1. A differential equation which is satisfied by the orthogonal trajectories to the curves $x = ky^3$ is

(a) $y' = \dfrac{y}{3x}$

(b) $y' = -\dfrac{y}{3x}$

(c) $y' = \dfrac{3x}{y}$

(d) $y' = -\dfrac{3x}{y}$

(e) $y' = -3ky^2$

2. Solve the initial value problem $y' = \dfrac{2x+y}{x}$, $y(1) = 0$ in order to obtain the value of $y(e)$.

(a) $y(e) = e^2$

(b) $y(e) = 2e$

(c) $y(e) = e$

(d) $y(e) = 2$

(e) $y(e) = -2e$

3. Solve the initial value problem $x^2y' + 2xy = \cos x$, $y(\pi) = 0$ in order to obtain the value of $y(2)$.

(a) $y(2) = \dfrac{\sin 2}{4}$

(b) $y(2) = \sin 2$

(c) $y(2) = \sin 2 - C$

(d) $y(2) = \dfrac{\sin 2 - 1}{4}$

(e) $y(2) = \dfrac{\sin 2 - C}{4}$

4. Consider the Bernoulli differential equation $y' + 2xy = xe^{-x^2}y^3$.
The transformation used to obtain the general solution is

(a) $u = \dfrac{1}{y}$

(b) $u = y^2$

(c) $u = \dfrac{1}{y^2}$

(d) $u = y^3$

(e) $u = \dfrac{1}{y^3}$

5. Let $f(x, y) = (2x^3y^2 - y)^4$. The value of $f_y(1, 1)$ is

(a) 6

(b) -6

(c) 12

(d) -12

(e) 24

6. Let $z = 2x^3 + y^2$, $x = s\ e^t$, $y = s^2 \cos t$.
When $s = 1$ and $t = 0$, $\dfrac{\partial z}{\partial s}$ is equal to

(a) 4

(b) 6

(c) 8

(d) 10

(e) 12

7. Let $xyz = \cos(x+y+z)$. Then $\dfrac{\partial z}{\partial x}$ is equal to

(a) $\dfrac{yz + \sin(x+y+z)}{xy + \sin(x+y+z)}$

(b) $-\left(\dfrac{yz + \sin(x+y+z)}{xy + \sin(x+y+z)}\right)$

(c) $\dfrac{yz - \sin(x+y+z)}{xy + \sin(x+y+z)}$

(d) $\dfrac{xz + \sin(x+y+z)}{xy + \sin(x+y+z)}$

(e) $-\left(\dfrac{xz + \sin(x+y+z)}{xy + \sin(x+y+z)}\right)$

8. The differential equation $2xy + (x^2-1)\dfrac{dy}{dx} = 0$ is

1) Homogeneous 2) Separable 3) Exact

(a) 1) and 2)

(b) 2) and 3)

(c) 1) and 3)

(d) 1), 2) and 3)

(e) None of the above

9. Consider the differential equation $2xy + 3x^2y + 3y^2 + (x^2+2y)y' = 0$.
Which of the following is an integrating factor?

(a) $\ln(3x)$

(b) $\dfrac{1}{3x}$

(c) $3x$

(d) e^{-3x}

(e) e^{3x}

10. The general solution of the differential equation $y'' + 4y = 0$ is

(a) $y = c_1 \cos(2x) + c_2 \sin(2x)$

(b) $y = c_1 \cos\left(\frac{x}{2}\right) + c_2 \sin\left(\frac{x}{2}\right)$

(c) $y = c_1 e^{-2x} + c_2 x e^{-2x}$

(d) $y = c_1 e^{4x} + c_2 x e^{4x}$

(e) $y = c_1 \cos(\sqrt{2}\, x) + c_2 \sin(\sqrt{2}\, x)$

11. Solve the initial value problem $y'' + 2y' + y = 0, \quad y(0) = 2, \quad y'(0) = 3$ in order to obtain $y(1)$.

(a) $y(1) = -e^{-1}$

(b) $y(1) = e^{-1}$

(c) $y(1) = 3e^{-1}$

(d) $y(1) = 5e^{-1}$

(e) $y(1) = 7e^{-1}$

12. The general solution of the differential equation $y'' + y' - 2y = \cos x$ is

(a) $y = c_1 e^{x} + c_2 e^{-2x} - \frac{3}{10}\cos x - \frac{1}{10}\sin x$

(b) $y = c_1 e^{x} + c_2 e^{-2x} - \frac{3}{10}\cos x + \frac{1}{10}\sin x$

(c) $y = c_1 e^{x} + c_2 e^{-2x} + \frac{3}{10}\cos x - \frac{1}{10}\sin x$

(d) $y = c_1 e^{-x} + c_2 e^{2x} + \frac{3}{10}\cos x - \frac{1}{10}\sin x$

(d) $y = c_1 e^{-x} + c_2 e^{2x} - \frac{3}{10}\cos x + \frac{1}{10}\sin x$

13. Let $y'' - 2y' + 2y = e^x \cos x$. An appropriate trial solution for the method of undetermined coefficients is

(a) $y_p = Ae^x + B\cos x$

(b) $y_p = A\cos x + B\sin x$

(c) $y_p = e^x(A\cos x + B\sin x)$

(d) $y_p = xe^x(A\cos x + B\sin x)$

(e) $y_p = x^2e^x(A\cos x + B\sin x)$

14. Let $y'' - 3y' + 2y = \cos(e^{-x})$. Given that the form of the particular solution is $y_p = u_1(x)e^x + u_2(x)e^{2x}$, find $u_1(x)$ by using the method of variation of parameters.

(a) $u_1 = e^{-x}\sin(e^{-x})$

(b) $u_1 = \sin(e^{-x})$

(c) $u_1 = -\sin(e^{-x})$

(d) $u_1 = \cos(e^{-x})$

(e) $u_1 = -\cos(e^{-x})$

15. The general solution of the differential equation $x^2y'' - xy' + y = 0$ is

(a) $y = c_1x^{-1} + c_2x^{-1}\ln x$

(b) $y = c_1x + c_2x\ln x$

(c) $y = c_1x + c_2x^{-1}$

(d) $y = x^{1/2}\Big(c_1\cos(\frac{\sqrt{3}}{2}\ln x) + c_2\sin(\frac{\sqrt{3}}{2}\ln x)\Big)$

(e) $y = x\Big(c_1\cos(\ln x) + c_2\sin(\ln x)\Big)$

16. Consider the system $\begin{cases} x' = 2x - y \\ y' = 3x - 2y \end{cases}$, where $x = x(t)$, and $y = y(t)$. Then the solution for $x(t)$ is given by

(a) $x(t) = c_1 e^t + c_2 e^{-t}$

(b) $x(t) = c_1 e^t + c_2 t e^t$

(c) $x(t) = c_1 e^{-t} + c_2 t e^{-t}$

(d) $x(t) = e^{\frac{t}{2}} \left[c_1 \cos\left(\frac{\sqrt{3}}{2}t\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}t\right) \right]$

(e) $x(t) = ce^t$

17. The sum of the series $\sum_{n=1}^{\infty} \frac{3^n}{\pi^{n-1}}$ is

(a) $\frac{\pi}{\pi - 3}$

(b) $\frac{3}{\pi - 3}$

(c) $\frac{3\pi}{\pi - 3}$

(d) 3π

(e) The series diverges.

18. The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if

(a) $p \geq 1$

(b) $p > 1$

(c) $p = 1$

(d) $p < 1$

(e) $-1 < p < 1$

19. Which of the three series below converge(s)?

1) $\sum_{n=1}^{\infty} \frac{4n+1}{5n+1}$ 2) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{e^n}}$ 3) $\sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$

(a) 1) and 2)

(b) 2) and 3)

(c) 1)

(d) 2)

(e) 3)

20. Which of the three series below converge?

1) $\sum_{n=1}^{\infty} \frac{1}{n^2+\sqrt{n}}$ 2) $\sum_{n=1}^{\infty} \frac{n}{n\sqrt{n}-1}$ 3) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+\ln n}$

(a) 1) and 2)

(b) 2) and 3)

(c) 1) and 3)

(d) 1), 2) and 3)

(e) None of the above

21. Which of the three series below are absolutely convergent?

1) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n}}$ 2) $\sum_{n=0}^{\infty} \frac{\sin n}{3^n}$ 3) $\sum_{n=1}^{\infty} \frac{1}{n^3}$

(a) 1) and 2)

(b) 2) and 3)

(c) 1) and 3)

(d) 1), 2) and 3)

(e) None of the above

22. Which of the three series below are absolutely convergent?

1) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ 2) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ 3) $\sum_{n=0}^{\infty} \frac{(-1)^{n}}{3^n}$

(a) 1) and 2)

(b) 2) and 3)

(c) 1) and 3)

(d) 1), 2) and 3)

(e) None of the above

23. Which of the three series below diverge?

1) $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ 2) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n}}$ 3) $\sum_{n=0}^{\infty} \left(\frac{3n^2+1}{2n^2+1}\right)^n$

(a) 1) and 2)

(b) 2) and 3)

(c) 1) and 3)

(d) 1), 2) and 3)

(e) None of the above

24. The radius of convergence R of the series $\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{3^n n!}$ is

(a) $R = 0$

(b) $R = 2$

(c) $R = 3$

(d) $R = 6$

(e) $R = \infty$

25. The interval of convergence I of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{\sqrt{n}\, 2^n}$ is

(a) $I = (1,\ 5)$

(b) $I = (1,\ 5]$

(c) $I = [1,\ 5)$

(d) $I = [1,\ 5]$

(e) None of the above

26. A power series representation of the function $f(x) = \dfrac{x}{1+x^3}$, along with its interval of converge I, is

(a) $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{3n}, \qquad I = (-1,\ 1)$

(b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{3n}, \qquad I = [0,\ 1)$

(c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{3n+1}, \quad I = (-1,\ 1)$

(d) $\displaystyle\sum_{n=0}^{\infty} x^{3n+1}, \qquad I = (-1,\ 1)$

(e) $\displaystyle\sum_{n=0}^{\infty} x^{3n+1}, \qquad I = [0,\ 1)$

27. The first four terms of the Maclaurin series for $f(x) = \dfrac{1}{\sqrt{1-x}}$ are

(a) $1 + \dfrac{1}{2}x + \dfrac{1}{4}x^2 + \dfrac{1}{6}x^3$

(b) $1 - \dfrac{1}{2}x + \dfrac{1}{6}x^2 + \dfrac{1}{24}x^3$

(c) $1 + \dfrac{1}{2}x + \dfrac{3}{4}x^2 + \dfrac{15}{16}x^3$

(d) $1 - \dfrac{1}{2}x + \dfrac{3}{8}x^2 + \dfrac{7}{24}x^3$

(e) $1 + \dfrac{1}{2}x + \dfrac{3}{8}x^2 + \dfrac{5}{16}x^3$

28. The coefficient of x^2 in the binomial series expansion of $f(x) = \sqrt[3]{1+x}$ is

(a) $\frac{1}{3}$

(b) $-\frac{1}{3}$

(c) $\frac{1}{9}$

(d) $-\frac{1}{9}$

(e) $\frac{10}{162}$

29. The coefficient of $(x-1)^3$ in the Taylor expansion of the function $f(x) = \ln x$ at the number $a = 1$ is

(a) 1

(b) $\frac{1}{2}$

(c) $-\frac{1}{2}$

(d) $\frac{1}{3}$

(e) $-\frac{1}{3}$

30. In the power series solution $y = \sum_{n=0}^{\infty} c_n x^n$ of the differential equation $y'' + xy' + y = 0$, the coefficients c_n $(n > 0)$ satisfy the recursion formula

(a) $c_{n+2} = \frac{1}{(n+2)(n+1)}\, c_{n-1}$

(b) $c_{n+2} = \frac{n}{(n+2)(n+1)}\, c_n$

(c) $c_{n+2} = \frac{-1}{(n+2)}\, c_n$

(d) $c_{n+2} = \frac{-1}{(n+2)(n+1)}\, c_n$

(e) $c_{n+1} = -\frac{n+1}{n}\, c_n$

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1. (d)
2. (b)
3. (a)
4. (c)
5. (c)
6. (d)
7. (b)
8. (b)
9. (e)
10. (a)
11. (e)
12. (b)
13. (d)
14. (b)
15. (b)
16. (a)
17. (c)
18. (b)
19. (b)
20. (c)
21. (b)
22. (e)
23. (c)
24. (e)
25. (e)
26. (c)

27. (e)
28. (d)
29. (d)
30. (c)