MAT 2377 (Winter 2017)

Assignment 1 - Solution

1. Consider the following events :
   M : a new car will require repairs on the engine.
   T : a new car will require repairs on the drive train.
   We have that
   $$P(M) = 0.85; \quad P(T) = 0.37; \quad P(M \cap T) = 0.25$$
   (a) We want to find $P(M \cup T) = P(M) + P(T) - P(M \cap T) = 0.97$
   (b) We want to find $P(M') = 1 - P(M) = 1 - 0.85 = 0.15$
   (c) We want to find $P(M' \cup T') = 1 - P(M \cap T) = 0.75$
2. Consider the following events :
   C : the strand has a high conductivity.
   R : the strand has a high strength.

   a) We have to find $P(C \cap R) = 73/98 = 0.7448$
   b) We have to find $P(C' \cup R') = 1 - P((C' \cup R')') = 1 - P(C \cap R) = 1 - 0.7448 = 0.2551$
   c) We have to find $P(R'|C') = \frac{P(R' \cap C')}{P(C')} = \frac{4/98}{20/98} = \frac{4}{20} = 0.2$
   d) Since $R' \cap C' \neq \emptyset$, R' and C' are not mutually exclusive.
   e) We have $P(R') = 9/98 = 0.0918$, but $P(R'|C') = 0.2$. It follows that, $P(R') \neq P(R'|C')$. Therefore, the data suggests that R' and C' are NOT independent.
3. Let A_i be the event such that among the five, we have i strands with hight resistance.
   (a)We want to find
   $$P(A_5) = \frac{\binom{89}{5}}{\binom{98}{5}} = 0.6112.$$
   (b) We want to find
   $$P(A_0) + P(A_1) = \frac{\binom{89}{0}\binom{9}{5}}{\binom{98}{5}} + \frac{\binom{89}{1}\binom{9}{4}}{\binom{98}{5}} = 0.00017.$$
4. Consider the following events :
   A : having a room at Ramada Inn.
   B : having a room at Sheraton.
   C : having a room at Lakeview Motor Lodge.
   D : the plumbing is defective.
   First, we have
   $$P(A) = 0.21; P(B) = 0.49; P(C) = 0.3; P(D|A) = 0.04; P(D|B) = 0.035; P(D|C) = 0.075.$$
   To identify the probabilities in the statement.
   (a) We want to find
   $$P(D) = P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C) = 0.04805.$$
   (b) We want to find
   $$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{P(D|C)P(C)}{P(D)} = \frac{(0.075)(0.3)}{0.04805} = 0.46826.$$

5. Let A_i be the event that the component i works. Consider the events $C = A_1 \cap A_2 \cap A_3$ and $D = A_4 \cup A_5 \cup A_6$. We compute

$$P(C) = P(A_1)P(A_2)P(A_3) = (0.9)^3 = 0.729$$

$$P(D) = 1 - P(A_4' \cap A_5' \cap A_6') = 1 - P(A_4')P(A_5')P(A_6') = 1 - (0.05)^3 = 0.999875.$$

Consider the event $E = D \cap A_7$. We have $P(E) = P(D \cap A_7) = P(D)P(A_7) = (0.999875)(0.9) = 0.8999$. The probability that the circuit will operate is

$$P(E \cup C) = 1 - P(E' \cap C') = 1 - P(E')P(C') = 1 - (1 - 0.8999)(1 - 0.729) = 0.973.$$

6. Here are two enumeration questions.

   (a) There are 60 choices for the first number, 57 choices for the second and 57 choices for the third. Then there are $60 \times 57 \times 57 = 194,940$ possible combinations.

   (b) For the first flavor, 4 of the 24 students are selected, then for the second flavor, 4 students are selected from the 20 students who remain and so on. So, the number of different ways to distribute the 6 flavors of candy to 24 children is

$$\binom{24}{4}\binom{20}{4}\binom{16}{4}\binom{12}{4}\binom{8}{4}\binom{4}{4} = \frac{24!}{(4!)^6} = 3.24 \times 10^{15}.$$