**Family Name:**__________________ **Given Name:**__________________ **I.D.#**_________

**MAT1339 - Fall 2017 - Assignment 1**
**Due: Monday, October 2, by 2 pm.**

**Grading Scheme:** Total: 10 points, where Q3: 4 points; Q4(b): 4 points; Q5(b): 2 points.

**You need to staple your assignment, and put your assignment into the box in the Hall of Math Building, 585 King Edward. Make sure the section on the box is your section.**

Instructions:
You should show your work for multiple choice questions.
For long questions you should write all the steps, similar to what we did in class.
You **should** write your own assignments. You are not allowed to copy another student's work. Note that plagiarism is taken very seriously at the University of Ottawa.

**Question 1** Find the average rate of change for $f(x) = 2x^3 + x^2 + 4x$ on $-1 \leq x \leq 3$.

(a) 40
(b) $\frac{35}{2}$
(c) 20
(d) 15

(C). $\frac{f(3)-f(-1)}{3-(-1)} = \frac{75-(-5)}{4} = 20.$

**Question 2** Find the instantaneous rate of change of the function in question 1 at $x = -1$.

(a) 8
(b) 12
(c) 4
(d) 0

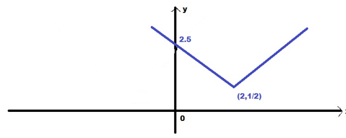
(a). $f'(x) = 6x^2 + 2x + 4,\ f'(-1) = 6 - 2 + 4 = 8.$

**Question 3** Use the definition of derivative (first principle) to find the derivative of $f(x) = \sqrt{x^2+3}$.

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{(x+h)^2+3}-\sqrt{x^2+3}}{h} \\
&= \lim_{h\to 0} \frac{(\sqrt{(x+h)^2+3}-\sqrt{x^2+3})(\sqrt{(x+h)^2+3}+\sqrt{x^2+3})}{h(\sqrt{(x+h)^2+3}+\sqrt{x^2+3})} \\
&= \lim_{h\to 0} \frac{(x+h)^2+3-[x^2+3]}{h(\sqrt{(x+h)^2+3}+\sqrt{x^2+3})} \\
&= \lim_{h\to 0} \frac{2xh+h^2}{h(\sqrt{(x+h)^2+3}+\sqrt{x^2+3})} \\
&= \lim_{h\to 0} \frac{2x+h}{\sqrt{(x+h)^2+3}+\sqrt{x^2+3}} \\
&= \frac{2x}{2\sqrt{x^2+3}} \\
&= \frac{x}{\sqrt{x^2+3}}
\end{aligned}
$$

**Question 4** Let $f(x) = |x-2| + 1/2$.

(a) Sketch the graph of $f(x)$.




(b) Is $f(x)$ continuous? Justify your answer. (Hint: write $f(x)$ as a piecewise defined function.)

$f(x) = \begin{cases} x - 3/2, & x \geq 2; \\ -x + 5/2, & x < 2. \end{cases}$ Obviously, $f(x)$ is continuous for any $x \neq 2$.

At $x = 2$,

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x - 3/2 = 1/2; \quad \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} -x + 5/2 = 1/2$$

Since $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$, $f(x)$ is also continuous at $x = 2$.

(c) Find $f'(1)$ and $f'(3)$.

When $x < 2$, $f'(x) = (x - 3/2)' = 1$, $f'(1) = 1$; when $x < 2$, $f'(x) = (-x + 5/2)' = -1$, $f'(3) = -1$.

(d) Show that $f'(2)$ does not exist. (Hint: use the definition of derivatives.)

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{f(2+h) - 1/2}{h}.$$

$$\lim_{h\to 0^+} \frac{f(2+h) - 1/2}{h} = \lim_{h\to 0^+} \frac{(2+h) - 3/2 - 1/2}{h} = \lim_{h\to 0^+} \frac{h}{h} = 1,$$

$$\lim_{h\to 0^-} \frac{f(2+h) - 1/2}{h} = \lim_{h\to 0^-} \frac{-(2+h) + 5/2 - 1/2}{h} = \lim_{h\to 0^+} \frac{-h}{h} = -1.$$

Since

$$\lim_{h\to 0^+} \frac{f(2+h) - 1/2}{h} \neq \lim_{h\to 0^-} \frac{f(2+h) - 1/2}{h},$$

$f'(2)$ does not exist.

**Question 5** Use differentiation rules to find $f'(x)$ (don't need to expand and simplify!):

(a) $f(x) = (2x^5 + 3x^2)(x^3 + 1)(\frac{1}{\sqrt{x}})$

By Product Rule,

$$f'(x) = (2x^5 + 3x^2)'(x^3 + 1)(\frac{1}{\sqrt{x}}) + (2x^5 + 3x^2)(x^3 + 1)'(\frac{1}{\sqrt{x}}) + (2x^5 + 3x^2)(x^3 + 1)(\frac{1}{\sqrt{x}})'$$

$$= (10x^4 + 6x)(x^3 + 1)(\frac{1}{\sqrt{x}}) + (2x^5 + 3x^2)(3x^2)(\frac{1}{\sqrt{x}}) + (2x^5 + 3x^2)(x^3 + 1)(-\frac{1}{2}x^{-3/2})$$

(b) $f(x) = \frac{\frac{3}{8}x^8+3x+5}{x^3+1}$

By Quotient Rule,

$$f'(x) = \frac{(\frac{3}{8}x^8 + 3x + 5)'(x^3 + 1) - (\frac{3}{8}x^8 + 3x + 5)(x^3 + 1)'}{(x^3 + 1)^2}$$

$$= \frac{(3x^7 + 3)(x^3 + 1) - (\frac{3}{8}x^8 + 3x + 5)(3x^2)}{(x^3 + 1)^2}$$

**Question 6** The position of a moving object is given by $s(t) = \frac{1}{3}t^3 + t^2 - 3t + 2$. We start observing the object at time $t = 0$.

(a) Find the velocity function $v(t)$.

$$v(t) = s'(t) = t^2 + 2t - 3$$

(b) Find the acceleration function $a(t)$.

$$a(t) = v'(t) = 2t + 2.$$

(c) Find the time at which we see the object stop.

$$v(t) = 0, \Rightarrow t^2 + 2t - 3 = 0, \Rightarrow t = 1.$$

$t = -3$ is not allowed.

(d) Find the position of the object when we see it stop.

$s(1) = \frac{1}{3} + 1 - 3 + 2 = \frac{1}{3}$.

(e) Determine the interval of $t$ on which the object is speeding up.

Speeding up means

$$v(t)a(t) > 0, \Rightarrow (t^2 + 2t - 3)(2t + 2) > 0, \Rightarrow t > 1.$$