Concordia University
Department of Computer Science and Software Engineering
COMP 232: Mathematics for Computer Science

Assignment 4: Fall 2013
Due date: November 29, midnight (hard deadline)

1. Let x be a real number. Show that
$$\lfloor 3x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{3} \rfloor + \lfloor x + \frac{2}{3} \rfloor$$

2. The Fibonacci numbers are defined as follows: $f_0 = 0$, $f_1 = 1$, and for $n \geq 2$, $f_n = f_{n-1} + f_{n-2}$. Prove that for every positive integer n,
$$f_3 + f_6 + \cdots + f_{3n} = \frac{1}{2}(f_{3n+2} - 1)$$

3. For the following relations on the set of all real numbers, state whether or not they are reflexive, symmetric, anti-symmetric, and/or transitive. Justify your answers.
   (a) $R = \{(x, y) \mid xy \geq 0\}$
   (b) $R = \{(x, y) \mid x = 1 \text{ or } y = 1\}$

4. Find the smallest relation containing the relation $\{(1, 2), (2, 1), (2, 3), (3, 4), (4, 1)\}$ that is
   (a) reflexive and transitive
   (b) reflexive, symmetric, and transitive

5. Given a relation R, is the symmetric closure of the transitive closure of R equal to the transitive closure of the symmetric closure of R? If yes, prove it. If no, then give a counter-example.

6. Let $S = \{u, v, w\}$. List all equivalence relations on S. How many of these are also partial orders?

7. Let R be the relation on $Z^+ \times Z^+$ such that $(a, b)R(c, d)$ if $gcd(a, b) = gcd(c, d)$.
   (a) Prove that R is an equivalence relation.
   (b) What is the equivalence class of $(1, 2)$?
   (c) Give an interpretation of the equivalence classes for R.

8. Let R be the relation on the set of all logical propositions defined as

$$aRb \text{ whenever } a \to b \equiv True$$

   (a) Is R an equivalence relation? Justify your answer.
   (b) Is R a partial order? Justify your answer.

9. Let R be the relation on Z such that xRy if and only if $x - y = c$.

   (a) Define R^2.
   (b) Define R^i for arbitrary $i \geq 1$.
   (c) Define R^*, the transitive closure of R.
   (d) Is R an equivalence relation? Justify your answer.
   (e) Is R^* an equivalence relation? Justify your answer.

10. Give proofs by induction for the following:

   (a) Let R and S be relations such that $R \subseteq S$. Prove that $R^n \subseteq S^n$ for all positive integers n.
   (b) Let R be a symmetric relation. Prove that R^n is symmetric for all positive integers n.