MAT 2379 B, Introduction to Biostatistics

Assignment 1 solutions (Total = 35)
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2.2 Let A be the event that the day is frigid and B the event that there is light snow that day. We know that $P(A) = 0.51$, $P(B) = 0.68$ and $P(A \cap B) = 0.35$. By the addition rule,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.51 + 0.68 - 0.35 = 0.84.$$

The desired probability is

$$P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.84 = 0.16.$$

[10] **2.4** Let A be the event that a randomly chosen woman in this study suffers from inoperable locoregional recurrent disease, and B the event that she has distant metastases. We know that

$$P(A) = \frac{87}{268} = 0.325, \quad P(B) = \frac{140}{268} = 0.522 \quad \text{and} \quad P(A \cap B) = \frac{41}{268} = 0.153.$$

(a) The probability that a randomly chosen woman in this study suffers from inoperable locoregional recurrent disease, but does not have distant metastases is:

$$P(A \cap B') = P(A) - P(A \cap B) = \frac{87 - 41}{268} = \frac{46}{268} = 0.172.$$

b) The probability that a randomly chosen woman in this study suffers from distant metastases but does not have inoperable locoregional recurrent disease is:

$$P(A' \cap B) = P(B) - P(A \cap B) = \frac{140 - 41}{268} = \frac{99}{268} = 0.369.$$

[5] **2.6** Let A be the event that the tree has been affected by a fire and B be the event that the tree has been affected by budworms. We know that $P(A' \cap B') = 0.75$, $P(A \cap B') = 0.12$, and $P(B \cap A') = 0.05$.
Since, $P(A' \cap B') = 0.75$, then $P(A \cup B) = 1 - 0.75 = 0.25$. From a Venn diagram, we observe that

$$\begin{aligned} 0.25 = P(A \cup B) &= P(A \cap B') + P(B \cap A') + P(A \cap B) \\ &= 0.12 + 0.05 + P(A \cap B) \end{aligned}$$

Hence,

$$P(A \cap B) = 0.25 - 0.12 - 0.05 = 0.08$$

3.2 We denote by D the event that a patient has sleep apnea, by A the event that the patient has symptom A and by B the event that the patient has symptom B. The four events $G_1 = A \cap B$, $G_2 = A \cap B'$, $G_3 = A' \cap B$ and $G_4 = A' \cap B'$ form a partition of the sample space S. We know that

$$P(G_1) = 0.56, \quad P(G_2) = 0.21, \quad P(G_3) = 0.19, \quad P(G_4) = 0.04$$

and

$$P(D|G_1) = 0.60, \quad P(D|G_2) = 0.45, \quad P(D|G_3) = 0.35, \quad P(D|G_4) = 0.03.$$

[5] (a) By the total probability rule,

$$\begin{aligned} P(D) &= P(D|G_1)P(G_1) + P(D|G_2)P(G_2) + P(D|G_3)P(G_3) + P(D|G_4)P(G_4) \\ &= (0.60)(0.56) + (0.45)(0.21) + (0.35)(0.19) + (0.03)(0.04) = 0.4982 \end{aligned}$$

[10] (b) We have to calculate $P(A|D)$. Note that

$$P(A|D) = P(G_1|D) + P(G_2|D).$$

By Bayes' rule,

$$\begin{aligned} P(G_1|D) &= \frac{P(D|G_1)P(G_1)}{P(D)} \\ &= \frac{(0.60)(0.56)}{0.4982} = 0.6744 \end{aligned}$$

and

$$\begin{aligned} P(G_2|D) &= \frac{P(D|G_2)P(G_2)}{P(D)} \\ &= \frac{(0.45)(0.21)}{0.4982} = 0.1896. \end{aligned}$$

Hence, $P(A|D) = 0.6744 + 0.1896 = 0.864$.

3.8 Let G and C be the respective events of guessing the answer and correctly choosing the right answer. We know that $P(C|G) = 1/5 = 0.2$, $P(C|G') = 0.8$ and $P(G) = 0.15$.
(a) We want

$$P(C) = P(C|G')P(G') + P(C|G)P(G) = 0.8(0.85) + 0.2(0.15) = 0.71.$$

(b) We want

$$P(G'|C) = \frac{P(G' \cap C)}{P(C)} = \frac{P(C|G')P(G')}{P(C)} = \frac{0.8(0.85)}{0.71} = 0.9577.$$

[5] **3.10** Let A be the event that the person is a non-smoker and B be the event that the person has emphysema. We know that $P(A) = 0.85$ and $P(A \cap B) = 0.24$. The desired probability is:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.24}{0.85} = 0.282.$$

3.3 (Adjusted with the given table)
(a) The false positive rate is: $\alpha = P(\text{test}+|\text{true}-) = 56/993 = 0.056$. The false negative rate is: $\beta = P(\text{test}-|\text{true}+) = 3/7 = 0.43$.

(b) The sensitivity is $P(\text{test}+|\text{true}+) = 4/7 = 0.571$. The specificity is $P(\text{test}-|\text{true}-) = 937/993 = 0.943$

(c) The positive predictive value is $P(\text{true}+|\text{test}+) = 4/60 = 0.067$. The positive predictive value is $P(\text{true}-|\text{test}-) = 937/940 = 0.996$.

AQ (Additional question)

Let B be the even that the randomly chosen student is a boy, therefore $P(B) = 40/100 = 0.4$.
Let E be the even that the randomly chosen student wears eyes glasses, therefore $P(E) = 0.4$.

$$P(E|B) = \frac{P(B \cap E)}{P(B)} = \frac{\frac{16}{100}}{0.4} = 0.4$$