# COMP 232, Fall 2017
# Assignment 1. Solution

1. For each of the following statements use a truth table to determine whether it is a tautology, a contradiction, or a contingency.

   (a) $\Big(\underbrace{(p \vee r)}_{\mathbf{a}} \wedge \underbrace{(q \vee r)}_{\mathbf{b}}\Big) \leftrightarrow \underbrace{\Big((p \wedge q) \vee r\Big)}_{\mathbf{c}}$

   **Solution:** Tautology.

| $p$ | $q$ | $r$ | $\overbrace{p \vee r}^{\mathbf{a}}$ | $\overbrace{q \vee r}^{\mathbf{b}}$ | $\mathbf{a} \wedge \mathbf{b}$ | $p \wedge q$ | $\overbrace{(p \wedge q) \vee r}^{\mathbf{c}}$ | $(\mathbf{a} \wedge \mathbf{b}) \leftrightarrow \mathbf{c}$ |
|---|---|---|---|---|---|---|---|---|
| $T$ | $T$ | $T$ | $T$ | $T$ | $T$ | $T$ | $T$ | $T$ |
| $T$ | $T$ | $F$ | $T$ | $T$ | $T$ | $T$ | $T$ | $T$ |
| $T$ | $F$ | $T$ | $T$ | $T$ | $T$ | $F$ | $T$ | $T$ |
| $F$ | $T$ | $T$ | $T$ | $T$ | $T$ | $F$ | $T$ | $T$ |
| $T$ | $F$ | $F$ | $T$ | $F$ | $F$ | $F$ | $F$ | $T$ |
| $F$ | $T$ | $F$ | $F$ | $T$ | $F$ | $F$ | $F$ | $T$ |
| $F$ | $F$ | $T$ | $T$ | $T$ | $T$ | $F$ | $T$ | $T$ |
| $F$ | $F$ | $F$ | $F$ | $F$ | $F$ | $F$ | $F$ | $T$ |

   (b) $(p \oplus q) \wedge (p \oplus \neg q)$

   **Solution:** Contradiction.

| $p$ | $q$ | $\neg q$ | $\overbrace{p \oplus q}^{\mathbf{a}}$ | $\overbrace{p \oplus \neg q}^{\mathbf{b}}$ | $\mathbf{a} \wedge \mathbf{b}$ |
|---|---|---|---|---|---|
| $T$ | $T$ | $F$ | $F$ | $T$ | $F$ |
| $T$ | $F$ | $T$ | $T$ | $F$ | $F$ |
| $F$ | $T$ | $F$ | $T$ | $F$ | $F$ |
| $F$ | $F$ | $T$ | $F$ | $T$ | $F$ |

(c) $\Big(p \to (q \to r)\Big) \leftrightarrow \Big(p \to (q \wedge r)\Big)$

**Solution:** Contingency.

| $p$ | $q$ | $r$ | $\overbrace{q \to r}^{\mathbf{a}}$ | $\overbrace{q \wedge r}^{\mathbf{b}}$ | $\overbrace{p \to \mathbf{a}}^{\mathbf{c}}$ | $\overbrace{p \to \mathbf{b}}^{\mathbf{d}}$ | $\mathbf{c} \leftrightarrow \mathbf{d}$ |
|---|---|---|---|---|---|---|---|
| $T$ | $T$ | $T$ | $T$ | $T$ | $T$ | $T$ | $T$ |
| $T$ | $T$ | $F$ | $F$ | $F$ | $F$ | $F$ | $T$ |
| $T$ | $F$ | $T$ | $T$ | $F$ | $T$ | $F$ | $F$ |
| $F$ | $T$ | $T$ | $T$ | $T$ | $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ | $T$ | $F$ | $T$ | $F$ | $F$ |
| $F$ | $T$ | $F$ | $F$ | $F$ | $T$ | $T$ | $T$ |
| $F$ | $F$ | $T$ | $T$ | $F$ | $T$ | $T$ | $T$ |
| $F$ | $F$ | $F$ | $T$ | $F$ | $T$ | $T$ | $T$ |

(d) $\underbrace{\Big(p \wedge \underbrace{(\neg q \to \neg p)}_{\mathbf{a}}\Big)}_{\mathbf{b}} \to q$

**Solution:** Tautology.

| $p$ | $q$ | $\neg p$ | $\neg q$ | $\overbrace{\neg q \to \neg p}^{\mathbf{a}}$ | $\overbrace{p \wedge \mathbf{a}}^{\mathbf{b}}$ | $\mathbf{b} \to q$ |
|---|---|---|---|---|---|---|
| $T$ | $T$ | $F$ | $F$ | $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ | $T$ | $F$ | $F$ | $T$ |
| $F$ | $T$ | $T$ | $F$ | $T$ | $F$ | $T$ |
| $F$ | $F$ | $T$ | $T$ | $T$ | $F$ | $T$ |

2. For each of the following logical equivalences state whether it is valid or invalid. If invalid then give a counterexample (*e.g.*, based on a truth table). If valid then give an algebraic proof using logical equivalences from Tables 6, 7, and 8 from Section 1.3 of textbook.

   (a) $(p \to r) \wedge (q \to r) \equiv (p \wedge q) \to r$

   **Solution:** Invalid.
   If $p = T$, $q = F$, and $r = F$ then the LHS is False, while the RHS is True.

   (b) $(p \to q) \vee (p \to r) \equiv (p \vee q) \to r$

   **Solution:** Invalid.
   If $p = T$, $q = T$, and $r = F$ then the LHS is True, while the RHS is False.

   (c) $\Big(\big((p \vee q) \wedge (p \to r) \wedge (q \to r)\big) \to r\Big) \equiv T$

   **Solution:** Valid.

$$
\begin{array}{rrl}
 & \Big((p \vee q) \wedge (p \to r) \wedge (q \to r)\Big) \to r & \\
\equiv & \neg\Big((p \vee q) \wedge (p \to r) \wedge (q \to r)\Big) \vee r & \text{law for conditional} \\
\equiv & \Big(\neg(p \vee q) \vee \neg(p \to r) \vee \neg(q \to r)\Big) \vee r & \text{generalized De Morgan} \\
\equiv & \Big(\neg(p \vee q) \vee \neg(\neg p \vee r) \vee \neg(\neg q \vee r)\Big) \vee r & \text{law for conditional, twice} \\
\equiv & \Big(\neg(p \vee q) \vee (\neg\neg p \wedge \neg r) \vee (\neg\neg q \wedge \neg r)\Big) \vee r & \text{deMorgan, twice} \\
\equiv & \Big(\neg(p \vee q) \vee (p \wedge \neg r) \vee (q \wedge \neg r)\Big) \vee r & \text{double negation, twice} \\
\equiv & \Big(\neg(p \vee q) \vee ((p \wedge \neg r) \vee (q \wedge \neg r))\Big) \vee r & \text{associativity} \\
\equiv & \Big(\neg(p \vee q) \vee ((p \vee q)) \wedge \neg r)\Big) \vee r & \text{distributivity} \\
\equiv & \Big(\neg(p \vee q) \vee (p \vee q)) \wedge (\neg(p \vee q) \vee \neg r)\Big) \vee r & \text{distributivity} \\
\equiv & \Big(T \wedge (\neg(p \vee q) \vee \neg r)\Big) \vee r & \text{excluded middle} \\
\equiv & \Big(\neg(p \vee q) \vee \neg r)\Big) \vee r & \text{identity} \\
\equiv & \neg(p \vee q) \vee (\neg r \vee r) & \text{associativity} \\
\equiv & \neg(p \vee q) \vee T & \text{excluded middle} \\
\equiv & T & \text{domination}
\end{array}
$$

(d) $\Big(\big((p \to q) \wedge (q \to r)\big) \to (p \to r)\Big) \;\equiv\; T$

**Solution:** Valid.
We shall instead prove $\neg\Big(\big((p \to q) \wedge (q \to r)\big) \to (p \to r)\Big) \;\equiv\; F$.
The claim then follows since $\neg A \equiv F$ if and only if $A \equiv T$.

$$\begin{aligned}
& \neg\Big(\big((p \to q) \wedge (q \to r)\big) \to (p \to r)\Big) & \\
\equiv\;& \neg\Big(\neg\big((p \to q) \wedge (q \to r)\big) \vee (p \to r)\Big) & \text{law for conditional} \\
\equiv\;& \neg\neg\big((p \to q) \wedge (q \to r)\big) \wedge \neg(p \to r) & \text{De Morgan} \\
\equiv\;& \big((p \to q) \wedge (q \to r)\big) \wedge \neg(p \to r) & \text{double negation} \\
\equiv\;& \big((\neg p \vee q) \wedge (\neg q \vee r)\big) \wedge \neg(\neg p \vee r) & \text{law for conditional, trice} \\
\equiv\;& \big((\neg p \vee q) \wedge (\neg q \vee r)\big) \wedge (\neg\neg p \wedge \neg r) & \text{De Morgan} \\
\equiv\;& \big((\neg p \vee q) \wedge (\neg q \vee r)\big) \wedge (p \wedge \neg r) & \text{double negation} \\
\equiv\;& (\neg p \vee q) \wedge \big((\neg q \vee r) \wedge p\big) \wedge \neg r & \text{associativity} \\
\equiv\;& (\neg p \vee q) \wedge \big(p \wedge (\neg q \vee r)\big) \wedge \neg r & \text{commutativity} \\
\equiv\;& \big((\neg p \vee q) \wedge p\big) \wedge \big((\neg q \vee r) \wedge \neg r\big) & \text{associativity} \\
\equiv\;& \big((\neg p \wedge p) \vee (q \wedge p)\big) \wedge \big((\neg q \wedge \neg r) \vee (r \wedge \neg r)\big) & \text{distributivity, twice} \\
\equiv\;& \big(F \vee (q \wedge p)\big) \wedge \big((\neg q \wedge \neg r) \vee F\big) & \text{excluded middle, twice} \\
\equiv\;& (q \wedge p) \wedge (\neg q \wedge \neg r) & \text{identity, twice} \\
\equiv\;& (p \wedge q) \wedge (\neg q \wedge \neg r) & \text{commutativity} \\
\equiv\;& p \wedge (q \wedge \neg q) \wedge \neg r & \text{associativity} \\
\equiv\;& p \wedge F \wedge \neg r & \text{excluded middle} \\
\equiv\;& (p \wedge F) \wedge \neg r & \text{associativity} \\
\equiv\;& F \wedge \neg r & \text{domination} \\
\equiv\;& F & \text{domination}
\end{aligned}$$

Here is an alternate simpler proof.

$$
\begin{aligned}
& \big((p \to q) \land (q \to r)\big) \to (p \to r) & \\
\equiv\ & \neg\big((p \to q) \land (q \to r)\big) \lor (p \to r) & \text{law for conditional} \\
\equiv\ & \neg\big((\neg p \lor q) \land (\neg q \lor r)\big) \lor (\neg p \lor r) & \text{law for conditional, trice} \\
\equiv\ & \big((p \land \neg q) \lor (q \land \neg r)\big) \lor (\neg p \lor r) & \text{De Morgan, trice} \\
\equiv\ & \big((p \land \neg q) \lor (\neg p \lor r)\big) \lor (q \land \neg r) & \text{commutativity, associativity} \\
\equiv\ & \big(((p \lor \neg p) \lor r) \land (\neg q \lor \neg p \lor r)\big) \lor (q \land \neg r) & \text{distributivity, associativity} \\
\equiv\ & \big((T \lor r) \land (\neg q \lor \neg p \lor r)\big) \lor (q \land \neg r) & \text{excluded middle} \\
\equiv\ & (\neg q \lor \neg p \lor r) \lor (q \land \neg r) & \text{domination, associativity} \\
\equiv\ & ((\neg q \lor q) \lor \neg p \lor r) \land (\neg q \lor \neg p \lor (r \lor \neg r)) & \text{distributivity, commutativity (twice), associativity (twice)} \\
\equiv\ & (T \lor \neg p \lor r) \land (\neg q \lor \neg p \lor T) & \text{excluded middle, twice} \\
\equiv\ & T \land T & \text{domination, twice} \\
\equiv\ & T & \text{conjunction}
\end{aligned}
$$

3. Which of the following conditions is *necessary* for the natural number $n$ to be divisible by 6. The natural numbers are $\mathbb{N} = \{0, 1, 2, \ldots, \}$.

   (a) $n$ is divisible by 3.

   (b) $n$ is divisible by 9.

   (c) $n$ is divisible by 12.

   (d) $n = 24$

   (e) $n^2$ is divisible by 3.

   (f) $n$ is even and divisible by 3.

**Solution:**

Necessary means *If $n$ is divisible by 6, then condition.* Conditions (a), (e), and (f) are necessary. We have

(a) If $n$ is divisible by 6, then $n$ is divisible by 3.

(e) If $n$ is divisible by 6, then $n^2$ is divisible by 3.

(f) If $n$ is divisible by 6, then $n$ is even and divisible by 3.

Sufficient means *If condition, then $n$ is divisible by 6.* Conditions (c), (d), and (f) are sufficient. We have

(c) If $n$ is divisible by 12, then $n$ is divisible by 6.

(d) If $n = 24$, then $n$ is divisible by 6.

(f) If $n$ is even and divisible by 3, then $n$ is divisible by 6.

4. A set of propositions is *consistent* if there is an assignment of truth values to each of the variables in the propositions that makes each proposition true. Is the following set of propositions consistent?

   (a) *If the file system is not locked, then new messages will be queued.*

   (b) *If the file system is not locked, then the system is functioning normally, and conversely.*

   (c) *If new messages are not queued, then they will be sent to the message buffer.*

   (d) *If the file system is not locked, then new messages will be sent to the message buffer.*

   (e) *New messages will not be sent to the message buffer.*

**Solution:**

Let us define the following propositions:

$FL =_{\texttt{def}}$ *The file system is locked.*

$NQ =_{\texttt{def}}$ *New messages will be queued.*

$FN =_{\texttt{def}}$ *The system is functioning normally.*

$NQ =_{\texttt{def}}$ *New messages will be queued.*

$NB =_{\texttt{def}}$ *New messages will be sent to the message buffer.*

We can now formalize propositions (a) – (e):

(a) $\neg FL \rightarrow NQ$

(b) $\neg FL \leftrightarrow FN$

(c) $\neg NQ \rightarrow NB$

(d) $\neg FL \rightarrow NB$

(e) $\neg NB$

The set (a) – (e) of propositions (the conjunction of the proposition in the set) is indeed satisfiable. A satisfying truth assignment is

$$FL = True,\ NQ = True,\ FN = False,\ NB = False$$

5. Suppose the domain of the propositional function $P(x, y)$ consists of pairs $x$ and $y$, where $x = 1$, 2, or 3, and $y = 1$, 2, or 3. Write out the propositions below using disjunctions and conjunctions only.

(a) $\exists x\, P(x, 3)$

**Solution:** $P(1,3) \vee P(2,3) \vee P(3,3)$

(b) $\forall y\, \neg P(2, y)$

**Solution:** $\neg P(2,1) \wedge \neg P(2,2) \wedge \neg P(2,3)$

(c) $\forall x\, \exists y\, P(x, y)$

**Solution:**

$$\begin{aligned}
& \forall x\, \exists y\, P(x,y) \\
\equiv \quad & \big(\exists y\, P(1,y)\big) \bigwedge \big(\exists y\, P(2,y)\big) \bigwedge \big(\exists y\, P(3,y)\big) \\
\equiv \quad & \big(P(1,1) \vee P(1,2) \vee P(1,3)\big) \\
\bigwedge \quad & \big(P(2,1) \vee P(2,2) \vee P(2,3)\big) \\
\bigwedge \quad & \big(P(3,1) \vee P(3,2) \vee P(3,3)\big)
\end{aligned}$$

(d) $\exists x\, \forall y\, \neg P(x, y)$

**Solution:**

$$\begin{aligned}
& \exists x\, \forall y\, \neg P(x,y) \\
\equiv \quad & \big(\forall y\, \neg P(1,y)\big) \bigvee \big(\forall y\, \neg P(2,y)\big) \bigvee \big(\forall y\, \neg P(3,y)\big) \\
\equiv \quad & \big(\neg P(1,1) \wedge \neg P(1,2) \wedge \neg P(1,3)\big) \\
\bigvee \quad & \big(\neg P(2,1) \wedge \neg P(2,2) \wedge \neg P(2,3)\big) \\
\bigvee \quad & \big(\neg P(3,1) \wedge \neg P(3,2) \wedge \neg P(3,3)\big)
\end{aligned}$$

6. Let the domain for $x$ be the set of all students in this class and the domain for $y$ be the set of all countries in the world. Let $P(x, y)$ denote student x has visited country y and $Q(x, y)$ denote student x has a friend in country y. Express each of the following using logical operations and quantifiers, and the propositional functions $P(x, y)$ and $Q(x, y)$.

   (a) *Carlos has visited Bulgaria.*

   **Solution:** $P(Carlos, Bulgaria)$

   (b) *Every student in this class has visited the United States.*

   **Solution:** $\forall x\, P(x, UnitedStates)$

   (c) *Every student in this class has visited some country in the world.*

   **Solution:** $\forall x\, \exists y\, P(x, y)$

   (d) *There is no country that every student in this class has visited.*

   **Solution:** $\forall y\, \exists x\, \neg P(x, y)$

   **Alternate Solution:** $\neg \exists y\, \forall x\, P(x, y)$

   (e) *There are two students in this class, who between them, have a friend in every country in the world.*

   **Solution:** $\exists x\, \exists y\, \Big(x \neq y \wedge \forall z [Q(x, z) \vee Q(y, z)]\Big)$

   (f) *Nobody in this class has visited a country in which they did not have a friend.*

   **Solution:** $\forall x\, \forall y\, \Big(P(x, y) \rightarrow Q(x, y)\Big)$

   Equivalent solution: $\neg \Big[\exists x\, \exists y\, \Big(P(x, y) \wedge \neg Q(x, y)\Big)\Big]$

7. For each part in the previous question, form the negation of the statement so that all negation symbols occur immediately in front of predicates. For example:

$$\neg\Big(\forall x\big(P(x) \wedge Q(x)\big)\Big) \equiv \exists x\Big(\neg\big((P(x) \wedge Q(x)\big)\Big) \equiv \exists x\Big(\big(\neg P(x)\big) \vee \big(\neg Q(x)\big)\Big)$$

   (a) **Solution:** $\neg P(Carlos, Bulgaria)$

   *Carlos has not visited Bulgaria*

   (b) **Solution:** $\neg\Big(\forall x\ P(x, UnitedStates)\Big) \equiv \exists x\ \Big(\neg P(x, UnitedStates)\Big)$

   *There is a student in this class who has not visited the United States*

   (c) **Solution:** $\neg\left(\forall x\left[\exists y\ P(x,y)\right]\right) \equiv \exists x\ \neg\Big[\exists y\ P(x,y)\Big] \equiv \exists x\ \forall y\ \Big[\neg P(x,y)\Big]$

   *There is a student in this class who has not visited any country*

   (d) **Solution:**

$$\begin{aligned} &\neg\left(\forall y\left[\exists x\ \neg P(x,y)\right]\right) && \equiv \\ &\exists y\ \neg\Big[\exists x \neg P(x,y)\Big] && \equiv \\ &\exists y\ \forall x\ \neg\Big[\neg P(x,y)\Big] && \equiv \\ &\exists y\ \forall x\ \Big[\neg\neg P(x,y)\Big] && \equiv \\ &\exists y\ \forall x\ P(x,y) \end{aligned}$$

   *There is a country that every student in this class has visited*

(e) **Solution:**

$$\neg\Big[\exists x \,\Big(\exists y\,\Big(x \neq y \wedge \forall z\Big[Q(x,z) \vee Q(y,z)\Big]\Big)\Big)\Big] \quad \equiv$$

$$\forall x\,\Big[\neg\Big(\exists y\,\Big(x \neq y \wedge \forall z\Big[Q(x,z) \vee Q(y,z)\Big]\Big)\Big)\Big] \quad \equiv$$

$$\forall x\,\Big[\forall y\,\neg\Big(x \neq y \wedge \forall z\Big[Q(x,z) \vee Q(y,z)\Big]\Big)\Big] \quad \equiv$$

$$\forall x\,\Big[\forall y\,\Big(x = y \vee \neg\Big(\forall z\Big[Q(x,z) \vee Q(y,z)\Big]\Big)\Big] \quad \equiv$$

$$\forall x\,\Big[\forall y\,\Big(x = y \vee \exists z\Big(\neg\Big[Q(x,z) \vee Q(y,z)\Big]\Big)\Big)\Big] \quad \equiv$$

$$\forall x\,\Big[\forall y\,\Big(x = y \vee \exists z\Big(\neg Q(x,z) \wedge \neg Q(y,z)\Big)\Big)\Big]$$

*For every pair of distinct students in this class, there is a country where neither one of them has a friend*

(f) **Solution:**

$$\neg\Big[\forall x\,\forall y\,\Big(P(x,y) \rightarrow Q(x,y)\Big)\Big] \quad \equiv$$

$$\exists x\Big[\neg\Big(\forall y\,\Big(P(x,y) \rightarrow Q(x,y)\Big)\Big)\Big] \quad \equiv$$

$$\exists x\Big[\exists y\,\Big(\neg\Big(P(x,y) \rightarrow Q(x,y)\Big)\Big)\Big] \quad \equiv$$

$$\exists x\Big[\exists y\,\Big(\neg\Big(\neg P(x,y) \vee Q(x,y)\Big)\Big)\Big] \quad \equiv$$

$$\exists x\Big[\exists y\,\Big(\neg\neg P(x,y) \wedge \neg Q(x,y)\Big)\Big] \quad \equiv$$

$$\exists x\Big[\exists y\,\Big(P(x,y) \wedge \neg Q(x,y)\Big)\Big]$$

*Somebody in this class has visited a country in which he/she doesn't have a friend.*

8. Negate the following statements and transform the negation so that negation symbols immediately precede predicates. (See example in Question 7.)

(a) $\exists x \exists y \Big(P(x,y)\Big) \lor \forall x \forall y \Big(Q(x,y)\Big)$

**Solution:**

$$\neg\Big[\exists x\, \exists y \Big(P(x,y)\Big) \bigvee \forall x\, \forall y \Big(Q(x,y)\Big)\Big]$$

$$\equiv \quad \neg\Big[\exists x\, \exists y \Big(P(x,y)\Big)\Big] \bigwedge \neg\Big[\forall x\, \forall y \Big(Q(x,y)\Big)\Big]$$

$$\equiv \quad \forall x\, \forall y \Big(\neg P(x,y)\Big) \bigwedge \exists x\, \exists y \Big(\neg Q(x,y)\Big)$$

(b) $\forall x \forall y \Big(Q(x,y) \leftrightarrow Q(y,x)\Big)$

**Solution:**

$$\neg\Big[\forall x\, \forall y \Big(Q(x,y) \leftrightarrow Q(y,x)\Big)\Big]$$

$$\equiv \quad \exists x\, \exists y \Big[\neg\Big(Q(x,y) \leftrightarrow Q(y,x)\Big)\Big]$$

$$\equiv \quad \exists x\, \exists y \Big[\neg\Big[\big(Q(x,y) \to Q(y,x)\big) \bigwedge \big(Q(y,x) \to Q(x,y)\big)\Big]\Big]$$

$$\equiv \quad \exists x\, \exists y \Big[\neg\big(Q(x,y) \to Q(y,x)\big) \bigvee \neg\big(Q(y,x) \to Q(x,y)\big)\Big]$$

$$\equiv \quad \exists x\, \exists y \Big[\neg\big(\neg Q(x,y) \lor Q(y,x)\big) \bigvee \neg\big(\neg Q(y,x) \lor Q(x,y)\big)\Big]$$

$$\equiv \quad \exists x\, \exists y \Big[\big(\neg\neg Q(x,y) \land \neg Q(y,x)\big) \bigvee \big(\neg\neg Q(y,x) \land \neg Q(x,y)\big)\Big]$$

$$\equiv \quad \exists x\, \exists y \Big[\big(Q(x,y) \land \neg Q(y,x)\big) \bigvee \big(Q(y,x) \land \neg Q(x,y)\big)\Big]$$

$$\equiv \quad \exists x\, \exists y \Big[Q(x,y) \oplus Q(y,x)\big)\Big]$$

(c) $\forall y \exists x \exists z \Big(T(x,y,z) \land Q(x,y)\Big)$

**Solution:**

$$\neg\Big[\forall y\, \exists x\, \exists z \Big(T(x,y,z) \land Q(x,y)\Big)\Big]$$

$$\equiv \quad \exists y\, \forall x\, \forall z\, \neg\Big(T(x,y,z) \land Q(x,y)\Big)$$

$$\equiv \quad \exists y\, \forall x\, \forall z \Big(\neg T(x,y,z) \lor \neg Q(x,y)\Big)$$