pm 2$

Ans: A(−3, 2), B(3, 2), C(2, −3), D(−2, −3)

#4: Let x caramels and y creams be in the box

Thus $x+y=30$

$0.75x+0.45y=(12.50-3)$

or $0.25x+0.25y=30\times 0.25$

$0.75x+0.45y=9.50$

$-0.20y=-2.00$

$x=20,\ y=10$

#9 (a) $d(AC)=\sqrt{(7-2)^2+(8-6)^2}=\sqrt{29}$

$d(BC)=\sqrt{(7-6)^2+(8-4)^2}=\sqrt{17}$

Point B is closer to C.

(b) $2x^2+2y^2-12x+8y=24$

$(x^2-6x+\frac{3}{3})+(y^2+4y+\frac{1}{2})=12+9+4$

$(x-3)^2+(y+2)^2=5^2$

Centre $(3,-2)$ rad $=5$

#15: (a) $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{(2x-1)(2x-5)}{(x-2)(x+4)}$

(c) $(fog)(x)=f(g(x))=\frac{2g(x)-1}{g(x)-2}$

$=\frac{\frac{9}{2x-5}}{\frac{-3x+14}{2x-5}}=\frac{9}{-3x+14}$.

Try part (a) and (d) as above.

#13. (a) $f(x)=\frac{2x-3}{x+4}$. replace $f(x)$ by x and x by y, solve for y

$x=\frac{2y-3}{y+4} \Rightarrow xy+4x=2y-3$

$\Rightarrow y(x-2)=-4x-3 \Rightarrow y=-\frac{4x+3}{x-2}$

or $f^{-1}(x)=-\frac{4x+3}{x-2}$

(b) H.A for $f(x)$: $y=2$

V.A for $f(x)$: $x=-4$

H.A for $f^{-1}(x)$: $y=-4$

V.A for $f^{-1}(x)$: $x=2$.

#15 Let x be the width and y the length

$y=2x,\ 2(x+3)+2(y+3)=51$

$\therefore 2x+6+2y+6=51$

$2x+2y=51 \Rightarrow 6x=39$

$x=6.5''$ and $y=13$ in.

#16 (a) $m(60)=40e^{-0.0277\times 60}$

$=40e^{-1.662}=7.59$ gm

(b) $10=40e^{-0.0277t}$

$\ln 10=\ln 40-0.0277t\ln e$

$0.0277t=\ln 40-\ln 10$

$t=\frac{\ln 4}{0.0277}=50$ days

(c) $20=40e^{-0.0277t}$

$t=\frac{\ln 2}{0.0277}=25$ days.

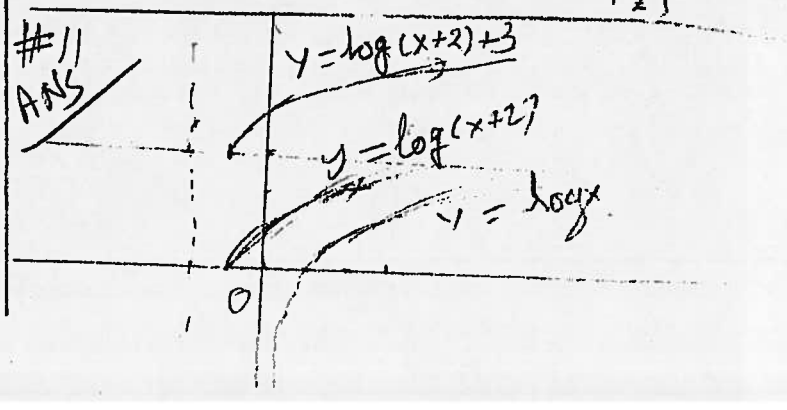
#12 Ans: (a) Domain: All R except $x=\frac{5}{2}$

Range: All reals IR except 2.

(b) $g(x)=\frac{x+4}{2x-5}$; D: All Nos except $x=\frac{5}{2}$

R: All $\mathbb{R}-\{\frac{1}{2}\}$

#11 Ans



Solution: MATH 206 FINAL EXAM APRIL 2016

#1 Simplify (a) $4\sqrt{12}+5\sqrt{27}-\sqrt{75} = 4\sqrt{4}\sqrt{3}+5\sqrt{9}\sqrt{3}-\sqrt{25}\sqrt{3}$

$= 8\sqrt{3}+15\sqrt{3}-5\sqrt{3} = 18\sqrt{3}$

(b) $\frac{1}{3}\log_3 27 - \log_3(27\div 18) = \frac{1}{3}\log_3 3^3 - \log_3 3^2 = 3-2 = 1$

#2. Rationalize the denominator

(a) $\frac{5}{2\sqrt{3}} = \frac{5\sqrt{3}}{2\sqrt{3}\sqrt{3}} = \frac{5\sqrt{3}}{6}$ (b) $\frac{2-\sqrt{5}}{2+3\sqrt{5}} = \frac{(2-\sqrt{5})(2-3\sqrt{5})}{(2+3\sqrt{5})(2-3\sqrt{5})} = \frac{19-8\sqrt{5}}{-41} = -\frac{19-8\sqrt{5}}{41}$

#3. Simplify: (a) $5x^4-25x^3-x^4+7x^3+5x^2 = 4x^4-18x^3+5x^2$

(b) $\frac{x^3-2^3}{x^3-2x^2} = \frac{(x-2)(x^2+2x+4)}{x^2(x-2)} = \frac{x^2+2x+4}{x^2}$

#4 Factor: (a) $4x^2-16x+16 = 4(x^2-4x+4) = 4(x-2)^2$

(b) $1-8x^2-9x^4 = 1+x^2-9x^2-9x^4 = (1+x^2)-9x^2(1+x^2)$

$= (1+x^2)(1-9x^2) = (1+x^2)(1-3x^2)(1+3x^2)$

#5: Simplify: $\frac{4x}{x^2-4} - \frac{2}{x^2+x-6} = \frac{4x}{(x-2)(x+2)} - \frac{2}{(x+3)(x-2)}$

$= \frac{4x[x+3]-2[x+2]}{(x-2)(x+2)(x+3)} = \frac{4x^2+10x-4}{(x-2)(x+2)(x+3)}$

#6 Solve: (a) $\frac{2x}{x^2-4} = \frac{4}{x^2-4} - \frac{3}{x+2} \Rightarrow \frac{2x-4}{x^2-4} + \frac{3}{x+2} = 0 \Rightarrow \frac{2(x-2)+3(x-2)}{(x^2-4)} = 0$

$\frac{5(x-2)}{(x-2)(x+2)} = 0 \Rightarrow \frac{5}{x+2} = 0$, $x \neq 2 \Rightarrow$ No solution

(b) $\log_3(3x-1) = 2 \Rightarrow 3x-1 = 3^2 \Rightarrow 3x = 10 \Rightarrow x = \frac{10}{3}$.

(c) $3^{x^3} = (3^3)^x \Rightarrow 3^{x^3} = 3^{3x} \Rightarrow x^3 = 3x \Rightarrow x^3-3x = 0 \Rightarrow$

$x(x^2-3) = 0 \Rightarrow x = 0$; $x^2-3 = 0$, $x = \pm\sqrt{3}$

#7 (a) $-2 \le 3-5x \le 18$

$2 \ge -3+5x \ge -18$

$5 \ge 5x \ge -15$

$1 \ge x \ge -3$

$-3 \le x \le 1$

(b) $\left|\frac{2x+3}{3} - \frac{1}{2}\right| < 1 \Rightarrow \left|\frac{4x+6-3}{6}\right| < 1$

$\left|\frac{4x+3}{6}\right| < 1$; $|4x+3| < 6$

$4x+3 < 6$ and $-(4x+3) < 6$

$4x < 3$ $\qquad 4x+3 > -6$

$x < \frac{3}{4}$ $\qquad 4x > -9$

$x > -\frac{9}{4}$

$-\frac{9}{4} < x < \frac{3}{4}$.

#8 Solve: $2x^2+y^2=18$, $xy=4$ $\Rightarrow y=\frac{4}{x}$ $\Rightarrow 2x^2+(\frac{4}{x})^2=18 \Rightarrow 2x^2+\frac{16}{x^2}=18 \Rightarrow$

$\Rightarrow 2x^4+16=18x^2 \Rightarrow x^4-9x^2+8=0 \Rightarrow (x^2-1)(x^2-8)=0 \Rightarrow$

$x^2-1=0$ or $x^2-8=0 \Rightarrow x=\pm 1, \pm 2\sqrt{2}$.

9 (a) Distance $=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=\sqrt{(-4-3)^2+(5-2)^2}=\sqrt{49+9}=\sqrt{58}$

(b) $2x^2+2y^2-12x+8y-24=0 \Rightarrow x^2-6x+9+y^2+4y+4=12+13$

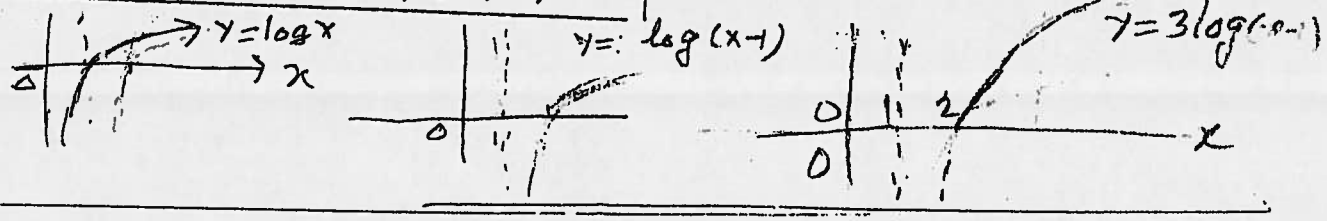
$(x-3)^2+(y+2)^2=5^2$ ∴ centre $(3,-2)$ radius $r=5$

10 (a) $f(x)=\frac{1}{\sqrt{x-1}}$ D: $x>1$ R: $(0,\infty)$

(b) $g(x)=4+\sqrt{x-3}$ D: $x\geq 3$. R: $[4,\infty)$

(c) $h(x)=|x|-4$ D: $(-\infty,\infty)$ R: $(-4,\infty)$

11.



12 $(fg)(x)=\frac{x^2}{(x+1)}\cdot\frac{2x}{(x^2-1)}=\frac{2x^3}{(x^3+x^2-x-1)}$; $(\frac{f}{g})x=\frac{x^2}{(x+1)}\div\frac{x^2-1}{2x}=\frac{2x^2}{x-1}$. $x\neq 0,-1$

$(f\circ g)(x)=f(g(x))=\frac{[g(x)]^2}{g(x)+1}=\frac{(\frac{2x}{x^2-1})^2}{\frac{2x}{x^2-1}+1}=\frac{4x^2}{(x^2-1)(x^2+2x-1)}$

$(g\circ f)x=g(f(x))=\frac{2f(x)}{[f(x)]^2-1}$ replace $f(x)$ by $\frac{x^2}{x+1}$ and simplify.

13 : Replace $f(x)$ by x and x by y : $x=\frac{2y+3}{y+2} \Rightarrow xy+2x=2y+3$

Solve for y : $y(x-2)=3-2x \Rightarrow y=\frac{-2x+3}{x-2} \Rightarrow f^{-1}(x)=\frac{-2x+3}{x-2}$

H.A of $f(x)$: $y=2$. and H.A of $f^{-1}(x)$ is $y=-2$.

V.A of $x=2$.

14 Let x \$ be invested in bonds and y \$ in stocks

$x = 2y$ and $x + y = 15000 \Rightarrow 3y = 15000$

$x = \$10,000$ and $y = \$5000$

15 Let x ml be added to get the required strength.

$$(153 + x) \times 0.13 = 153 \times 0.30$$

$$0.13x = 153 \times 0.30 - 153 \times 0.13$$

$$x = \frac{153 \times 0.17}{0.13} = 200.08 \text{ ml.}$$

#16 $$P(t) = \frac{2000}{1 + 1999 e^{0.8905t}}$$

(a) $$P(5) = \frac{2000}{1 + 1999 e^{0.8905 \times 5}} = 82.35$$ 82 persons

(b) $$1000 \left[1 + 1999 e^{-0.8905t}\right] = 2000$$

$$1 + 1999 e^{0.8905t} = 2$$

$$e^{-0.8905t} = \frac{1}{1999}$$

$$-0.8905t \ln e = \ln\left(\frac{1}{1999}\right)$$

$$t = \frac{\ln 1999}{0.8905} = 8.535 \text{ days.}$$

(c) When $t = \infty$ then $e^{0.8905 \times \infty} = 0$

Thus $P = \frac{2000}{1+0} = 2000$ entire popula...

35

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	206/1	AA
Examination	**Date**	**Pages**
Final	June 2008	2
Instructor		**Course Examiner**
L. Dube		D. Sen

Special Instructions

▷ **Only approved calculators are allowed.**

MARKS

[4] **1.** Simplify the expressions below. Do not use a calculator.

(a) $\sqrt{125}+\sqrt{27}+\sqrt{12}$ $5(\sqrt{5}+\sqrt{3})$ Ans (b) $\log_3 15+\log_3 4-\log_3 30$ $\log_3 2$ Ans

[4] **2.** Rationalize the denominator:

(a) $\frac{5}{3\sqrt{7}}$ $\frac{5\sqrt{7}}{21}$ (b) $\frac{-3}{\sqrt{5}+2}$ $-3(\sqrt{5}-2)$ Ans

[6] **3.** Simplify the expressions:

(a) $2x(x^3+2x-5)+2x^2(-x^2+3x+1)$ $2x(3x^2+3x-5)$ (b) $\frac{2t+8}{t^2-16}$ $=\frac{2}{t-4}$ Ans

[8] **4.** Factor the polynomials completely:

(a) $x^5+6x^4+9x^3$ $x^3(x+3)^2$ (b) $2x^4-32$ $=2(x-2)(x+2)(x^2+4)$ Ans.

[4] **5.** Perform the arithmetic operations and simplify:

$$\frac{2x+1}{x^2+7x+12}-\frac{3x}{x+4} = \frac{-3x^2-7x+1}{x^2+7x+12}$$

[9] **6.** Solve the equations:

(a) $\frac{4}{(2x-1)^2}+\frac{12}{2x-1}+9=0$ $x=\frac{1}{6}$ (b) $\log_3(x+1)+\log_3(x+4)=2$ $x=\frac{-5\pm\sqrt{5}}{2}$ (c) $2\cdot 5^x=250$ $x=3$

[8] **7.** Solve the inequalities, express your answer using set notation or interval notation:

(a) $\frac{1}{2}(x-4)\geq(2x+6)$ $x\leq -\frac{16}{3}$ (b) $|1-4x|-7<-2$ $-1<x<\frac{3}{2}$

[4] **8.** Solve the system of equations:

$$\begin{aligned} x^2 + y^2 &= 10 \\ x - y &= -2 \end{aligned}$$

Ans: $\begin{cases} x=1, y=3 \\ x=-3, y=-1 \end{cases}$

[8] **9.** (a) Write down the equation of the line passing through the point $(-2,3)$, and perpendicular to the line $2x+y=3$. Ans $y=\frac{1}{2}x+4$

(b) Show that the equation $x^2+y^2-6x+2y+1=0$ represents a circle. Find coordinates of the center, and radius of the circle.

Center. $(3,-1)$ rad $=3$

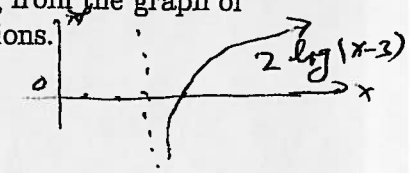
[6] **10.** Find the domain and range of the functions (do not graph):

(a) $f(x)=\dfrac{x+1}{x^4-16}$ D: All Reals except $x=\pm 2$ R: $(-\infty,\infty)$

(b) $g(x)=\sqrt{9-x^2}$ D: $-3\le x\le 3$ R: $[0,3]$

(c) $h(x)=|2x-3|+1$ D: $(-\infty,\infty)$ R: $[0,\infty)$

[5] **11.** Sketch the graph of the function $f(x)=2\log(x-3)$, starting from the graph of the function $g(x)=\log x$ and using appropriate transformations.



[8] **12.** Let $f(x)=\dfrac{1}{2x+3}$ and $g(x)=2x^2$. Find:

(a) fg $\dfrac{2x^2}{2x+3}$ (b) $\dfrac{f}{g}(x)$ $=\dfrac{1}{2x^2(2x+3)}$ (c) $f\circ g$ $\dfrac{1}{4x^2+3}$ (d) $g\circ f(-x)=\dfrac{2}{(2x+3)^2}$

[8] **13.** (a) Find the inverse of the function $f(x)=\dfrac{3x+5}{x+2}$. $f^{-1}(x)=\dfrac{5-2x}{x-3}$

(b) Find the vertical and horizontal asymptotes of both f and f^{-1} above.

f: VA: $x=-3$; HA $y=3$ | f^{-1}: VA: $x=3$, HA $y=-2$

[5] **14.** John has \$70,000 to invest and requires an overall rate of return of 9%. He invests in a safe, government-insured certificate of deposit, but it only pays 8%. To obtain 9%, John agrees to invest some of his amount in noninsured corporate bonds paying 12%. How much should be placed in each investment to achieve his goal? \$17500 At 12% and \$52500 at 8%.

[5] **15.** A rectangle has the area of 300 $meter^2$. If the length is 20 meter larger than the width, find the length and width of the rectangle. $w=10\,m$, $l=30$

[8] **16.** The mass $m(t)$ remaining after t days from a 40 gm sample of thorium-234 is given by,

$$m(t)=40e^{-0.0277t}$$

(a) How much of the sample will remain after 60 days? 16.133 gm

(b) After how long will only 10 gm of the sample remain? 50 yr

(c) Find the half life of thorium-234. 25.023 yr.

Solution: Math 206 Final Exam June 2008

1 (a) $\sqrt{25\cdot 5}+\sqrt{9\cdot 3}+\sqrt{4\cdot 3}$ Ans

$= 5\sqrt{5}+3\sqrt{3}+2\sqrt{3} = 5\sqrt{5}+5\sqrt{3}$

$= 5(\sqrt{5}+\sqrt{3})$ ANS.

(b) $\log_3 15+\log_3 4-\log_3 30 = \log_3 \frac{60}{30}$

$= \log_3 2$. ANS

2. (a) $\frac{5}{3\sqrt{7}} = \frac{5}{3\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{21}$ ANS

(b) $-\frac{3}{\sqrt{5}+2} = -\frac{3}{\sqrt{5}+2}\cdot\frac{\sqrt{5}-2}{\sqrt{5}-2} = -\frac{3(\sqrt{5}-2)}{5-4}$

$= -3(\sqrt{5}-2)$ ANS

4 (a) $x^3(x^2+6x+9) = x^3(x+3)^2$

(b) $2(x^4-4^2) = 2(x^2-4)(x^2+4)$

$= 2(x-2)(x+2)(x^2+4)$ ANS

3 (a) $2x^4+4x^2-10x-2x^4+6x^3+2x^2$

$= 6x^3+6x^2-10x = 2x(3x^2+3x-5)$

(b) $\frac{2t+8}{t^2-16} = \frac{2(t+4)}{(t-4)(t+4)} = \frac{2}{t-4}$ ANS

5 $\frac{2x+1}{(x+3)(x+4)} - \frac{3x}{x+4} = \frac{2x+1-3x(x+3)}{(x+3)(x+4)}$

$= \frac{-3x^2-7x+1}{(x+3)(x+4)} = \frac{-3x^2-7x+1}{x^2+7x+12}$ ANS

6. (a) $\frac{4}{(2x-1)^2}+\frac{12(2x-1)}{(2x-1)^2}+\frac{9(2x-1)^2}{(2x-1)^2}=0$

$\frac{4+12(12x-1)+9(2x-1)^2}{(2x-1)^2} = 0$ $2x-1\neq 0$

$9(2x-1)^2+12(12x-1)+4 = 0$

$[3(2x-1)+2]^2 = 0$

$6x-3+2 = 0$, $\boxed{x=\frac{1}{6}}$ ANS

(b) $\log_3(x+1)(x+4) = \log_3 9$

$(x+1)(x+4) = 9$

$x^2+5x+4 = 9 \Rightarrow x^2+5x-5=0$

$\boxed{x=\frac{-5\pm\sqrt{5}}{2}}$ ANS

(c) $2\cdot 5^x = 250 \Rightarrow 5^x = 125$

$5^x = 5^3 \Rightarrow \boxed{x=3}$ ANS

7 (a) $x-4 \geq 4x+12 \Rightarrow -3x \geq 16 \Rightarrow x \leq -\frac{16}{3}$

(b) $|1-4x|<5 \Leftrightarrow -5<1-4x<5 \Rightarrow -1<x<\frac{3}{2}$

8 $x^2+y^2=10$, $(y-2)^2+y^2=10 \Rightarrow y^2-2y-3=0$

$x-y=-2 \Rightarrow x=y-2$ $y=-1,3$

When $y=-1$, $x=-3$ when $y=3$, $x=1$

Ans $x=1, y=3$ or $x=-3, y=-1$

9 (a) Slope of $y=-2x+3$ is -2

Slope of required line $\frac{1}{2}$.

Eqn of the line $y-3=\frac{1}{2}(x+2)$

or $y=\frac{1}{2}x+4$

(b) $(x^2-6x+9)+(y^2-2y+1) = -1+10$

$(x-3)^2+(y+1)^2 = 9$

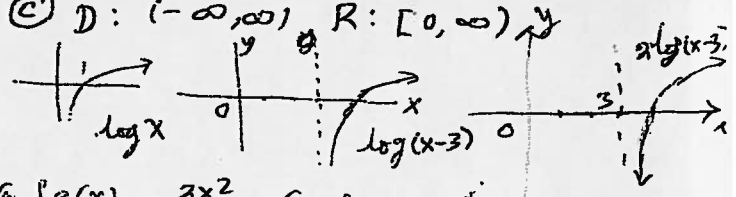
Centre $(3,-1)$ rad $r=3$.

10 (a) D: All $\mathbb{R}$ $(x\neq\pm 2)$ R: $(-\infty,\infty)$

(b) D: $-3\leq x\leq 3$ R: $[0,3]$

(c) D: $(-\infty,\infty)$ R: $[0,\infty)$

11



12 (a) $fg(x) = \frac{2x^2}{2x+3}$ (b) $\frac{f}{g}(x) = \frac{x^2}{2x^2(2x+3)}$ $\frac{f}{g}(1) = \frac{2}{(2+3)}$

(c) $(f\circ g)(x) = f(g(x)) = \frac{1}{2g(x)+3} = \frac{1}{2x^2+3}$

13 (a) Write $y=\frac{3x+5}{x+2}$ switch $x;y$ $x=\frac{3y+5}{y+2}$, Solve for y

$y=\frac{5-2x}{x-3}$ or $f^{-1}(x)=\frac{5-2x}{x-3}$.

V.A of $f(x)$: $x=-2$, and H.A: $y=3$

for $f^{-1}(x)$: V.A: $x=3$ and H.A: $y=-2$

14 let $x be invested at 12%. rest at 8%.

$0.12x+0.08(70000-x) = 0.09\times 70000$

$\therefore x = \$17500$ at 12% $52500 at 8%. Solve for

15 $x(x+20) = 300$ let x be smaller

$x^2+20x-300=0$ side

$(x-10)(x+30)=0 \Rightarrow x=10, \cancel{x=-30}$

Ans 10 and 30 meters.

16 (a) $m(40) = 40e^{-0.027\times 40} = 16.133$ gm

(b) $10 = 40e^{-0.0277t} \Rightarrow e^{-0.0277t} = \frac{1}{4}$

Take ln: $+0.0277t = \ln 4$

(c) $20 = 40e^{-0.0277t}$ $t = \frac{\ln 4}{.0277} = 50$ yr.

$t = \frac{\ln 2}{.0277} = 25.023$ yr.