4. *LINGO User's Guide,* LINDO Systems, Inc., Chicago, IL, e-mail: info@lindo.com, 1999.
5. *MPL Modeling System* (*Release* 4.0) manual, Maximal Software, Inc., Arlington, VA, e-mail: info@maximal-usa.com, 1998.
6. Williams, H. P.: *Model Building in Mathematical Programming,* 3d ed., Wiley, New York, 1990.

LEARNING AIDS FOR THIS CHAPTER IN YOUR OR COURSEWARE

A Demonstration Example in OR Tutor:

Graphical Method

An Excel Add-In:

Premium Solver

"Ch. 3—Intro to LP" Files for Solving the Examples:

Excel File
LINGO/LINDO File
MPL/CPLEX File

Supplement to Appendix 3.1:

More about LINGO (appears on the book's website, www.mhhe.com/hillier).

See Appendix 1 for documentation of the software.

PROBLEMS

The symbols to the left of some of the problems (or their parts) have the following meaning:

D: The demonstration example listed above may be helpful.
C: Use the computer to solve the problem by applying the simplex method. The available software options for doing this include the Excel Solver or Premium Solver (Sec. 3.6), MPL/CPLEX (Sec. 3.7), LINGO (Appendix 3.1), and LINDO (Appendix 4.1), but follow any instructions given by your instructor regarding the option to use.

An asterisk on the problem number indicates that at least a partial answer is given in the back of the book.

D **3.1-1.*** For each of the following constraints, draw a separate graph to show the nonnegative solutions that satisfy this constraint.
(a) $x_1 + 3x_2 \le 6$
(b) $4x_1 + 3x_2 \le 12$
(c) $4x_1 + x_2 \le 8$
(d) Now combine these constraints into a single graph to show the feasible region for the entire set of functional constraints plus nonnegativity constraints.

D **3.1-2.** Consider the following objective function for a linear programming model:

Maximize $Z = 2x_1 + 3x_2$

(a) Draw a graph that shows the corresponding objective function lines for $Z = 6$, $Z = 12$, and $Z = 18$.
(b) Find the slope-intercept form of the equation for each of these three objective function lines. Compare the slope for these three lines. Also compare the intercept with the x_2 axis.

3.1-3. Consider the following equation of a line:

$$20x_1 + 40x_2 = 400$$

(a) Find the slope-intercept form of this equation.

(b) Use this form to identify the slope and the intercept with the x_2 axis for this line.
(c) Use the information from part (*b*) to draw a graph of this line.

D **3.1-4.*** Use the graphical method to solve the problem:

$$\text{Maximize} \quad Z = 2x_1 + x_2,$$

subject to

$$\begin{aligned} x_2 &\le 10 \\ 2x_1 + 5x_2 &\le 60 \\ x_1 + x_2 &\le 18 \\ 3x_1 + x_2 &\le 44 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

D **3.1-5.** Use the graphical method to solve the problem:

$$\text{Maximize} \quad Z = 10x_1 + 20x_2,$$

subject to

$$\begin{aligned} -x_1 + 2x_2 &\le 15 \\ x_1 + x_2 &\le 12 \\ 5x_1 + 3x_2 &\le 45 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

3.1-6. The Whitt Window Company is a company with only three employees which makes two different kinds of hand-crafted windows: a wood-framed and an aluminum-framed window. They earn \$60 profit for each wood-framed window and \$30 profit for each aluminum-framed window. Doug makes the wood frames, and can make 6 per day. Linda makes the aluminum frames, and can make 4 per day. Bob forms and cuts the glass, and can make 48 square feet of glass per day. Each wood-framed window uses 6 square feet of glass and each aluminum-framed window uses 8 square feet of glass.

The company wishes to determine how many windows of each type to produce per day to maximize total profit.
(a) Describe the analogy between this problem and the Wyndor Glass Co. problem discussed in Sec. 3.1. Then construct and fill in a table like Table 3.1 for this problem, identifying both the activities and the resources.
(b) Formulate a linear programming model for this problem.
D **(c)** Use the graphical model to solve this model.
(d) A new competitor in town has started making wood-framed windows as well. This may force the company to lower the price they charge and so lower the profit made for each wood-framed window. How would the optimal solution change (if at all) if the profit per wood-framed window decreases from \$60 to \$40? From \$60 to \$20?
(e) Doug is considering lowering his working hours, which would decrease the number of wood frames he makes per day. How would the optimal solution change if he makes only 5 wood frames per day?

3.1-7. The Apex Television Company has to decide on the number of 27- and 20-inch sets to be produced at one of its factories. Market research indicates that at most 40 of the 27-inch sets and 10 of the 20-inch sets can be sold per month. The maximum number of work-hours available is 500 per month. A 27-inch set requires 20 work-hours and a 20-inch set requires 10 work-hours. Each 27-inch set sold produces a profit of \$120 and each 20-inch set produces a profit of \$80. A wholesaler has agreed to purchase all the television sets produced if the numbers do not exceed the maxima indicated by the market research.
(a) Formulate a linear programming model for this problem.
D **(b)** Use the graphical method to solve this model.

3.1-8. The WorldLight Company produces two light fixtures (products 1 and 2) that require both metal frame parts and electrical components. Management wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of frame parts and 2 units of electrical components are required. For each unit of product 2, 3 units of frame parts and 2 units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of \$1, and each unit of product 2, up to 60 units, gives a profit of \$2. Any excess over 60 units of product 2 brings no profit, so such an excess has been ruled out.
(a) Formulate a linear programming model for this problem.
D **(b)** Use the graphical method to solve this model. What is the resulting total profit?

3.1-9. The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is \$5 per unit on special risk insurance and \$2 per unit on mortgages.

Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

	Work-Hours per Unit		
Department	**Special Risk**	**Mortgage**	**Work-Hours Available**
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

(a) Formulate a linear programming model for this problem.
D **(b)** Use the graphical method to solve this model.
(c) Verify the exact value of your optimal solution from part (*b*) by solving algebraically for the simultaneous solution of the relevant two equations.

3.1-10. Weenies and Buns is a food processing plant which manufactures hot dogs and hot dog buns. They grind their own flour for the hot dog buns at a maximum rate of 200 pounds per week. Each hot dog bun requires 0.1 pound of flour. They currently have a contract with Pigland, Inc., which specifies that a delivery of 800 pounds of pork product is delivered every Monday. Each hot dog requires $\frac{1}{4}$ pound of pork product. All the other ingredients in the hot dogs and hot dog buns are in plentiful supply. Finally, the labor force at Weenies and Buns consists of 5 employees working full time (40 hours per week each). Each hot dog requires 3 minutes of labor, and each hot dog bun requires 2 minutes of labor. Each hot dog yields a profit of \$0.20, and each bun yields a profit of \$0.10.

Weenies and Buns would like to know how many hot dogs and how many hot dog buns they should produce each week so as to achieve the highest possible profit.
(a) Formulate a linear programming model for this problem.
D **(b)** Use the graphical method to solve this model.

3.1-11.* The Omega Manufacturing Company has discontinued the production of a certain unprofitable product line. This act created considerable excess production capacity. Management is considering devoting this excess capacity to one or more of three products; call them products 1, 2, and 3. The available capacity on the machines that might limit output is summarized in the following table:

Machine Type	Available Time (Machine Hours per Week)
Milling machine	500
Lathe	350
Grinder	150

The number of machine hours required for each unit of the respective products is

Productivity coefficient (in machine hours per unit)

Machine Type	Product 1	Product 2	Product 3
Milling machine	9	3	5
Lathe	5	4	0
Grinder	3	0	2

The sales department indicates that the sales potential for products 1 and 2 exceeds the maximum production rate and that the sales potential for product 3 is 20 units per week. The unit profit would be \$50, \$20, and \$25, respectively, on products 1, 2, and 3. The objective is to determine how much of each product Omega should produce to maximize profit.
(a) Formulate a linear programming model for this problem.
C **(b)** Use a computer to solve this model by the simplex method.

D **3.1-12.** Consider the following problem, where the value of c_1 has not yet been ascertained.

$$\text{Maximize} \quad Z = c_1x_1 + x_2,$$

subject to

$$\begin{aligned} x_1 + x_2 &\le 6 \\ x_1 + 2x_2 &\le 10 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of $c_1(-\infty < c_1 < \infty)$.

D **3.1-13.** Consider the following problem, where the value of k has not yet been ascertained.

$$\text{Maximize} \quad Z = x_1 + 2x_2,$$

subject to

$$\begin{aligned} -x_1 + x_2 &\le 2 \\ x_2 &\le 3 \\ kx_1 + x_2 &\le 2k + 3, \qquad \text{where } k \ge 0 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

The solution currently being used is $x_1 = 2$, $x_2 = 3$. Use graphical analysis to determine the values of k such that this solution actually is optimal.

D **3.1-14.** Consider the following problem, where the values of c_1 and c_2 have not yet been ascertained.

$$\text{Maximize} \quad Z = c_1x_1 + c_2x_2,$$

subject to

$$\begin{aligned} 2x_1 + x_2 &\le 11 \\ -x_1 + 2x_2 &\le 2 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of c_1 and c_2. (*Hint:* Separate the cases where $c_2 = 0$, $c_2 > 0$, and $c_2 < 0$. For the latter two cases, focus on the ratio of c_1 to c_2.)

3.2-1. The following table summarizes the key facts about two products, A and B, and the resources, Q, R, and S, required to produce them.

Resource	Resource Usage per Unit Produced		Amount of Resource Available
	Product A	Product B	
Q	2	1	2
R	1	2	2
S	3	3	4
Profit per unit	3	2	

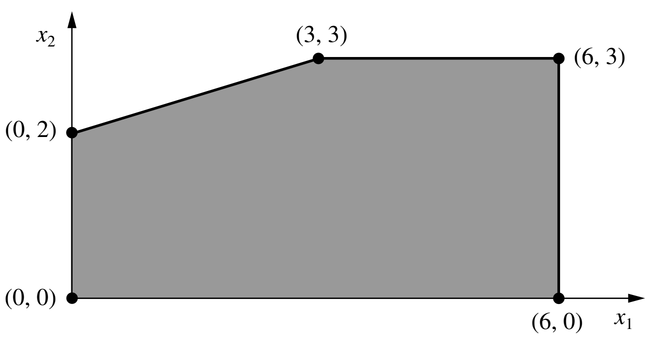
All the assumptions of linear programming hold.

(a) Formulate a linear programming model for this problem.

D **(b)** Solve this model graphically.

(c) Verify the exact value of your optimal solution from part (*b*) by solving algebraically for the simultaneous solution of the relevant two equations.

3.2-2. The shaded area in the following graph represents the feasible region of a linear programming problem whose objective function is to be maximized.



Label each of the following statements as True or False, and then justify your answer based on the graphical method. In each case, give an example of an objective function that illustrates your answer.

(a) If (3, 3) produces a larger value of the objective function than (0, 2) and (6, 3), then (3, 3) must be an optimal solution.

(b) If (3, 3) is an optimal solution and multiple optimal solutions exist, then either (0, 2) or (6, 3) must also be an optimal solution.

(c) The point (0, 0) cannot be an optimal solution.

3.2-3.* This is your lucky day. You have just won a \$10,000 prize. You are setting aside \$4,000 for taxes and partying expenses, but you have decided to invest the other \$6,000. Upon hearing this news, two different friends have offered you an opportunity to become a partner in two different entrepreneurial ventures, one planned by each friend. In both cases, this investment would involve expending some of your time next summer as well as putting up cash. Becoming a *full* partner in the first friend's venture would require an investment of \$5,000 and 400 hours, and your estimated profit (ignoring the value of your time) would be \$4,500. The corresponding figures for the second friend's venture are \$4,000 and 500 hours, with an estimated profit to you of \$4,500. However, both friends are flexible and would allow you to come in at any *fraction* of a full partnership you would like. If you choose a fraction of a full partnership, all the above figures given for a full partnership (money investment, time investment, and your profit) would be multiplied by this same fraction.

Because you were looking for an interesting summer job anyway (maximum of 600 hours), you have decided to participate in one or both friends' ventures in whichever combination would maximize your total estimated profit. You now need to solve the problem of finding the best combination.

(a) Describe the analogy between this problem and the Wyndor Glass Co. problem discussed in Sec. 3.1. Then construct and fill in a table like Table 3.1 for this problem, identifying both the activities and the resources.

(b) Formulate a linear programming model for this problem.

D **(c)** Use the graphical method to solve this model. What is your total estimated profit?

D **3.2-4.** Use the graphical method to find all optimal solutions for the following model:

$$\text{Maximize} \quad Z = 500x_1 + 300x_2,$$

subject to

$$\begin{aligned} 15x_1 + 5x_2 &\le 300 \\ 10x_1 + 6x_2 &\le 240 \\ 8x_1 + 12x_2 &\le 450 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

D **3.2-5.** Use the graphical method to demonstrate that the following model has no feasible solutions.

$$\text{Maximize} \quad Z = 5x_1 + 7x_2,$$

subject to

$$\begin{aligned} 2x_1 - x_2 &\le -1 \\ -x_1 + 2x_2 &\le -1 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

D **3.2-6.** Suppose that the following constraints have been provided for a linear programming model.

$$-x_1 + 3x_2 \leq 30$$
$$-3x_1 + x_2 \leq 30$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

(a) Demonstrate that the feasible region is unbounded.
(b) If the objective is to maximize $Z = -x_1 + x_2$, does the model have an optimal solution? If so, find it. If not, explain why not.
(c) Repeat part (*b*) when the objective is to maximize $Z = x_1 - x_2$.
(d) For objective functions where this model has no optimal solution, does this mean that there are no good solutions according to the model? Explain. What probably went wrong when formulating the model?

3.3-1. Reconsider Prob. 3.2-3. Indicate why each of the four assumptions of linear programming (Sec. 3.3) appears to be reasonably satisfied for this problem. Is one assumption more doubtful than the others? If so, what should be done to take this into account?

3.3-2. Consider a problem with two decision variables, x_1 and x_2, which represent the levels of activities 1 and 2, respectively. For each variable, the permissible values are 0, 1, and 2, where the feasible combinations of these values for the two variables are determined from a variety of constraints. The objective is to maximize a certain measure of performance denoted by Z. The values of Z for the possibly feasible values of (x_1, x_2) are estimated to be those given in the following table:

	x_2		
x_1	0	1	2
0	0	4	8
1	3	8	13
2	6	12	18

Based on this information, indicate whether this problem completely satisfies each of the four assumptions of linear programming. Justify your answers.

3.4-1.* For each of the four assumptions of linear programming discussed in Sec. 3.3, write a one-paragraph analysis of how well you feel it applies to each of the following examples given in Sec. 3.4:
(a) Design of radiation therapy (Mary).
(b) Regional planning (Southern Confederation of Kibbutzim).
(c) Controlling air pollution (Nori & Leets Co.).

3.4-2. For each of the four assumptions of linear programming discussed in Sec. 3.3, write a one-paragraph analysis of how well it applies to each of the following examples given in Sec. 3.4.
(a) Reclaiming solid wastes (Save-It Co.).
(b) Personnel scheduling (Union Airways).
(c) Distributing goods through a distribution network (Distribution Unlimited Co.).

D **3.4-3.** Use the graphical method to solve this problem:

$$\text{Maximize} \qquad Z = 15x_1 + 20x_2,$$

subject to

$$x_1 + 2x_2 \geq 10$$
$$2x_1 - 3x_2 \leq 6$$
$$x_1 + x_2 \geq 6$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

D **3.4-4.** Use the graphical method to solve this problem:

$$\text{Minimize} \qquad Z = 3x_1 + 2x_2,$$

subject to

$$x_1 + 2x_2 \leq 12$$
$$2x_1 + 3x_2 = 12$$
$$2x_1 + x_2 \geq 8$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

D **3.4-5.** Consider the following problem, where the value of c_1 has not yet been ascertained.

$$\text{Maximize} \qquad Z = c_1x_1 + 2x_2,$$

subject to

$$4x_1 + x_2 \leq 12$$
$$x_1 - x_2 \geq 2$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of c_1.

D **3.4-6.** Consider the following model:

$$\text{Minimize} \qquad Z = 40x_1 + 50x_2,$$

subject to

$$2x_1 + 3x_2 \geq 30$$
$$x_1 + x_2 \geq 12$$
$$2x_1 + x_2 \geq 20$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

(a) Use the graphical method to solve this model.
(b) How does the optimal solution change if the objective function is changed to $Z = 40x_1 + 70x_2$?
(c) How does the optimal solution change if the third functional constraint is changed to $2x_1 + x_2 \geq 15$?

3.4-7. Ralph Edmund loves steaks and potatoes. Therefore, he has decided to go on a steady diet of only these two foods (plus some liquids and vitamin supplements) for all his meals. Ralph realizes that this isn't the healthiest diet, so he wants to make sure that he eats the right quantities of the two foods to satisfy some key nutritional requirements. He has obtained the following nutritional and cost information:

Ingredient	Grams of Ingredient per Serving: Steak	Grams of Ingredient per Serving: Potatoes	Daily Requirement (Grams)
Carbohydrates	5	15	≥ 50
Protein	20	5	≥ 40
Fat	15	2	≤ 60
Cost per serving	$4	$2	

Ralph wishes to determine the number of daily servings (may be fractional) of steak and potatoes that will meet these requirements at a minimum cost.
(a) Formulate a linear programming model for this problem.
D **(b)** Use the graphical method to solve this model.
C **(c)** Use a computer to solve this model by the simplex method.

3.4-8. Dwight is an elementary school teacher who also raises pigs for supplemental income. He is trying to decide what to feed his pigs. He is considering using a combination of pig feeds available from local suppliers. He would like to feed the pigs at minimum cost while also making sure each pig receives an adequate supply of calories and vitamins. The cost, calorie content, and vitamin content of each feed is given in the table below.

Contents	Feed Type A	Feed Type B
Calories (per pound)	800	1,000
Vitamins (per pound)	140 units	70 units
Cost (per pound)	$0.40	$0.80

Each pig requires at least 8,000 calories per day and at least 700 units of vitamins. A further constraint is that no more than one-third of the diet (by weight) can consist of Feed Type A, since it contains an ingredient which is toxic if consumed in too large a quantity.
(a) Formulate a linear programming model for this problem.
D **(b)** Use the graphical method to solve this model. What is the resulting daily cost per pig?

3.4-9. Web Mercantile sells many household products through an on-line catalog. The company needs substantial warehouse space for storing its goods. Plans now are being made for leasing warehouse storage space over the next 5 months. Just how much space will be required in each of these months is known. However, since these space requirements are quite different, it may be most economical to lease only the amount needed each month on a month-by-month basis. On the other hand, the additional cost for leasing space for additional months is much less than for the first month, so it may be less expensive to lease the maximum amount needed for the entire 5 months. Another option is the intermediate approach of changing the total amount of space leased (by adding a new lease and/or having an old lease expire) at least once but not every month.

The space requirement and the leasing costs for the various leasing periods are as follows:

Month	Required Space (Sq. Ft.)
1	30,000
2	20,000
3	40,000
4	10,000
5	50,000

Leasing Period (Months)	Cost per Sq. Ft. Leased
1	$ 65
2	$100
3	$135
4	$160
5	$190

The objective is to minimize the total leasing cost for meeting the space requirements.
(a) Formulate a linear programming model for this problem.
C **(b)** Solve this model by the simplex method.

3.4-10. Larry Edison is the director of the Computer Center for Buckly College. He now needs to schedule the staffing of the center. It is open from 8 A.M. until midnight. Larry has monitored the usage of the center at various times of the day, and determined that the following number of computer consultants are required:

Time of Day	Minimum Number of Consultants Required to Be on Duty
8 A.M.–noon	4
Noon–4 P.M.	8
4 P.M.–8 P.M.	10
8 P.M.–midnight	6

Two types of computer consultants can be hired: full-time and part-time. The full-time consultants work for 8 consecutive hours in any of the following shifts: morning (8 A.M.–4 P.M.), afternoon (noon–8 P.M.), and evening (4 P.M.–midnight). Full-time consultants are paid $14 per hour.

Part-time consultants can be hired to work any of the four shifts listed in the above table. Part-time consultants are paid $12 per hour.

An additional requirement is that during every time period, there must be at least 2 full-time consultants on duty for every part-time consultant on duty.

Larry would like to determine how many full-time and how many part-time workers should work each shift to meet the above requirements at the minimum possible cost.

(a) Formulate a linear programming model for this problem.

C **(b)** Solve this model by the simplex method.

3.4-11.* The Medequip Company produces precision medical diagnostic equipment at two factories. Three medical centers have placed orders for this month's production output. The table to the right shows what the cost would be for shipping each unit from each factory to each of these customers. Also shown are the number of units that will be produced at each factory and the number of units ordered by each customer. (Go to the next column.)

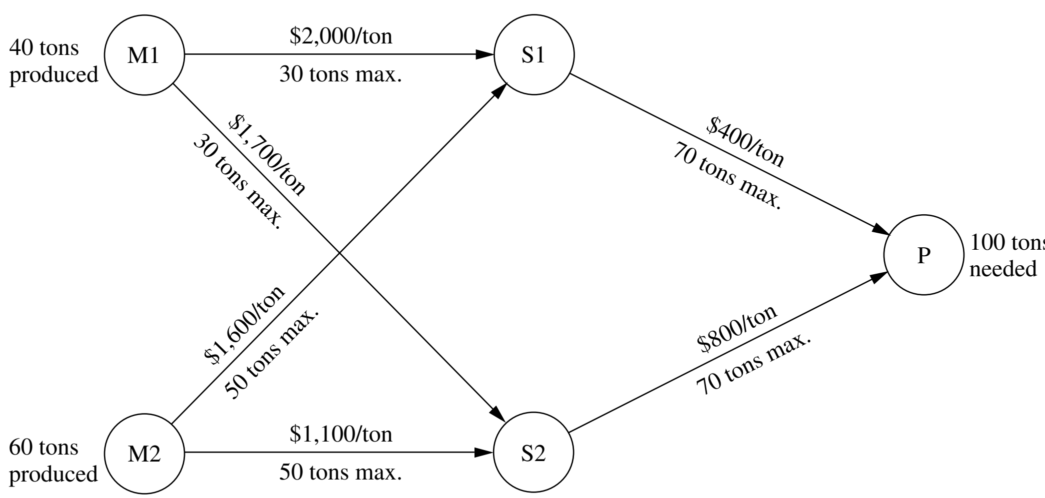
To / From	Unit Shipping Cost			
	Customer 1	**Customer 2**	**Customer 3**	**Output**
Factory 1	$600	$800	$700	400 units
Factory 2	$400	$900	$600	500 units
Order size	300 units	200 units	400 units	

A decision now needs to be made about the shipping plan for how many units to ship from each factory to each customer.

(a) Formulate a linear programming model for this problem.

C **(b)** Solve this model by the simplex method.

3.4-12. The Fagersta Steelworks currently is working two mines to obtain its iron ore. This iron ore is shipped to either of two storage facilities. When needed, it then is shipped on to the company's steel plant. The diagram below depicts this distribution network, where M1 and M2 are the two mines, S1 and S2 are the two storage facilities, and P is the steel plant. The diagram also shows the monthly amounts produced at the mines and needed at the plant, as well as the shipping cost and the maximum amount that can be shipped per month through each shipping lane. (Go to the left column below the diagram.)



Management now wants to determine the most economical plan for shipping the iron ore from the mines through the distribution network to the steel plant.

(a) Formulate a linear programming model for this problem.

C **(b)** Solve this model by the simplex method.

3.4-13.* Al Ferris has $60,000 that he wishes to invest now in order to use the accumulation for purchasing a retirement annuity in 5 years. After consulting with his financial adviser, he has been offered four types of fixed-income investments, which we will label as investments A, B, C, D.

Investments A and B are available at the beginning of each of the next 5 years (call them years 1 to 5). Each dollar invested in A at the beginning of a year returns \$1.40 (a profit of \$0.40) 2 years later (in time for immediate reinvestment). Each dollar invested in B at the beginning of a year returns \$1.70 three years later.

Investments C and D will each be available at one time in the future. Each dollar invested in C at the beginning of year 2 returns \$1.90 at the end of year 5. Each dollar invested in D at the beginning of year 5 returns \$1.30 at the end of year 5.

Al wishes to know which investment plan maximizes the amount of money that can be accumulated by the beginning of year 6.

(a) All the functional constraints for this problem can be expressed as equality constraints. To do this, let A_t, B_t, C_t, and D_t be the amount invested in investment A, B, C, and D, respectively, at the beginning of year t for each t where the investment is available and will mature by the end of year 5. Also let R_t be the number of available dollars *not* invested at the beginning of year t (and so available for investment in a later year). Thus, the amount invested at the beginning of year t *plus* R_t must equal the number of dollars available for investment at that time. Write such an equation in terms of the relevant variables above for the beginning of each of the 5 years to obtain the five functional constraints for this problem.

(b) Formulate a complete linear programming model for this problem.

C **(c)** Solve this model by the simplex model.

3.4-14. The Metalco Company desires to blend a new alloy of 40 percent tin, 35 percent zinc, and 25 percent lead from several available alloys having the following properties:

	Alloy				
Property	**1**	**2**	**3**	**4**	**5**
Percentage of tin	60	25	45	20	50
Percentage of zinc	10	15	45	50	40
Percentage of lead	30	60	10	30	10
Cost (\$/lb)	22	20	25	24	27

The objective is to determine the proportions of these alloys that should be blended to produce the new alloy at a minimum cost.

(a) Formulate a linear programming model for this problem.

C **(b)** Solve this model by the simplex method.

3.4-15. The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes—large, medium, and small—that yield a net unit profit of \$420, \$360, and \$300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

(a) Formulate a linear programming model for this problem.

C **(b)** Solve this model by the simplex method.

3.4-16* A cargo plane has three compartments for storing cargo: front, center, and back. These compartments have capacity limits on both *weight* and *space,* as summarized below:

Compartment	Weight Capacity (Tons)	Space Capacity (Cubic Feet)
Front	12	7,000
Center	18	9,000
Back	10	5,000

Furthermore, the weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the airplane.

The following four cargoes have been offered for shipment on an upcoming flight as space is available:

Cargo	Weight (Tons)	Volume (Cubic Feet/Ton)	Profit (\$/Ton)
1	20	500	320
2	16	700	400
3	25	600	360
4	13	400	290

Any portion of these cargoes can be accepted. The objective is to determine how much (if any) of each cargo should be accepted and how to distribute each among the compartments to maximize the total profit for the flight.

(a) Formulate a linear programming model for this problem.

C **(b)** Solve this model by the simplex method to find one of its multiple optimal solutions.

3.4-17. Comfortable Hands is a company which features a product line of winter gloves for the entire family—men, women, and children. They are trying to decide what mix of these three types of gloves to produce.

Comfortable Hands' manufacturing labor force is unionized. Each full-time employee works a 40-hour week. In addition, by union contract, the number of full-time employees can never drop below 20. Nonunion part-time workers can also be hired with the following union-imposed restrictions: (1) each part-time worker works 20 hours per week, and (2) there must be at least 2 full-time employees for each part-time employee.

All three types of gloves are made out of the same 100 percent genuine cowhide leather. Comfortable Hands has a long-term contract with a supplier of the leather, and receives a 5,000 square feet shipment of the material each week. The material requirements and labor requirements, along with the *gross profit* per glove sold (not considering labor costs) is given in the following table.

Glove	Material Required (Square Feet)	Labor Required (Minutes)	Gross Profit (per Pair)
Men's	2	30	$8
Women's	1.5	45	$10
Children's	1	40	$6

Each full-time employee earns $13 per hour, while each part-time employee earns $10 per hour. Management wishes to know what mix of each of the three types of gloves to produce per week, as well as how many full-time and how many part-time workers to employ. They would like to maximize their *net profit*—their gross profit from sales minus their labor costs.

(a) Formulate a linear programming model for this problem.

C **(b)** Solve this model by the simplex method.

3.4-18. Oxbridge University maintains a powerful mainframe computer for research use by its faculty, Ph.D. students, and research associates. During all working hours, an operator must be available to operate and maintain the computer, as well as to perform some programming services. Beryl Ingram, the director of the computer facility, oversees the operation.

It is now the beginning of the fall semester, and Beryl is confronted with the problem of assigning different working hours to her operators. Because all the operators are currently enrolled in the university, they are available to work only a limited number of hours each day, as shown in the following table.

		Maximum Hours of Availability				
Operators	**Wage Rate**	**Mon.**	**Tue.**	**Wed.**	**Thurs.**	**Fri.**
K. C.	$10.00/hour	6	0	6	0	6
D. H.	$10.10/hour	0	6	0	6	0
H. B.	$ 9.90/hour	4	8	4	0	4
S. C.	$ 9.80/hour	5	5	5	0	5
K. S.	$10.80/hour	3	0	3	8	0
N. K.	$11.30/hour	0	0	0	6	2

There are six operators (four undergraduate students and two graduate students). They all have different wage rates because of differences in their experience with computers and in their programming ability. The above table shows their wage rates, along with the maximum number of hours that each can work each day.

Each operator is guaranteed a certain minimum number of hours per week that will maintain an adequate knowledge of the operation. This level is set arbitrarily at 8 hours per week for the undergraduate students (K. C., D. H., H. B., and S. C.) and 7 hours per week for the graduate students (K. S. and N. K.).

The computer facility is to be open for operation from 8 A.M. to 10 P.M. Monday through Friday with exactly one operator on duty during these hours. On Saturdays and Sundays, the computer is to be operated by other staff.

Because of a tight budget, Beryl has to minimize cost. She wishes to determine the number of hours she should assign to each operator on each day.

(a) Formulate a linear programming model for this problem.

C **(b)** Solve this model by the simplex method.

3.4-19. Slim-Down Manufacturing makes a line of nutritionally complete, weight-reduction beverages. One of their products is a strawberry shake which is designed to be a complete meal. The strawberry shake consists of several ingredients. Some information about each of these ingredients is given below.

Ingredient	Calories from Fat (per tbsp)	Total Calories (per tbsp)	Vitamin Content (mg/tbsp)	Thickeners (mg/tbsp)	Cost (¢/tbsp)
Strawberry flavoring	1	50	20	3	10
Cream	75	100	0	8	8
Vitamin supplement	0	0	50	1	25
Artificial sweetener	0	120	0	2	15
Thickening agent	30	80	2	25	6

The nutritional requirements are as follows. The beverage must total between 380 and 420 calories (inclusive). No more than 20 percent of the total calories should come from fat. There must be at least 50 milligrams (mg) of vitamin content. For taste reasons, there must be at least 2 tablespoons (tbsp) of strawberry flavoring for each tablespoon of artificial sweetener. Finally, to maintain proper thickness, there must be exactly 15 mg of thickeners in the beverage.

Management would like to select the quantity of each ingredient for the beverage which would minimize cost while meeting the above requirements.

(a) Formulate a linear programming model for this problem.

C **(b)** Solve this model by the simplex method.

3.4-20. Joyce and Marvin run a day care for preschoolers. They are trying to decide what to feed the children for lunches. They would like to keep their costs down, but also need to meet the nutritional requirements of the children. They have already decided to go with peanut butter and jelly sandwiches, and some combination of graham crackers, milk, and orange juice. The nutritional content of each food choice and its cost are given in the table below.

Food Item	Calories from Fat	Total Calories	Vitamin C (mg)	Protein (g)	Cost (¢)
Bread (1 slice)	10	70	0	3	5
Peanut butter (1 tbsp)	75	100	0	4	4
Strawberry jelly (1 tbsp)	0	50	3	0	7
Graham cracker (1 cracker)	20	60	0	1	8
Milk (1 cup)	70	150	2	8	15
Juice (1 cup)	0	100	120	1	35

The nutritional requirements are as follows. Each child should receive between 400 and 600 calories. No more than 30 percent of the total calories should come from fat. Each child should consume at least 60 milligrams (mg) of vitamin C and 12 grams (g) of protein. Furthermore, for practical reasons, each child needs exactly 2 slices of bread (to make the sandwich), at least twice as much peanut butter as jelly, and at least 1 cup of liquid (milk and/or juice).

Joyce and Marvin would like to select the food choices for each child which minimize cost while meeting the above requirements.

(a) Formulate a linear programming model for this problem.

C **(b)** Solve this model by the simplex method.

3.5-1. Read the article footnoted in Sec. 3.5 that describes the first case study presented in that section: "Choosing the Product Mix at Ponderosa Industrial."

(a) Describe the two factors which, according to the article, often hinder the use of optimization models by managers.

(b) Section 3.5 indicates without elaboration that using linear programming at Ponderosa "led to a dramatic shift in the types of plywood products emphasized by the company." Identify this shift.

(c) With the success of this application, management then was eager to use optimization for other problems as well. Identify these other problems.

(d) Photocopy the two pages of appendixes that give the mathematical formulation of the problem and the structure of the linear programming model.

3.5-2. Read the article footnoted in Sec. 3.5 that describes the second case study presented in that section: "Personnel Scheduling at United Airlines."

(a) Describe how United Airlines prepared shift schedules at airports and reservations offices prior to this OR study.

(b) When this study began, the *problem definition phase* defined five specific project requirements. Identify these project requirements.

(c) At the end of the presentation of the corresponding example in Sec. 3.4 (personnel scheduling at Union Airways), we pointed out that the divisibility assumption does not hold for this kind of application. An integer solution is needed, but linear programming may provide an optimal solution that is noninteger. How does United Airlines deal with this problem?

(d) Describe the flexibility built into the scheduling system to satisfy the group culture at each office. Why was this flexibility needed?

(e) Briefly describe the tangible and intangible benefits that resulted from the study.

3.5-3. Read the 1986 article footnoted in Sec. 2.1 that describes the third case study presented in Sec. 3.5: "Planning Supply, Distribution, and Marketing at Citgo Petroleum Corporation."

(a) What happened during the years preceding this OR study that made it vastly more important to control the amount of capital tied up in inventory?

(b) What geographical area is spanned by Citgo's distribution network of pipelines, tankers, and barges? Where do they market their products?

(c) What time periods are included in the model?

(d) Which computer did Citgo use to solve the model? What were typical run times?

(e) Who are the four types of model users? How does each one use the model?

(f) List the major types of reports generated by the SDM system.

(g) What were the major implementation challenges for this study?

(h) List the direct and indirect benefits that were realized from this study.

3.6-1.* You are given the following data for a linear programming problem where the objective is to maximize the profit from allocating three resources to two nonnegative activities.

Resource	Resource Usage per Unit of Each Activity		Amount of Resource Available
	Activity 1	Activity 2	
1	2	1	10
2	3	3	20
3	2	4	20
Contribution per unit	\$20	\$30	

Contribution per unit = profit per unit of the activity.

(a) Formulate a linear programming model for this problem.
D **(b)** Use the graphical method to solve this model.
(c) Display the model on an Excel spreadsheet.
(d) Use the spreadsheet to check the following solutions: $(x_1, x_2) = (2, 2), (3, 3), (2, 4), (4, 2), (3, 4), (4, 3)$. Which of these solutions are feasible? Which of these feasible solutions has the best value of the objective function?
C **(e)** Use the Excel Solver to solve the model by the simplex method.

3.6-2. Ed Butler is the production manager for the Bilco Corporation, which produces three types of spare parts for automobiles. The manufacture of each part requires processing on each of two machines, with the following processing times (in hours):

Machine	Part		
	A	B	C
1	0.02	0.03	0.05
2	0.05	0.02	0.04

Each machine is available 40 hours per month. Each part manufactured will yield a unit profit as follows:

	Part		
	A	B	C
Profit	\$50	\$40	\$30

Ed wants to determine the mix of spare parts to produce in order to maximize total profit.
(a) Formulate a linear programming model for this problem.
(b) Display the model on an Excel spreadsheet.
(c) Make three guesses of your own choosing for the optimal solution. Use the spreadsheet to check each one for feasibility and, if feasible, to find the value of the objective function. Which feasible guess has the best objective function value?
(d) Use the Excel Solver to solve the model by the simplex method.

3.6-3. You are given the following data for a linear programming problem where the objective is to minimize the cost of conducting two nonnegative activities so as to achieve three benefits that do not fall below their minimum levels.

Benefit	Benefit Contribution per Unit of Each Activity		Minimum Acceptable Level
	Activity 1	Activity 2	
1	5	3	60
2	2	2	30
3	7	9	126
Unit cost	\$60	\$50	

(a) Formulate a linear programming model for this problem.
D **(b)** Use the graphical method to solve this model.
(c) Display the model on an Excel spreadsheet.
(d) Use the spreadsheet to check the following solutions: $(x_1, x_2) = (7, 7), (7, 8), (8, 7), (8, 8), (8, 9), (9, 8)$. Which of these solutions are feasible? Which of these feasible solutions has the best value of the objective function?
C **(e)** Use the Excel Solver to solve this model by the simplex method.

3.6-4.* Fred Jonasson manages a family-owned farm. To supplement several food products grown on the farm, Fred also raises pigs for market. He now wishes to determine the quantities of the available types of feed (corn, tankage, and alfalfa) that should be given to each pig. Since pigs will eat any mix of these feed types, the objective is to determine which mix will meet certain nutritional requirements at a *minimum cost.* The number of units of each type of basic nutritional ingredient contained within a kilogram of each feed type is given in the following table, along with the daily nutritional requirements and feed costs:

Nutritional Ingredient	Kilogram of Corn	Kilogram of Tankage	Kilogram of Alfalfa	Minimum Daily Requirement
Carbohydrates	90	20	40	200
Protein	30	80	60	180
Vitamins	10	20	60	150
Cost (¢)	84	72	60	

(a) Formulate a linear programming model for this problem.
(b) Display the model on an Excel spreadsheet.
(c) Use the spreadsheet to check if $(x_1, x_2, x_3) = (1, 2, 2)$ is a feasible solution and, if so, what the daily cost would be for this diet. How many units of each nutritional ingredient would this diet provide daily?
(d) Take a few minutes to use a trial-and-error approach with the spreadsheet to develop your best guess for the optimal solution. What is the daily cost for your solution?
C **(e)** Use the Excel Solver to solve the model by the simplex method.

3.6-5. Maureen Laird is the chief financial officer for the Alva Electric Co., a major public utility in the midwest. The company has scheduled the construction of new hydroelectric plants 5, 10, and 20 years from now to meet the needs of the growing population in the region served by the company. To cover at least the construction costs, Maureen needs to invest some of the company's money now to meet these future cash-flow needs. Maureen may purchase only three kinds of financial assets, each of which costs \$1 million per unit. Fractional units may be purchased. The assets produce income 5, 10, and 20 years from now, and that income is needed to cover at least minimum cash-flow requirements in those years. (Any excess income above the minimum requirement for each time period will be used to increase dividend payments to shareholders rather than saving it to help meet the minimum cash-flow requirement in the next time period.) The following table shows both the amount of income generated by each unit of each asset and the minimum amount of income needed for each of the future time periods when a new hydroelectric plant will be constructed.

	Income per Unit of Asset			Minimum Cash
Year	**Asset 1**	**Asset 2**	**Asset 3**	**Flow Required**
5	\$2 million	\$1 million	\$0.5 million	\$400 million
10	\$0.5 million	\$0.5 million	\$1 million	\$100 million
20	0	\$1.5 million	\$2 million	\$300 million

Maureen wishes to determine the mix of investments in these assets that will cover the cash-flow requirements while minimizing the total amount invested.
(a) Formulate a linear programming model for this problem.
(b) Display the model on a spreadsheet.
(c) Use the spreadsheet to check the possibility of purchasing 100 units of Asset 1, 100 units of Asset 2, and 200 units of Asset 3. How much cash flow would this mix of investments generate 5, 10, and 20 years from now? What would be the total amount invested?
(d) Take a few minutes to use a trial-and-error approach with the spreadsheet to develop your best guess for the optimal solution. What is the total amount invested for your solution?
C **(e)** Use the Excel Solver to solve the model by the simplex method.

3.7-1. The Philbrick Company has two plants on opposite sides of the United States. Each of these plants produces the same two products and then sells them to wholesalers within its half of the country. The orders from wholesalers have already been received for the next 2 months (February and March), where the number of units requested are shown below. (The company is not obligated to completely fill these orders but will do so if it can without decreasing its profits.)

	Plant 1		Plant 2	
Product	**February**	**March**	**February**	**March**
1	3,600	6,300	4,900	4,200
2	4,500	5,400	5,100	6,000

Each plant has 20 production days available in February and 23 production days available in March to produce and ship these products. Inventories are depleted at the end of January, but each plant has enough inventory capacity to hold 1,000 units total of the two products if an excess amount is produced in February for sale in March. In either plant, the cost of holding inventory in this way is \$3 per unit of product 1 and \$4 per unit of product 2.

Each plant has the same two production processes, each of which can be used to produce either of the two products. The production cost per unit produced of each product is shown below for each process in each plant.

	Plant 1		Plant 2	
Product	**Process 1**	**Process 2**	**Process 1**	**Process 2**
1	\$62	\$59	\$61	\$65
2	\$78	\$85	\$89	\$86

The production rate for each product (number of units produced per day devoted to that product) also is given below for each process in each plant.

	Plant 1		Plant 2	
Product	**Process 1**	**Process 2**	**Process 1**	**Process 2**
1	100	140	130	110
2	120	150	160	130

The net sales revenue (selling price minus normal shipping costs) the company receives when a plant sells the products to its own customers (the wholesalers in its half of the country) is \$83 per unit of product 1 and \$112 per unit of product 2. However, it also is possible (and occasionally desirable) for a plant to make a shipment to the other half of the country to help fill the sales of the other plant. When this happens, an extra shipping cost of \$9 per unit of product 1 and \$7 per unit of product 2 is incurred.

Management now needs to determine how much of each product should be produced by each production process in each plant during each month, as well as how much each plant should sell of each product in each month and how much each plant should ship of each product in each month to the other plant's customers. The objective is to determine which feasible plan would maximize the total profit (total net sales revenue minus the sum of the production costs, inventory costs, and extra shipping costs).

(a) Formulate a complete linear programming model in algebraic form that shows the individual constraints and decision variables for this problem.

C **(b)** Formulate this same model on an Excel spreadsheet instead. Then use the Excel Solver to solve the model.

C **(c)** Use MPL to formulate this model in a compact form. Then use the MPL solver CPLEX to solve the model.

C **(d)** Use LINGO to formulate this model in a compact form. Then use the LINGO solver to solve the model.

C **3.7-2.** Reconsider Prob. 3.1-11.

(a) Use MPL/CPLEX to formulate and solve the model for this problem.

(b) Use LINGO to formulate and solve this model.

C **3.7-3.** Reconsider Prob. 3.4-11.

(a) Use MPL/CPLEX to formulate and solve the model for this problem.

(b) Use LINGO to formulate and solve this model.

C **3.7-4.** Reconsider Prob. 3.4-15.

(a) Use MPL/CPLEX to formulate and solve the model for this problem.

(b) Use LINGO to formulate and solve this model.

C **3.7-5.** Reconsider Prob. 3.4-18.

(a) Use MPL/CPLEX to formulate and solve the model for this problem.

(b) Use LINGO to formulate and solve this model.

C **3.7-6.** Reconsider Prob. 3.6-4.

(a) Use MPL/CPLEX to formulate and solve the model for this problem.

(b) Use LINGO to formulate and solve this model.

C **3.7-7.** Reconsider Prob. 3.6-5.

(a) Use MPL/CPLEX to formulate and solve the model for this problem.

(b) Use LINGO to formulate and solve this model.

3.7-8. A large paper manufacturing company, the Quality Paper Corporation, has 10 paper mills from which it needs to supply 1,000 customers. It uses three alternative types of machines and four types of raw materials to make five different types of paper. Therefore, the company needs to develop a detailed production distribution plan on a monthly basis, with an objective of minimizing the total cost of producing and distributing the paper during the month. Specifically, it is necessary to determine jointly the amount of each type of paper to be made at each paper mill on each type of machine *and* the amount of each type of paper to be shipped from each paper mill to each customer.

The relevant data can be expressed symbolically as follows:

D_{jk} = number of units of paper type k demanded by customer j,

r_{klm} = number of units of raw material m needed to produce 1 unit of paper type k on machine type l,

R_{im} = number of units of raw material m available at paper mill i,

c_{kl} = number of capacity units of machine type l that will produce 1 unit of paper type k,

C_{il} = number of capacity units of machine type l available at paper mill i,

P_{ikl} = production cost for each unit of paper type k produced on machine type l at paper mill i,

T_{ijk} = transportation cost for each unit of paper type k shipped from paper mill i to customer j.

(a) Using these symbols, formulate a linear programming model for this problem by hand.

(b) How many functional constraints and decision variables does this model have?

C **(c)** Use MPL to formulate this problem.

C **(d)** Use LINGO to formulate this problem.

CASE 3.1 AUTO ASSEMBLY

Automobile Alliance, a large automobile manufacturing company, organizes the vehicles it manufactures into three families: a family of trucks, a family of small cars, and a family of midsized and luxury cars. One plant outside Detroit, MI, assembles two models from the family of midsized and luxury cars. The first model, the Family Thrillseeker, is a four-door sedan with vinyl seats, plastic interior, standard features, and excellent gas mileage. It is marketed as a smart buy for middle-class families with tight budgets, and each Family Thrillseeker sold generates a modest profit of $3,600 for the company. The second model, the Classy Cruiser, is a two-door luxury sedan with leather seats, wooden interior, custom features, and navigational capabilities. It is marketed as a privilege of affluence for upper-middle-class families, and each Classy Cruiser sold generates a healthy profit of $5,400 for the company.

Rachel Rosencrantz, the manager of the assembly plant, is currently deciding the production schedule for the next month. Specifically, she must decide how many Family Thrillseekers and how many Classy Cruisers to assemble in the plant to maximize profit for the company. She knows that the plant possesses a capacity of 48,000 labor-hours during the month. She also knows that it takes 6 labor-hours to assemble one Family Thrillseeker and 10.5 labor-hours to assemble one Classy Cruiser.

Because the plant is simply an assembly plant, the parts required to assemble the two models are not produced at the plant. They are instead shipped from other plants around the Michigan area to the assembly plant. For example, tires, steering wheels, windows, seats, and doors all arrive from various supplier plants. For the next month, Rachel knows that she will be able to obtain only 20,000 doors (10,000 left-hand doors and 10,000 right-hand doors) from the door supplier. A recent labor strike forced the shutdown of that particular supplier plant for several days, and that plant will not be able to meet its production schedule for the next month. Both the Family Thrillseeker and the Classy Cruiser use the same door part.

In addition, a recent company forecast of the monthly demands for different automobile models suggests that the demand for the Classy Cruiser is limited to 3,500 cars. There is no limit on the demand for the Family Thrillseeker within the capacity limits of the assembly plant.

(a) Formulate and solve a linear programming problem to determine the number of Family Thrillseekers and the number of Classy Cruisers that should be assembled.

Before she makes her final production decisions, Rachel plans to explore the following questions independently except where otherwise indicated.

(b) The marketing department knows that it can pursue a targeted $500,000 advertising campaign that will raise the demand for the Classy Cruiser next month by 20 percent. Should the campaign be undertaken?

(c) Rachel knows that she can increase next month's plant capacity by using overtime labor. She can increase the plant's labor-hour capacity by 25 percent. With the new assembly plant capacity, how many Family Thrillseekers and how many Classy Cruisers should be assembled?

(d) Rachel knows that overtime labor does not come without an extra cost. What is the maximum amount she should be willing to pay for all overtime labor beyond the cost of this labor at regular time rates? Express your answer as a lump sum.

(e) Rachel explores the option of using both the targeted advertising campaign and the overtime labor-hours. The advertising campaign raises the demand for the Classy Cruiser by 20 percent, and the overtime labor increases the plant's labor-hour capacity by 25 percent. How many Family Thrillseekers and how many Classy Cruisers should be assembled using the advertising campaign and overtime labor-hours if the profit from each Classy Cruiser sold continues to be 50 percent more than for each Family Thrillseeker sold?

(f) Knowing that the advertising campaign costs $500,000 and the maximum usage of overtime labor-hours costs $1,600,000 beyond regular time rates, is the solution found in part (*e*) a wise decision compared to the solution found in part (*a*)?

(g) Automobile Alliance has determined that dealerships are actually heavily discounting the price of the Family Thrillseekers to move them off the lot. Because of a profit-sharing agreement with its dealers, the company is therefore not making a profit of $3,600 on the Family Thrillseeker but is instead making a profit of $2,800. Determine the number of Family Thrillseekers and the number of Classy Cruisers that should be assembled given this new discounted price.

(h) The company has discovered quality problems with the Family Thrillseeker by randomly testing Thrillseekers at the end of the assembly line. Inspectors have discovered that in over 60 percent of the cases, two of the four doors on a Thrillseeker do not seal properly. Because the percentage of defective Thrillseekers determined by the random testing is so high, the floor supervisor has decided to perform quality control tests on every Thrillseeker at the end of the line. Because of the added tests, the time it takes to assemble one Family Thrillseeker has increased from 6 to 7.5 hours. Determine the number of units of each model that should be assembled given the new assembly time for the Family Thrillseeker.

(i) The board of directors of Automobile Alliance wishes to capture a larger share of the luxury sedan market and therefore would like to meet the full demand for Classy Cruisers. They ask Rachel to determine by how much the profit of her assembly plant would decrease as compared to the profit found in part (*a*). They then ask her to meet the full demand for Classy Cruisers if the decrease in profit is not more than $2,000,000.

(j) Rachel now makes her final decision by combining all the new considerations described in parts (*f*), (*g*), and (*h*). What are her final decisions on whether to undertake the advertising campaign, whether to use overtime labor, the number of Family Thrillseekers to assemble, and the number of Classy Cruisers to assemble?

CASE 3.2 CUTTING CAFETERIA COSTS

A cafeteria at All-State University has one special dish it serves like clockwork every Thursday at noon. This supposedly tasty dish is a casserole that contains sautéed onions, boiled sliced potatoes, green beans, and cream of mushroom soup. Unfortunately, students fail to see the special quality of this dish, and they loathingly refer to it as the Killer Casserole. The students reluctantly eat the casserole, however, because the cafeteria provides only a limited selection of dishes for Thursday's lunch (namely, the casserole).

Maria Gonzalez, the cafeteria manager, is looking to cut costs for the coming year, and she believes that one sure way to cut costs is to buy less expensive and perhaps lower-quality ingredients. Because the casserole is a weekly staple of the cafeteria menu, she concludes that if she can cut costs on the ingredients purchased for the casserole, she can significantly reduce overall cafeteria operating costs. She therefore decides to invest time in determining how to minimize the costs of the casserole while maintaining nutritional and taste requirements.

Maria focuses on reducing the costs of the two main ingredients in the casserole, the potatoes and green beans. These two ingredients are responsible for the greatest costs, nutritional content, and taste of the dish.

Maria buys the potatoes and green beans from a wholesaler each week. Potatoes cost \$0.40 per pound, and green beans cost \$1.00 per pound.

All-State University has established nutritional requirements that each main dish of the cafeteria must meet. Specifically, the total amount of the dish prepared for all the students for one meal must contain 180 grams (g) of protein, 80 milligrams (mg) of iron, and 1,050 mg of vitamin C. (There are 453.6 g in 1 lb and 1,000 mg in 1 g.) For simplicity when planning, Maria assumes that only the potatoes and green beans contribute to the nutritional content of the casserole.

Because Maria works at a cutting-edge technological university, she has been exposed to the numerous resources on the World Wide Web. She decides to surf the Web to find the nutritional content of potatoes and green beans. Her research yields the following nutritional information about the two ingredients:

	Potatoes	**Green Beans**
Protein	1.5 g per 100 g	5.67 g per 10 ounces
Iron	0.3 mg per 100 g	3.402 mg per 10 ounces
Vitamin C	12 mg per 100 g	28.35 mg per 10 ounces

(There are 28.35 g in 1 ounce.)

Edson Branner, the cafeteria cook who is surprisingly concerned about taste, informs Maria that an edible casserole must contain at least a six to five ratio in the weight of potatoes to green beans.

Given the number of students who eat in the cafeteria, Maria knows that she must purchase enough potatoes and green beans to prepare a minimum of 10 kilograms (kg) of casserole each week. (There are 1,000 g in 1 kg.) Again for simplicity in planning, she assumes that only the potatoes and green beans determine the amount of casserole that can be prepared. Maria does not establish an upper limit on the amount of casserole to prepare, since she knows all leftovers can be served for many days thereafter or can be used creatively in preparing other dishes.

(a) Determine the amount of potatoes and green beans Maria should purchase each week for the casserole to minimize the ingredient costs while meeting nutritional, taste, and demand requirements.

Before she makes her final decision, Maria plans to explore the following questions independently except where otherwise indicated.

(b) Maria is not very concerned about the taste of the casserole; she is only concerned about meeting nutritional requirements and cutting costs. She therefore forces Edson to change the recipe to allow for only at least a one to two ratio in the weight of potatoes to green beans. Given the new recipe, determine the amount of potatoes and green beans Maria should purchase each week.

(c) Maria decides to lower the iron requirement to 65 mg since she determines that the other ingredients, such as the onions and cream of mushroom soup, also provide iron. Determine the amount of potatoes and green beans Maria should purchase each week given this new iron requirement.

(d) Maria learns that the wholesaler has a surplus of green beans and is therefore selling the green beans for a lower price of $0.50 per lb. Using the same iron requirement from part (*c*) and the new price of green beans, determine the amount of potatoes and green beans Maria should purchase each week.

(e) Maria decides that she wants to purchase lima beans instead of green beans since lima beans are less expensive and provide a greater amount of protein and iron than green beans. Maria again wields her absolute power and forces Edson to change the recipe to include lima beans instead of green beans. Maria knows she can purchase lima beans for $0.60 per lb from the wholesaler. She also knows that lima beans contain 22.68 g of protein per 10 ounces of lima beans, 6.804 mg of iron per 10 ounces of lima beans, and no vitamin C. Using the new cost and nutritional content of lima beans, determine the amount of potatoes and lima beans Maria should purchase each week to minimize the ingredient costs while meeting nutritional, taste, and demand requirements. The nutritional requirements include the reduced iron requirement from part (*c*).

(f) Will Edson be happy with the solution in part (*e*)? Why or why not?

(g) An All-State student task force meets during Body Awareness Week and determines that All-State University's nutritional requirements for iron are too lax and that those for vitamin C are too stringent. The task force urges the university to adopt a policy that requires each serving of an entrée to contain at least 120 mg of iron and at least 500 mg of vitamin C. Using potatoes and lima beans as the ingredients for the dish and using the new nutritional requirements, determine the amount of potatoes and lima beans Maria should purchase each week.

CASE 3.3 STAFFING A CALL CENTER[1]

California Children's Hospital has been receiving numerous customer complaints because of its confusing, decentralized appointment and registration process. When customers want to make appointments or register child patients, they must contact the clinic or department they plan to visit. Several problems exist with this current strategy. Parents do not always know the most appropriate clinic or department they must visit to address their children's ailments. They therefore spend a significant amount of time on the phone being transferred from clinic to clinic until they reach the most appropriate clinic for their needs. The hospital also does not publish the phone numbers of all clinic and departments, and parents must therefore invest a large amount of time in detective work to track down the correct phone number. Finally, the various clinics and departments do not communicate with each other. For example, when a doctor schedules a referral with a colleague located in another department or clinic, that department or clinic almost never receives word of the referral. The parent must contact the correct department or clinic and provide the needed referral information.

[1]This case is based on an actual project completed by a team of master's students in the Department of Engineering-Economic Systems and Operations Research at Stanford University.

In efforts to reengineer and improve its appointment and registration process, the children's hospital has decided to centralize the process by establishing one call center devoted exclusively to appointments and registration. The hospital is currently in the middle of the planning stages for the call center. Lenny Davis, the hospital manager, plans to operate the call center from 7 A.M. to 9 P.M. during the weekdays.

Several months ago, the hospital hired an ambitious management consulting firm, Creative Chaos Consultants, to forecast the number of calls the call center would receive each hour of the day. Since all appointment and registration-related calls would be received by the call center, the consultants decided that they could forecast the calls at the call center by totaling the number of appointment and registration-related calls received by all clinics and departments. The team members visited all the clinics and departments, where they diligently recorded every call relating to appointments and registration. They then totaled these calls and altered the totals to account for calls missed during data collection. They also altered totals to account for repeat calls that occurred when the same parent called the hospital many times because of the confusion surrounding the decentralized process. Creative Chaos Consultants determined the average number of calls the call center should expect during each hour of a weekday. The following table provides the forecasts.

Work Shift	Average Number of Calls
7 A.M.–9 A.M.	40 calls per hour
9 A.M.–11 A.M.	85 calls per hour
11 A.M.–1 P.M.	70 calls per hour
1 P.M.–3 P.M.	95 calls per hour
3 P.M.–5 P.M.	80 calls per hour
5 P.M.–7 P.M.	35 calls per hour
7 P.M.–9 P.M.	10 calls per hour

After the consultants submitted these forecasts, Lenny became interested in the percentage of calls from Spanish speakers since the hospital services many Spanish patients. Lenny knows that he has to hire some operators who speak Spanish to handle these calls. The consultants performed further data collection and determined that on average, 20 percent of the calls were from Spanish speakers.

Given these call forecasts, Lenny must now decide how to staff the call center during each 2 hour shift of a weekday. During the forecasting project, Creative Chaos Consultants closely observed the operators working at the individual clinics and departments and determined the number of calls operators process per hour. The consultants informed Lenny that an operator is able to process an average of six calls per hour. Lenny also knows that he has both full-time and part-time workers available to staff the call center. A full-time employee works 8 hours per day, but because of paperwork that must also be completed, the employee spends only 4 hours per day on the phone. To balance the schedule, the employee alternates the 2-hour shifts between answering phones and completing paperwork. Full-time employees can start their day either by answering phones or by completing paperwork on the first shift. The full-time employees speak either Spanish or English, but none of them are bilingual. Both Spanish-speaking and English-speaking employees are paid \$10 per hour for work before 5 P.M. and \$12 per hour for work after 5 P.M. The full-time employees can begin work at the beginning of the 7 A.M. to 9 A.M. shift, 9 A.M. to 11 A.M. shift, 11 A.M. to 1 P.M. shift, or 1 P.M. to 3 P.M. shift. The part-time employees work for 4 hours, only answer calls, and only speak English. They can start work at the beginning of the 3 P.M. to 5 P.M. shift or the 5 P.M. to 7 P.M. shift, and like the full-time employees, they are paid \$10 per hour for work before 5 P.M. and \$12 per hour for work after 5 P.M.

For the following analysis consider only the labor cost for the time employees spend answering phones. The cost for paperwork time is charged to other cost centers.

(a) How many Spanish-speaking operators and how many English-speaking operators does the hospital need to staff the call center during each 2-hour shift of the day in order to answer all calls? Please provide an integer number since half a human operator makes no sense.

(b) Lenny needs to determine how many full-time employees who speak Spanish, full-time employees who speak English, and part-time employees he should hire to begin on each shift. Creative Chaos Consultants advise him that linear programming can be used to do this in such a way as to minimize operating costs while answering all calls. Formulate a linear programming model of this problem.

(c) Obtain an optimal solution for the linear programming model formulated in part (*b*) to guide Lenny's decision.

(d) Because many full-time workers do not want to work late into the evening, Lenny can find only one qualified English-speaking operator willing to begin work at 1 P.M. Given this new constraint, how many full-time English-speaking operators, full-time Spanish-speaking operators, and part-time operators should Lenny hire for each shift to minimize operating costs while answering all calls?

(e) Lenny now has decided to investigate the option of hiring bilingual operators instead of monolingual operators. If all the operators are bilingual, how many operators should be working during each 2-hour shift to answer all phone calls? As in part (*a*), please provide an integer answer.

(f) If all employees are bilingual, how many full-time and part-time employees should Lenny hire to begin on each shift to minimize operating costs while answering all calls? As in part (*b*), formulate a linear programming model to guide Lenny's decision.

(g) What is the maximum percentage increase in the hourly wage rate that Lenny can pay bilingual employees over monolingual employees without increasing the total operating costs?

(h) What other features of the call center should Lenny explore to improve service or minimize operating costs?

3. Dantzig, G.B., and M.N. Thapa: *Linear Programming 1: Introduction,* Springer, New York, 1997.
4. *LINDO User's Manual,* LINDO Systems, Inc., Chicago, IL, e-mail: info@lindo.com, 1999.
5. Vanderbei, R. J.: *Linear Programming: Foundations and Extensions,* Kluwer Academic Publishers, Boston, MA, 1996.

LEARNING AIDS FOR THIS CHAPTER IN YOUR OR COURSEWARE

Demonstration Examples in OR Tutor:

Interpretation of the Slack Variables
Simplex Method—Algebraic Form
Simplex Method—Tabular Form

Interactive Routines:

Enter or Revise a General Linear Programming Model
Set Up for the Simplex Method—Interactive Only
Solve Interactively by the Simplex Method

An Automatic Routine:

Solve Automatically by the Interior-Point Algorithm

An Excel Add-In:

Premium Solver

Files (Chapter 3) for Solving the Wyndor and Radiation Therapy Examples:

Excel File
LINGO/LINDO File
MPL/CPLEX File

See Appendix 1 for documentation of the software.

PROBLEMS

The symbols to the left of some of the problems (or their parts) have the following meaning:

D: The corresponding demonstration example listed above may be helpful.

I: We suggest that you use the corresponding interactive routine listed above (the printout records your work).

C: Use the computer with any of the software options available to you (or as instructed by your instructor) to solve the problem automatically. (See Sec. 4.8 for a listing of the options featured in this book and on the CD-ROM.)

An asterisk on the problem number indicates that at least a partial answer is given in the back of the book.

4.1-1. Consider the following problem.

$$\text{Maximize} \quad Z = x_1 + 2x_2,$$

subject to

$$\begin{aligned} x_1 \quad\; &\le 2 \\ x_2 &\le 2 \\ x_1 + x_2 &\le 3 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

(a) Plot the feasible region and circle all the CPF solutions.

(b) For each CPF solution, identify the pair of constraint boundary equations that it satisfies.
(c) For each CPF solution, use this pair of constraint boundary equations to solve algebraically for the values of x_1 and x_2 at the corner point.
(d) For each CPF solution, identify its adjacent CPF solutions.
(e) For each pair of adjacent CPF solutions, identify the constraint boundary they share by giving its equation.

4.1-2. Consider the following problem.

Maximize $\quad Z = 3x_1 + 2x_2,$

subject to

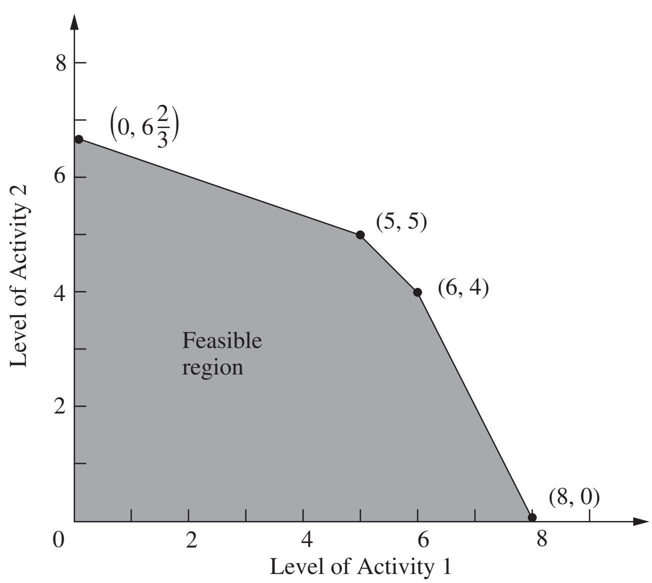
$$2x_1 + x_2 \le 6$$
$$x_1 + 2x_2 \le 6$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

(a) Use the graphical method to solve this problem. Circle all the corner points on the graph.
(b) For each CPF solution, identify the pair of constraint boundary equations it satisfies.
(c) For each CPF solution, identify its adjacent CPF solutions.
(d) Calculate Z for each CPF solution. Use this information to identify an optimal solution.
(e) Describe graphically what the simplex method does step by step to solve the problem.

4.1-3. A certain linear programming model involving two activities has the feasible region shown below.



The objective is to maximize the total profit from the two activities. The unit profit for activity 1 is \$1,000 and the unit profit for activity 2 is \$2,000.
(a) Calculate the total profit for each CPF solution. Use this information to find an optimal solution.
(b) Use the solution concepts of the simplex method given in Sec. 4.1 to identify the sequence of CPF solutions that would be examined by the simplex method to reach an optimal solution.

4.1-4.* Consider the linear programming model (given in the back of the book) that was formulated for Prob. 3.2-3.
(a) Use graphical analysis to identify all the *corner-point solutions* for this model. Label each as either feasible or infeasible.
(b) Calculate the value of the objective function for each of the CPF solutions. Use this information to identify an optimal solution.
(c) Use the solution concepts of the simplex method given in Sec. 4.1 to identify which sequence of CPF solutions might be examined by the simplex method to reach an optimal solution. (*Hint*: There are *two* alternative sequences to be identified for this particular model.)

4.1-5. Repeat Prob. 4.1-4 for the following problem.

Maximize $\quad Z = x_1 + 2x_2,$

subject to

$$x_1 + 3x_2 \le 8$$
$$x_1 + x_2 \le 4$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

4.1-6. Repeat Prob. 4.1-4 for the following problem.

Maximize $\quad Z = 3x_1 + 2x_2,$

subject to

$$x_1 \le 4$$
$$x_1 + 3x_2 \le 15$$
$$2x_1 + x_2 \le 10$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

4.1-7. Describe graphically what the simplex method does step by step to solve the following problem.

Maximize $\quad Z = 2x_1 + 3x_2,$

subject to

$$-3x_1 + x_2 \le 1$$
$$4x_1 + 2x_2 \le 20$$

$$\begin{aligned} 4x_1 - x_2 &\le 10 \\ -x_1 + 2x_2 &\le 5 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

4.1-8. Describe graphically what the simplex method does step by step to solve the following problem.

$$\text{Minimize} \qquad Z = 5x_1 + 7x_2,$$

subject to

$$\begin{aligned} 2x_1 + 3x_2 &\ge 42 \\ 3x_1 + 4x_2 &\ge 60 \\ x_1 + x_2 &\ge 18 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

4.1-9. Label each of the following statements about linear programming problems as true or false, and then justify your answer.

(a) For minimization problems, if the objective function evaluated at a CPF solution is no larger than its value at every adjacent CPF solution, then that solution is optimal.

(b) Only CPF solutions can be optimal, so the number of optimal solutions cannot exceed the number of CPF solutions.

(c) If multiple optimal solutions exist, then an optimal CPF solution may have an adjacent CPF solution that also is optimal (the same value of Z).

4.1-10. The following statements give inaccurate paraphrases of the six solution concepts presented in Sec. 4.1. In each case, explain what is wrong with the statement.

(a) The best CPF solution always is an optimal solution.

(b) An iteration of the simplex method checks whether the current CPF solution is optimal and, if not, moves to a new CPF solution.

(c) Although any CPF solution can be chosen to be the initial CPF solution, the simplex method always chooses the origin.

(d) When the simplex method is ready to choose a new CPF solution to move to from the current CPF solution, it only considers adjacent CPF solutions because one of them is likely to be an optimal solution.

(e) To choose the new CPF solution to move to from the current CPF solution, the simplex method identifies all the adjacent CPF solutions and determines which one gives the largest rate of improvement in the value of the objective function.

4.2-1. Reconsider the model in Prob. 4.1-4.

(a) Introduce slack variables in order to write the functional constraints in augmented form.

(b) For each CPF solution, identify the corresponding BF solution by calculating the values of the slack variables. For each BF solution, use the values of the variables to identify the nonbasic variables and the basic variables.

(c) For each BF solution, demonstrate (by plugging in the solution) that, after the nonbasic variables are set equal to zero, this BF solution also is the simultaneous solution of the system of equations obtained in part (*a*).

4.2-2. Reconsider the model in Prob. 4.1-5. Follow the instructions of Prob. 4.2-1 for parts (*a*), (*b*), and (*c*).

(d) Repeat part (*b*) for the corner-point infeasible solutions and the corresponding basic infeasible solutions.

(e) Repeat part (*c*) for the basic infeasible solutions.

4.2-3. Follow the instructions of Prob. 4.2-1 for the model in Prob. 4.1-6.

D,I **4.3-1.** Work through the simplex method (in algebraic form) step by step to solve the model in Prob. 4.1-4.

4.3-2. Reconsider the model in Prob. 4.1-5.

(a) Work through the simplex method (in algebraic form) *by hand* to solve this model.

D,I **(b)** Repeat part (*a*) with the corresponding interactive routine in your OR Tutor.

C **(c)** Verify the optimal solution you obtained by using a software package based on the simplex method.

4.3-3. Follow the instructions of Prob. 4.3-2 for the model in Prob. 4.1-6.

D,I **4.3-4.*** Work through the simplex method (in algebraic form) step by step to solve the following problem.

$$\text{Maximize} \qquad Z = 4x_1 + 3x_2 + 6x_3,$$

subject to

$$\begin{aligned} 3x_1 + x_2 + 3x_3 &\le 30 \\ 2x_1 + 2x_2 + 3x_3 &\le 40 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0, \qquad x_3 \ge 0.$$

D,I **4.3-5.** Work through the simplex method (in algebraic form) step by step to solve the following problem.

$$\text{Maximize} \qquad Z = x_1 + 2x_2 + 4x_3,$$

subject to

$$\begin{aligned} 3x_1 + x_2 + 5x_3 &\le 10 \\ x_1 + 4x_2 + x_3 &\le 8 \\ 2x_1 \phantom{{}+4x_2} + 2x_3 &\le 7 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0.$$

D,I **4.3-6.** Work through the simplex method (in algebraic form) step by step to solve the following problem.

$$\text{Maximize} \qquad Z = x_1 + 2x_2 + 2x_3,$$

subject to

$$\begin{aligned} 5x_1 + 2x_2 + 3x_3 &\leq 15 \\ x_1 + 4x_2 + 2x_3 &\leq 12 \\ 2x_1 \qquad\quad + x_3 &\leq 8 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0.$$

4.3-7. Consider the following problem.

$$\text{Maximize} \qquad Z = 5x_1 + 3x_2 + 4x_3,$$

subject to

$$\begin{aligned} 2x_1 + x_2 + x_3 &\leq 20 \\ 3x_1 + x_2 + 2x_3 &\leq 30 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0.$$

You are given the information that the *nonzero* variables in the optimal solution are x_2 and x_3.

(a) Describe how you can use this information to adapt the simplex method to solve this problem in the minimum possible number of iterations (when you start from the usual initial BF solution). Do *not* actually perform any iterations.

(b) Use the procedure developed in part (*a*) to solve this problem by hand. (Do *not* use your OR Courseware.)

4.3-8. Consider the following problem.

$$\text{Maximize} \qquad Z = 2x_1 + 4x_2 + 3x_3,$$

subject to

$$\begin{aligned} x_1 + 3x_2 + 2x_3 &\leq 30 \\ x_1 + x_2 + x_3 &\leq 24 \\ 3x_1 + 5x_2 + 3x_3 &\leq 60 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0.$$

You are given the information that $x_1 > 0$, $x_2 = 0$, and $x_3 > 0$ in the optimal solution.

(a) Describe how you can use this information to adapt the simplex method to solve this problem in the minimum possible number of iterations (when you start from the usual initial BF solution). Do *not* actually perform any iterations.

(b) Use the procedure developed in part (*a*) to solve this problem by hand. (Do *not* use your OR Courseware.)

4.3-9. Label each of the following statements as true or false, and then justify your answer by referring to specific statements (with page citations) in the chapter.

(a) The simplex method's rule for choosing the entering basic variable is used because it always leads to the *best* adjacent BF solution (largest Z).

(b) The simplex method's minimum ratio rule for choosing the leaving basic variable is used because making another choice with a larger ratio would yield a basic solution that is not feasible.

(c) When the simplex method solves for the next BF solution, elementary algebraic operations are used to eliminate each nonbasic variable from all but one equation (*its* equation) and to give it a coefficient of +1 in that one equation.

D,I **4.4-1.** Repeat Prob. 4.3-1, using the tabular form of the simplex method.

D,I,C **4.4-2.** Repeat Prob. 4.3-2, using the tabular form of the simplex method.

D,I,C **4.4-3.** Repeat Prob. 4.3-3, using the tabular form of the simplex method.

4.4-4. Consider the following problem.

$$\text{Maximize} \qquad Z = 2x_1 + x_2,$$

subject to

$$\begin{aligned} x_1 + x_2 &\leq 40 \\ 4x_1 + x_2 &\leq 100 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

(a) Solve this problem graphically in a freehand manner. Also identify all the CPF solutions.

(b) Now repeat part (*a*) when using a ruler to draw the graph carefully.

D **(c)** Use hand calculations to solve this problem by the simplex method in algebraic form.

D,I **(d)** Now use your OR Courseware to solve this problem interactively by the simplex method in algebraic form.

D **(e)** Use hand calculations to solve this problem by the simplex method in tabular form.

D,I **(f)** Now use your OR Courseware to solve this problem interactively by the simplex method in tabular form.

C **(g)** Use a software package based on the simplex method to solve the problem.

4.4-5. Repeat Prob. 4.4-4 for the following problem.

Maximize $Z = 2x_1 + 3x_2$,

subject to

$$\begin{aligned} x_1 + 2x_2 &\le 30 \\ x_1 + x_2 &\le 20 \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0.$$

4.4-6. Consider the following problem.

Maximize $Z = 2x_1 + 4x_2 + 3x_3$,

subject to

$$\begin{aligned} 3x_1 + 4x_2 + 2x_3 &\le 60 \\ 2x_1 + x_2 + 2x_3 &\le 40 \\ x_1 + 3x_2 + 2x_3 &\le 80 \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0.$$

D,I **(a)** Work through the simplex method step by step in algebraic form.

D,I **(b)** Work through the simplex method step by step in tabular form.

C **(c)** Use a software package based on the simplex method to solve the problem.

4.4-7. Consider the following problem.

Maximize $Z = 3x_1 + 5x_2 + 6x_3$,

subject to

$$\begin{aligned} 2x_1 + x_2 + x_3 &\le 4 \\ x_1 + 2x_2 + x_3 &\le 4 \\ x_1 + x_2 + 2x_3 &\le 4 \\ x_1 + x_2 + x_3 &\le 3 \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0.$$

D,I **(a)** Work through the simplex method step by step in algebraic form.

D,I **(b)** Work through the simplex method in tabular form.

C **(c)** Use a computer package based on the simplex method to solve the problem.

4.4-8. Consider the following problem.

Maximize $Z = 2x_1 - x_2 + x_3$,

subject to

$$\begin{aligned} x_1 - x_2 + 3x_3 &\le 4 \\ 2x_1 + x_2 &\le 10 \\ x_1 - x_2 - x_3 &\le 7 \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0.$$

D,I **(a)** Work through the simplex method step by step in algebraic form to solve this problem.

D,I **(b)** Work through the simplex method step by step in tabular form to solve the problem.

C **(c)** Use a computer package based on the simplex method to solve the problem.

D,I **4.4-9.** Work through the simplex method step by step (in tabular form) to solve the following problem.

Maximize $Z = 2x_1 - x_2 + x_3$,

subject to

$$\begin{aligned} 3x_1 + x_2 + x_3 &\le 6 \\ x_1 - x_2 + 2x_3 &\le 1 \\ x_1 + x_2 - x_3 &\le 2 \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0.$$

D,I **4.4-10.** Work through the simplex method step by step to solve the following problem.

Maximize $Z = -x_1 + x_2 + 2x_3$,

subject to

$$\begin{aligned} x_1 + 2x_2 - x_3 &\le 20 \\ -2x_1 + 4x_2 + 2x_3 &\le 60 \\ 2x_1 + 3x_2 + x_3 &\le 50 \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0.$$

4.5-1. Consider the following statements about linear programming and the simplex method. Label each statement as true or false, and then justify your answer.

(a) In a particular iteration of the simplex method, if there is a tie for which variable should be the leaving basic variable, then the next BF solution must have at least one basic variable equal to zero.

(b) If there is no leaving basic variable at some iteration, then the problem has no feasible solutions.

(c) If at least one of the basic variables has a coefficient of zero in row 0 of the final tableau, then the problem has multiple optimal solutions.

(d) If the problem has multiple optimal solutions, then the problem must have a bounded feasible region.

4.5-2. Suppose that the following constraints have been provided for a linear programming model with decision variables x_1 and x_2.

$$-x_1 + 3x_2 \leq 30$$
$$-3x_1 + x_2 \leq 30$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

(a) Demonstrate graphically that the feasible region is unbounded.
(b) If the objective is to maximize $Z = -x_1 + x_2$, does the model have an optimal solution? If so, find it. If not, explain why not.
(c) Repeat part (*b*) when the objective is to maximize $Z = x_1 - x_2$.
(d) For objective functions where this model has no optimal solution, does this mean that there are no good solutions according to the model? Explain. What probably went wrong when formulating the model?
D,I **(e)** Select an objective function for which this model has no optimal solution. Then work through the simplex method step by step to demonstrate that Z is unbounded.
C **(f)** For the objective function selected in part (*e*), use a software package based on the simplex method to determine that Z is unbounded.

4.5-3. Follow the instructions of Prob. 4.5-2 when the constraints are the following:

$$2x_1 - x_2 \leq 20$$
$$x_1 - 2x_2 \leq 20$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

D,I **4.5-4.** Consider the following problem.

$$\text{Maximize} \quad Z = 5x_1 + x_2 + 3x_3 + 4x_4,$$

subject to

$$x_1 - 2x_2 + 4x_3 + 3x_4 \leq 20$$
$$-4x_1 + 6x_2 + 5x_3 - 4x_4 \leq 40$$
$$2x_1 - 3x_2 + 3x_3 + 8x_4 \leq 50$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0, \qquad x_4 \geq 0.$$

Work through the simplex method step by step to demonstrate that Z is unbounded.

4.5-5. A basic property of any linear programming problem with a bounded feasible region is that every feasible solution can be expressed as a convex combination of the CPF solutions (perhaps in more than one way). Similarly, for the augmented form of the problem, every feasible solution can be expressed as a convex combination of the BF solutions.

(a) Show that *any* convex combination of *any* set of feasible solutions must be a feasible solution (so that any convex combination of CPF solutions must be feasible).
(b) Use the result quoted in part (*a*) to show that any convex combination of BF solutions must be a feasible solution.

4.5-6. Using the facts given in Prob. 4.5-5, show that the following statements must be true for any linear programming problem that has a bounded feasible region and multiple optimal solutions:
(a) Every convex combination of the optimal BF solutions must be optimal.
(b) No other feasible solution can be optimal.

4.5-7. Consider a two-variable linear programming problem whose CPF solutions are (0, 0), (6, 0), (6, 3), (3, 3), and (0, 2). (See Prob. 3.2-2 for a graph of the feasible region.)
(a) Use the graph of the feasible region to identify all the constraints for the model.
(b) For each pair of adjacent CPF solutions, give an example of an objective function such that all the points on the line segment between these two corner points are multiple optimal solutions.
(c) Now suppose that the objective function is $Z = -x_1 + 2x_2$. Use the graphical method to find all the optimal solutions.
D,I **(d)** For the objective function in part (*c*), work through the simplex method step by step to find all the optimal BF solutions. Then write an algebraic expression that identifies all the optimal solutions.

D,I **4.5-8.** Consider the following problem.

$$\text{Maximize} \quad Z = x_1 + x_2 + x_3 + x_4,$$

subject to

$$x_1 + x_2 \leq 3$$
$$x_3 + x_4 \leq 2$$

and

$$x_j \geq 0, \qquad \text{for } j = 1, 2, 3, 4.$$

Work through the simplex method step by step to find *all* the optimal BF solutions.

4.6-1.* Consider the following problem.

$$\text{Maximize} \quad Z = 2x_1 + 3x_2,$$

subject to

$$x_1 + 2x_2 \leq 4$$
$$x_1 + x_2 = 3$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

(a) Solve this problem graphically.
(b) Using the Big M method, construct the complete first simplex tableau for the simplex method and identify the corresponding initial (artificial) BF solution. Also identify the initial entering basic variable and the leaving basic variable.
I **(c)** Continue from part (*b*) to work through the simplex method step by step to solve the problem.

4.6-2. Consider the following problem.

$$\text{Maximize} \quad Z = 4x_1 + 2x_2 + 3x_3 + 5x_4,$$

subject to

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 + 2x_4 &= 300 \\ 8x_1 + x_2 + x_3 + 5x_4 &= 300 \end{aligned}$$

and

$$x_j \geq 0, \qquad \text{for } j = 1, 2, 3, 4.$$

(a) Using the Big M method, construct the complete first simplex tableau for the simplex method and identify the corresponding initial (artificial) BF solution. Also identify the initial entering basic variable and the leaving basic variable.
I **(b)** Work through the simplex method step by step to solve the problem.
(c) Using the two-phase method, construct the complete first simplex tableau for phase 1 and identify the corresponding initial (artificial) BF solution. Also identify the initial entering basic variable and the leaving basic variable.
I **(d)** Work through phase 1 step by step.
(e) Construct the complete first simplex tableau for phase 2.
I **(f)** Work through phase 2 step by step to solve the problem.
(g) Compare the sequence of BF solutions obtained in part (*b*) with that in parts (*d*) and (*f*). Which of these solutions are feasible only for the artificial problem obtained by introducing artificial variables and which are actually feasible for the real problem?
C **(h)** Use a software package based on the simplex method to solve the problem.

4.6-3. Consider the following problem.

$$\text{Minimize} \quad Z = 3x_1 + 2x_2,$$

subject to

$$\begin{aligned} 2x_1 + x_2 &\geq 10 \\ -3x_1 + 2x_2 &\leq 6 \\ x_1 + x_2 &\geq 6 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

(a) Solve this problem graphically.
(b) Using the Big M method, construct the complete first simplex tableau for the simplex method and identify the corresponding initial (artificial) BF solution. Also identify the initial entering basic variable and the leaving basic variable.
I **(c)** Work through the simplex method step by step to solve the problem.

4.6-4.* Consider the following problem.

$$\text{Minimize} \quad Z = 2x_1 + 3x_2 + x_3,$$

subject to

$$\begin{aligned} x_1 + 4x_2 + 2x_3 &\geq 8 \\ 3x_1 + 2x_2 \qquad &\geq 6 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0.$$

(a) Reformulate this problem to fit our standard form for a linear programming model presented in Sec. 3.2.
I **(b)** Using the Big M method, work through the simplex method step by step to solve the problem.
I **(c)** Using the two-phase method, work through the simplex method step by step to solve the problem.
(d) Compare the sequence of BF solutions obtained in parts (*b*) and (*c*). Which of these solutions are feasible only for the artificial problem obtained by introducing artificial variables and which are actually feasible for the real problem?
C **(e)** Use a software package based on the simplex method to solve the problem.

4.6-5. For the Big M method, explain why the simplex method never would choose an artificial variable to be an entering basic variable once all the artificial variables are nonbasic.

4.6-6. Consider the following problem.

$$\text{Maximize} \quad Z = 90x_1 + 70x_2,$$

subject to

$$\begin{aligned} 2x_1 + x_2 &\leq 2 \\ x_1 - x_2 &\geq 2 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

(a) Demonstrate graphically that this problem has no feasible solutions.
C **(b)** Use a computer package based on the simplex method to determine that the problem has no feasible solutions.
I **(c)** Using the Big M method, work through the simplex method step by step to demonstrate that the problem has no feasible solutions.
I **(d)** Repeat part (*c*) when using phase 1 of the two-phase method.

4.6-7. Follow the instructions of Prob. 4.6-6 for the following problem.

Minimize $Z = 5{,}000x_1 + 7{,}000x_2$,

subject to

$$\begin{aligned} -2x_1 + x_2 &\geq 1 \\ x_1 - 2x_2 &\geq 1 \end{aligned}$$

and

$x_1 \geq 0, \quad x_2 \geq 0.$

4.6-8. Consider the following problem.

Maximize $Z = 2x_1 + 5x_2 + 3x_3$,

subject to

$$\begin{aligned} x_1 - 2x_2 + x_3 &\geq 20 \\ 2x_1 + 4x_2 + x_3 &= 50 \end{aligned}$$

and

$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$

(a) Using the Big M method, construct the complete first simplex tableau for the simplex method and identify the corresponding initial (artificial) BF solution. Also identify the initial entering basic variable and the leaving basic variable.

I **(b)** Work through the simplex method step by step to solve the problem.

I **(c)** Using the two-phase method, construct the complete first simplex tableau for phase 1 and identify the corresponding initial (artificial) BF solution. Also identify the initial entering basic variable and the leaving basic variable.

I **(d)** Work through phase 1 step by step.

(e) Construct the complete first simplex tableau for phase 2.

I **(f)** Work through phase 2 step by step to solve the problem.

(g) Compare the sequence of BF solutions obtained in part (*b*) with that in parts (*d*) and (*f*). Which of these solutions are feasible only for the artificial problem obtained by introducing artificial variables and which are actually feasible for the real problem?

C **(h)** Use a software package based on the simplex method to solve the problem.

4.6-9. Consider the following problem.

Minimize $Z = 2x_1 + x_2 + 3x_3$,

subject to

$$\begin{aligned} 5x_1 + 2x_2 + 7x_3 &= 420 \\ 3x_1 + 2x_2 + 5x_3 &\geq 280 \end{aligned}$$

and

$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$

I **(a)** Using the two-phase method, work through phase 1 step by step.

C **(b)** Use a software package based on the simplex method to formulate and solve the phase 1 problem.

I **(c)** Work through phase 2 step by step to solve the original problem.

C **(d)** Use a computer code based on the simplex method to solve the original problem.

4.6-10.* Consider the following problem.

Minimize $Z = 3x_1 + 2x_2 + 4x_3$,

subject to

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 60 \\ 3x_1 + 3x_2 + 5x_3 &\geq 120 \end{aligned}$$

and

$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$

I **(a)** Using the Big M method, work through the simplex method step by step to solve the problem.

I **(b)** Using the two-phase method, work through the simplex method step by step to solve the problem.

(c) Compare the sequence of BF solutions obtained in parts (*a*) and (*b*). Which of these solutions are feasible only for the artificial problem obtained by introducing artificial variables and which are actually feasible for the real problem?

C **(d)** Use a software package based on the simplex method to solve the problem.

4.6-11. Follow the instructions of Prob. 4.6-10 for the following problem.

Minimize $Z = 3x_1 + 2x_2 + 7x_3$,

subject to

$$\begin{aligned} -x_1 + x_2 &= 10 \\ 2x_1 - x_2 + x_3 &\geq 10 \end{aligned}$$

and

$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$

4.6-12. Follow the instructions of Prob. 4.6-10 for the following problem.

Minimize $Z = 3x_1 + 2x_2 + x_3$,

subject to

$$\begin{aligned} x_1 + x_2 &= 7 \\ 3x_1 + x_2 + x_3 &\geq 10 \end{aligned}$$

and

$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$

4.6-13. Label each of the following statements as true or false, and then justify your answer.

(a) When a linear programming model has an equality constraint, an artificial variable is introduced into this constraint in order to start the simplex method with an obvious initial basic solution that is feasible for the original model.

(b) When an artificial problem is created by introducing artificial variables and using the Big M method, if all artificial variables in an optimal solution for the artificial problem are equal to zero, then the real problem has no feasible solutions.

(c) The two-phase method is commonly used in practice because it usually requires fewer iterations to reach an optimal solution than the Big M method does.

4.6-14. Consider the following problem.

$$\text{Maximize} \quad Z = x_1 + 4x_2 + 2x_3,$$

subject to

$$\begin{aligned} 4x_1 - x_2 + x_3 &\le 5 \\ -x_1 - x_2 + 2x_3 &\le 10 \end{aligned}$$

and

$$x_2 \ge 0, \qquad x_3 \ge 0$$

(no nonnegativity constraint for x_1).

(a) Reformulate this problem so all variables have nonnegativity constraints.

D,I **(b)** Work through the simplex method step by step to solve the problem.

C **(c)** Use a software package based on the simplex method to solve the problem.

4.6-15.* Consider the following problem.

$$\text{Maximize} \quad Z = -x_1 + 4x_2,$$

subject to

$$\begin{aligned} -3x_1 + x_2 &\le 6 \\ x_1 + 2x_2 &\le 4 \\ x_2 &\ge -3 \end{aligned}$$

(no lower bound constraint for x_1).

(a) Solve this problem graphically.

(b) Reformulate this problem so that it has only two functional constraints and all variables have nonnegativity constraints.

D,I **(c)** Work through the simplex method step by step to solve the problem.

4.6-16. Consider the following problem.

$$\text{Maximize} \quad Z = -x_1 + 2x_2 + x_3,$$

subject to

$$\begin{aligned} 3x_2 + x_3 &\le 120 \\ x_1 - x_2 - 4x_3 &\le 80 \\ -3x_1 + x_2 + 2x_3 &\le 100 \end{aligned}$$

(no nonnegativity constraints).

(a) Reformulate this problem so that all variables have nonnegativity constraints.

D,I **(b)** Work through the simplex method step by step to solve the problem.

C **(c)** Use a computer package based on the simplex method to solve the problem.

4.6-17. This chapter has described the simplex method as applied to linear programming problems where the objective function is to be maximized. Section 4.6 then described how to convert a minimization problem to an equivalent maximization problem for applying the simplex method. Another option with minimization problems is to make a few modifications in the instructions for the simplex method given in the chapter in order to apply the algorithm directly.

(a) Describe what these modifications would need to be.

(b) Using the Big M method, apply the modified algorithm developed in part (*a*) to solve the following problem directly by hand. (Do not use your OR Courseware.)

$$\text{Minimize} \quad Z = 3x_1 + 8x_2 + 5x_3,$$

subject to

$$\begin{aligned} 3x_2 + 4x_3 &\ge 70 \\ 3x_1 + 5x_2 + 2x_3 &\ge 70 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0, \qquad x_3 \ge 0.$$

4.6-18. Consider the following problem.

$$\text{Maximize} \quad Z = -2x_1 + x_2 - 4x_3 + 3x_4,$$

subject to

$$\begin{aligned} x_1 + x_2 + 3x_3 + 2x_4 &\le 4 \\ x_1 - x_3 + x_4 &\ge -1 \\ 2x_1 + x_2 &\le 2 \\ x_1 + 2x_2 + x_3 + 2x_4 &= 2 \end{aligned}$$

and

$$x_2 \ge 0, \qquad x_3 \ge 0, \qquad x_4 \ge 0$$

(no nonnegativity constraint for x_1).

(a) Reformulate this problem to fit our standard form for a linear programming model presented in Sec. 3.2.

(b) Using the Big M method, construct the complete first simplex tableau for the simplex method and identify the corresponding initial (artificial) BF solution. Also identify the initial entering basic variable and the leaving basic variable.

(c) Using the two-phase method, construct row 0 of the first simplex tableau for phase 1.

C **(d)** Use a computer package based on the simplex method to solve the problem.

I **4.6-19.** Consider the following problem.

Maximize $\quad Z = 4x_1 + 5x_2 + 3x_3,$

subject to

$$\begin{aligned} x_1 + x_2 + 2x_3 &\geq 20 \\ 15x_1 + 6x_2 - 5x_3 &\leq 50 \\ x_1 + 3x_2 + 5x_3 &\leq 30 \end{aligned}$$

and

$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$

Work through the simplex method step by step to demonstrate that this problem does not possess any feasible solutions.

4.7-1. Refer to Fig. 4.10 and the resulting *allowable range to stay feasible* for the respective right-hand sides of the Wyndor Glass Co. problem given in Sec. 3.1. Use graphical analysis to demonstrate that each given allowable range is correct.

4.7-2. Reconsider the model in Prob. 4.1-5. Interpret the right-hand side of the respective functional constraints as the amount available of the respective resources.

(a) Use graphical analysis as in Fig. 4.8 to determine the shadow prices for the respective resources.
(b) Use graphical analysis to perform sensitivity analysis on this model. In particular, check each parameter of the model to determine whether it is a *sensitive* parameter (a parameter whose value cannot be changed without changing the optimal solution) by examining the graph that identifies the optimal solution.
(c) Use graphical analysis as in Fig. 4.9 to determine the allowable range for each c_j value (coefficient of x_j in the objective function) over which the current optimal solution will remain optimal.
(d) Changing just one b_i value (the right-hand side of functional constraint i) will shift the corresponding constraint boundary. If the current optimal CPF solution lies on this constraint boundary, this CPF solution also will shift. Use graphical analysis to determine the allowable range for each b_i value over which this CPF solution will remain feasible.
C **(e)** Verify your answers in parts (*a*), (*c*), and (*d*) by using a computer package based on the simplex method to solve the problem and then to generate sensitivity analysis information.

4.7-3. Repeat Prob. 4.7-2 for the model in Prob. 4.1-6.

4.7-4. You are given the following linear programming problem.

Maximize $\quad Z = 4x_1 + 2x_2,$

subject to

$$\begin{aligned} 2x_1 &\leq 16 \qquad \text{(resource 1)} \\ x_1 + 3x_2 &\leq 17 \qquad \text{(resource 2)} \\ x_2 &\leq 5 \qquad \text{(resource 3)} \end{aligned}$$

and

$x_1 \geq 0, \quad x_2 \geq 0.$

(a) Solve this problem graphically.
(b) Use graphical analysis to find the shadow prices for the resources.
(c) Determine how many additional units of resource 1 would be needed to increase the optimal value of Z by 15.

4.7-5. Consider the following problem.

Maximize $\quad Z = x_1 - 7x_2 + 3x_3,$

subject to

$$\begin{aligned} 2x_1 + x_2 - x_3 &\leq 4 \qquad \text{(resource 1)} \\ 4x_1 - 3x_2 &\leq 2 \qquad \text{(resource 2)} \\ -3x_1 + 2x_2 + x_3 &\leq 3 \qquad \text{(resource 3)} \end{aligned}$$

and

$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$

D,I **(a)** Work through the simplex method step by step to solve the problem.
(b) Identify the shadow prices for the three resources and describe their significance.
C **(c)** Use a software package based on the simplex method to solve the problem and then to generate sensitivity information. Use this information to identify the shadow price for each resource, the allowable range to stay optimal for each objective function coefficient, and the allowable range to stay feasible for each right-hand side.

4.7-6.* Consider the following problem.

Maximize $\quad Z = 2x_1 - 2x_2 + 3x_3,$

subject to

$$\begin{aligned} -x_1 + x_2 + x_3 &\leq 4 \qquad \text{(resource 1)} \\ 2x_1 - x_2 + x_3 &\leq 2 \qquad \text{(resource 2)} \\ x_1 + x_2 + 3x_3 &\leq 12 \qquad \text{(resource 3)} \end{aligned}$$

and

$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$

D,I **(a)** Work through the simplex method step by step to solve the problem.
(b) Identify the shadow prices for the three resources and describe their significance.
C **(c)** Use a software package based on the simplex method to solve the problem and then to generate sensitivity information. Use this information to identify the shadow price for each resource, the allowable range to stay optimal for each objective function coefficient and the allowable range to stay feasible for each right-hand side.

4.7-7. Consider the following problem.

Maximize $Z = 2x_1 + 4x_2 - x_3,$

subject to

$$\begin{aligned} 3x_2 - x_3 &\le 30 \qquad \text{(resource 1)} \\ 2x_1 - x_2 + x_3 &\le 10 \qquad \text{(resource 2)} \\ 4x_1 + 2x_2 - 2x_3 &\le 40 \qquad \text{(resource 3)} \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0.$$

D,I **(a)** Work through the simplex method step by step to solve the problem.

(b) Identify the shadow prices for the three resources and describe their significance.

C **(c)** Use a software package based on the simplex method to solve the problem and then to generate sensitivity information. Use this information to identify the shadow price for each resource, the allowable range to stay optimal for each objective function coefficient, and the allowable range to stay feasible for each right-hand side.

4.7-8. Consider the following problem.

Maximize $Z = 5x_1 + 4x_2 - x_3 + 3x_4,$

subject to

$$\begin{aligned} 3x_1 + 2x_2 - 3x_3 + x_4 &\le 24 \qquad \text{(resource 1)} \\ 3x_1 + 3x_2 + x_3 + 3x_4 &\le 36 \qquad \text{(resource 2)} \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0, \quad x_4 \ge 0.$$

D,I **(a)** Work through the simplex method step by step to solve the problem.

(b) Identify the shadow prices for the two resources and describe their significance.

C **(c)** Use a software package based on the simplex method to solve the problem and then to generate sensitivity information. Use this information to identify the shadow price for each resource, the allowable range to stay optimal for each objective function coefficient, and the allowable range to stay feasible for each right-hand side.

4.9.1. Use the interior-point algorithm in your OR Courseware to solve the model in Prob. 4.1-4. Choose $\alpha = 0.5$ from the Option menu, use $(x_1, x_2) = (0.1, 0.4)$ as the initial trial solution, and run 15 iterations. Draw a graph of the feasible region, and then plot the trajectory of the trial solutions through this feasible region.

4.9-2. Repeat Prob. 4.9-1 for the model in Prob. 4.1-5.

4.9-3. Repeat Prob. 4.9-1 for the model in Prob. 4.1-6.

CASE 4.1 FABRICS AND FALL FASHIONS

From the tenth floor of her office building, Katherine Rally watches the swarms of New Yorkers fight their way through the streets infested with yellow cabs and the sidewalks littered with hot dog stands. On this sweltering July day, she pays particular attention to the fashions worn by the various women and wonders what they will choose to wear in the fall. Her thoughts are not simply random musings; they are critical to her work since she owns and manages TrendLines, an elite women's clothing company.

Today is an especially important day because she must meet with Ted Lawson, the production manager, to decide upon next month's production plan for the fall line. Specifically, she must determine the quantity of each clothing item she should produce given the plant's production capacity, limited resources, and demand forecasts. Accurate planning for next month's production is critical to fall sales since the items produced next month will appear in stores during September, and women generally buy the majority of the fall fashions when they first appear in September.

She turns back to her sprawling glass desk and looks at the numerous papers covering it. Her eyes roam across the clothing patterns designed almost six months ago, the lists of materials requirements for each pattern, and the lists of demand forecasts for each pattern determined by customer surveys at fashion shows. She remembers the hectic and sometimes nightmarish days of designing the fall line and presenting it at fashion shows in New York, Milan, and Paris. Ultimately, she paid her team of six designers a total of $860,000 for their work on her fall line. With the cost of hiring runway models, hair stylists, and makeup artists, sewing and fitting clothes, building the set, choreographing and rehearsing the show, and renting the conference hall, each of the three fashion shows cost her an additional $2,700,000.

She studies the clothing patterns and material requirements. Her fall line consists of both professional and casual fashions. She determined the prices for each clothing item by taking into account the quality and cost of material, the cost of labor and machining, the demand for the item, and the prestige of the TrendLines brand name.

The fall professional fashions include:

Clothing Item	Materials Requirements	Price	Labor and Machine Cost
Tailored wool slacks	3 yards of wool 2 yards of acetate for lining	$300	$160
Cashmere sweater	1.5 yards of cashmere	$450	$150
Silk blouse	1.5 yards of silk	$180	$100
Silk camisole	0.5 yard of silk	$120	$ 60
Tailored skirt	2 yards of rayon 1.5 yards of acetate for lining	$270	$120
Wool blazer	2.5 yards of wool 1.5 yards of acetate for lining	$320	$140

The fall casual fashions include:

Clothing Item	Materials Requirements	Price	Labor and Machine Cost
Velvet pants	3 yards of velvet 2 yards of acetate for lining	$350	$175
Cotton sweater	1.5 yards of cotton	$130	$ 60
Cotton miniskirt	0.5 yard of cotton	$ 75	$ 40
Velvet shirt	1.5 yards of velvet	$200	$160
Button-down blouse	1.5 yards of rayon	$120	$ 90

She knows that for the next month, she has ordered 45,000 yards of wool, 28,000 yards of acetate, 9,000 yards of cashmere, 18,000 yards of silk, 30,000 yards of rayon, 20,000 yards of velvet, and 30,000 yards of cotton for production. The prices of the materials are listed on the next page.

Material	Price per yard
Wool	$ 9.00
Acetate	$ 1.50
Cashmere	$60.00
Silk	$13.00
Rayon	$ 2.25
Velvet	$12.00
Cotton	$ 2.50

Any material that is not used in production can be sent back to the textile wholesaler for a full refund, although scrap material cannot be sent back to the wholesaler.

She knows that the production of both the silk blouse and cotton sweater leaves leftover scraps of material. Specifically, for the production of one silk blouse or one cotton sweater, 2 yards of silk and cotton, respectively, are needed. From these 2 yards, 1.5 yards are used for the silk blouse or the cotton sweater and 0.5 yard is left as scrap material. She does not want to waste the material, so she plans to use the rectangular scrap of silk or cotton to produce a silk camisole or cotton miniskirt, respectively. Therefore, whenever a silk blouse is produced, a silk camisole is also produced. Likewise, whenever a cotton sweater is produced, a cotton miniskirt is also produced. Note that it is possible to produce a silk camisole without producing a silk blouse and a cotton miniskirt without producing a cotton sweater.

The demand forecasts indicate that some items have limited demand. Specifically, because the velvet pants and velvet shirts are fashion fads, TrendLines has forecasted that it can sell only 5,500 pairs of velvet pants and 6,000 velvet shirts. TrendLines does not want to produce more than the forecasted demand because once the pants and shirts go out of style, the company cannot sell them. TrendLines can produce less than the forecasted demand, however, since the company is not required to meet the demand. The cashmere sweater also has limited demand because it is quite expensive, and TrendLines knows it can sell at most 4,000 cashmere sweaters. The silk blouses and camisoles have limited demand because many women think silk is too hard to care for, and TrendLines projects that it can sell at most 12,000 silk blouses and 15,000 silk camisoles.

The demand forecasts also indicate that the wool slacks, tailored skirts, and wool blazers have a great demand because they are basic items needed in every professional wardrobe. Specifically, the demand for wool slacks is 7,000 pairs of slacks, and the demand for wool blazers is 5,000 blazers. Katherine wants to meet at least 60 percent of the demand for these two items in order to maintain her loyal customer base and not lose business in the future. Although the demand for tailored skirts could not be estimated, Katherine feels she should make at least 2,800 of them.

(a) Ted is trying to convince Katherine not to produce any velvet shirts since the demand for this fashion fad is quite low. He argues that this fashion fad alone accounts for $500,000 of the fixed design and other costs. The net contribution (price of clothing item − materials cost − labor cost) from selling the fashion fad should cover these fixed costs. Each velvet shirt generates a net contribution of $22. He argues that given the net contribution, even satisfying the maximum demand will not yield a profit. What do you think of Ted's argument?

(b) Formulate and solve a linear programming problem to maximize profit given the production, resource, and demand constraints.

Before she makes her final decision, Katherine plans to explore the following questions independently except where otherwise indicated.

(c) The textile wholesaler informs Katherine that the velvet cannot be sent back because the demand forecasts show that the demand for velvet will decrease in the future. Katherine can therefore get no refund for the velvet. How does this fact change the production plan?

(d) What is an intuitive economic explanation for the difference between the solutions found in parts (*b*) and (*c*)?

(e) The sewing staff encounters difficulties sewing the arms and lining into the wool blazers since the blazer pattern has an awkward shape and the heavy wool material is difficult to cut and sew. The increased labor time to sew a wool blazer increases the labor and machine cost for each blazer by $80. Given this new cost, how many of each clothing item should TrendLines produce to maximize profit?

(f) The textile wholesaler informs Katherine that since another textile customer canceled his order, she can obtain an extra 10,000 yards of acetate. How many of each clothing item should TrendLines now produce to maximize profit?

(g) TrendLines assumes that it can sell every item that was not sold during September and October in a big sale in November at 60 percent of the original price. Therefore, it can sell all items in unlimited quantity during the November sale. (The previously mentioned upper limits on demand concern only the sales during September and October.) What should the new production plan be to maximize profit?

CASE 4.2 NEW FRONTIERS

Rob Richman, president of AmeriBank, takes off his glasses, rubs his eyes in exhaustion, and squints at the clock in his study. It reads 3 A.M. For the last several hours, Rob has been poring over AmeriBank's financial statements from the last three quarters of operation. AmeriBank, a medium-sized bank with branches throughout the United States, is headed for dire economic straits. The bank, which provides transaction, savings, and investment and loan services, has been experiencing a steady decline in its net income over the past year, and trends show that the decline will continue. The bank is simply losing customers to nonbank and foreign bank competitors.

AmeriBank is not alone in its struggle to stay out of the red. From his daily industry readings, Rob knows that many American banks have been suffering significant losses because of increasing competition from nonbank and foreign bank competitors offering services typically in the domain of American banks. Because the nonbank and foreign bank competitors specialize in particular services, they are able to better capture the market for those services by offering less expensive, more efficient, more convenient services. For example, large corporations now turn to foreign banks and commercial paper offerings for loans, and affluent Americans now turn to money-market funds for investment. Banks face the daunting challenge of distinguishing themselves from nonbank and foreign bank competitors.

Rob has concluded that one strategy for distinguishing AmeriBank from its competitors is to improve services that nonbank and foreign bank competitors do not readily provide: transaction services. He has decided that a more convenient transaction method must logically succeed the automatic teller machine, and he believes that electronic banking over the Internet allows this convenient transaction method. Over the Internet, customers are able to perform transactions on their desktop computers either at home or at work. The explosion of the Internet means that many potential customers understand and use the World Wide Web. He therefore feels that if AmeriBank offers Web banking (as the practice of Internet banking is commonly called), the bank will attract many new customers.

Before Rob undertakes the project to make Web banking possible, however, he needs to understand the market for Web banking and the services AmeriBank should provide over the Internet. For example, should the bank only allow customers to access account balances and historical transaction information over the Internet, or should the bank develop a strategy to allow customers to make deposits and withdrawals over the Internet? Should the bank try to recapture a portion of the investment market by continuously running stock prices and allowing customers to make stock transactions over the Internet for a minimal fee?

Because AmeriBank is not in the business of performing surveys, Rob has decided to outsource the survey project to a professional survey company. He has opened the project up for bidding by several survey companies and will award the project to the company which is willing to perform the survey for the least cost.

Sophisticated Surveys is one of three survey companies competing for the project. Rob provided each survey company with a list of survey requirements to ensure that AmeriBank receives the needed information for planning the Web banking project.

Because different age groups require different services, AmeriBank is interested in surveying four different age groups. The first group encompasses customers who are 18 to 25 years old. The bank assumes that this age group has limited yearly income and performs minimal transactions. The second group encompasses customers who are 26 to 40 years old. This age group has significant sources of income, performs many transactions, requires numerous loans for new houses and cars, and invests in various securities. The third group encompasses customers who are 41 to 50 years old. These customers typically have the same level of income and perform the same number of transactions as the second age group, but the bank assumes that these customers are less likely to use Web banking since they have not become as comfortable with the explosion of computers or the Internet. Finally, the fourth group encompasses customers who are 51 years of age and over. These customers commonly crave security and require continuous information on retirement funds. The banks believes that it is highly unlikely that customers in this age group will use Web banking, but the bank desires to learn the needs of this age group for the future. AmeriBank wants to interview 2,000 customers with at least 20 percent from the first age group, at least 27.5 percent from the second age group, at least 15 percent from the third age group, and at least 15 percent from the fourth age group.

Rob understands that the Internet is a recent phenomenon and that some customers may not have heard of the World Wide Web. He therefore wants to ensure that the survey includes a mix of customers who know the Internet well and those that have less exposure to the Internet. To ensure that AmeriBank obtains the correct mix, he wants to interview at least 15 percent of customers from the Silicon Valley where Internet use is high, at least 35 percent of customers from big cities where Internet use is medium, and at least 20 percent of customers from small towns where Internet use is low.

Sophisticated Surveys has performed an initial analysis of these survey requirements to determine the cost of surveying different populations. The costs per person surveyed are listed in the following table:

	Age Group			
Region	**18 to 25**	**26 to 40**	**41 to 50**	**51 and over**
Silicon Valley	$4.75	$6.50	$6.50	$5.00
Big cities	$5.25	$5.75	$6.25	$6.25
Small towns	$6.50	$7.50	$7.50	$7.25

Sophisticated Surveys explores the following options cumulatively.

(a) Formulate a linear programming model to minimize costs while meeting all survey constraints imposed by AmeriBank.
(b) If the profit margin for Sophisticated Surveys is 15 percent of cost, what bid will they submit?
(c) After submitting its bid, Sophisticated Surveys is informed that it has the lowest cost but that AmeriBank does not like the solution. Specifically, Rob feels that the selected survey population is not representative enough of the banking customer population. Rob wants at least 50 people of each age group surveyed in each region. What is the new bid made by Sophisticated Surveys?
(d) Rob feels that Sophisticated Survey oversampled the 18- to 25-year-old population and the Silicon Valley population. He imposes a new constraint that no more than 600 individuals can be surveyed from the 18- to 25-year-old population and no more than 650 individuals can be surveyed from the Silicon Valley population. What is the new bid?
(e) When Sophisticated Surveys calculated the cost of reaching and surveying particular individuals, the company thought that reaching individuals in young populations would be easiest. In a recently completed survey, however, Sophisticated Surveys learned that this assumption was wrong. The new costs for surveying the 18- to 25-year-old population are listed below.

Region survey cost per person	
Silicon Valley	$6.50
Big cities	$6.75
Small towns	$7.00

Given the new costs, what is the new bid?

(f) To ensure the desired sampling of individuals, Rob imposes even stricter requirements. He fixes the exact percentage of people that should be surveyed from each population. The requirements are listed below:

Population	percentage of people surveyed
18 to 25	25%
26 to 40	35%
41 to 50	20%
51 and over	20%
Silicon Valley	20%
Big cities	50%
Small towns	30%

By how much would these new requirements increase the cost of surveying for Sophisticated Surveys? Given the 15 percent profit margin, what would Sophisticated Surveys bid?

CASE 4.3 ASSIGNING STUDENTS TO SCHOOLS

The Springfield school board has made the decision to close one of its middle schools (sixth, seventh, and eighth grades) at the end of this school year and reassign all of next year's middle school students to the three remaining middle schools. The school district provides bussing for all middle school students who must travel more than approximately a mile, so the school board wants a plan for reassigning the students that will minimize the total bussing cost. The annual cost per student of bussing from each of the six residential areas of the city to each of the schools is shown in the following table (along with other basic data for next year), where 0 indicates that bussing is not needed and a dash indicates an infeasible assignment.

Area	No. of Students	Percentage in 6th Grade	Percentage in 7th Grade	Percentage in 8th Grade	Bussing Cost per Student		
					School 1	School 2	School 3
1	450	32	38	30	$300	0	$700
2	600	37	28	35	—	$400	$500
3	550	30	32	38	$600	$300	$200
4	350	28	40	32	$200	$500	—
5	500	39	34	27	0	—	$400
6	450	34	28	38	$500	$300	0
				School capacity:	900	1,100	1,000

The school board also has imposed the restriction that each grade must constitute between 30 and 36 percent of each school's population. The above table shows the percentage of each area's middle school population for next year that falls into each of the three grades. The school attendance zone boundaries can be drawn so as to split any given area among more than one school, but assume that the percentages shown in the table will continue to hold for any partial assignment of an area to a school.

You have been hired as an operations research consultant to assist the school board in determining how many students in each area should be assigned to each school.

(a) Formulate a linear programming model for this problem.
(b) Solve the model.
(c) What is your resulting recommendation to the school board?

After seeing your recommendation, the school board expresses concern about all the splitting of residential areas among multiple schools. They indicate that they "would like to keep each neighborhood together."

(d) Adjust your recommendation as well as you can to enable each area to be assigned to just one school. (Adding this restriction may force you to fudge on some other constraints.) How much does this increase the total bussing cost? (This line of analysis will be pursued more rigorously in Case 12.4.)

The school board is considering eliminating some bussing to reduce costs. Option 1 is to eliminate bussing only for students traveling 1 to 1.5 miles, where the cost per student is given in the table as $200. Option 2 is to also eliminate bussing for students traveling 1.5 to 2 miles, where the estimated cost per student is $300.

(e) Revise the model from part (*a*) to fit Option 1, and solve. Compare these results with those from part (*c*), including the reduction in total bussing cost.
(f) Repeat part (*e*) for Option 2.

The school board now needs to choose among the three alternative bussing plans (the current one or Option 1 or Option 2). One important factor is bussing costs. However, the school board also wants to place equal weight on a second factor: the inconvenience and safety problems caused by forcing students to travel by foot or bicycle a substantial distance (more than a mile, and especially more than 1.5 miles). Therefore, they want to choose a plan that provides the best trade-off between these two factors.

(g) Use your results from parts (*c*), (*e*), and (*f*) to summarize the key information related to these two factors that the school board needs to make this decision.
(h) Which decision do you think should be made? Why?

Note: This case will be continued in later chapters (Cases 6.3 and 12.4), so we suggest that you save your analysis, including your basic model.

LEARNING AIDS FOR THIS CHAPTER IN YOUR OR COURSEWARE

A Demonstration Example in OR Tutor:

Fundamental Insight

Interactive Routines:

Enter or Revise a General Linear Programming Model
Set Up for the Simplex Method—Interactive Only
Solve Interactively by the Simplex Method

Files (Chapter 3) for Solving the Wyndor Example:

Excel File
LINGO/LINDO File
MPL/CPLEX File

See Appendix 1 for documentation of the software.

PROBLEMS

The symbols to the left of some of the problems (or their parts) have the following meaning:

D: The demonstration example listed above may be helpful.
I: You can check some of your work by using the interactive routines listed above for the original simplex method.

An asterisk on the problem number indicates that at least a partial answer is given in the back of the book.

5.1-1.* Consider the following problem.

Maximize $Z = 3x_1 + 2x_2$,

subject to

$$2x_1 + x_2 \leq 6$$
$$x_1 + 2x_2 \leq 6$$

and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

(a) Solve this problem graphically. Identify the CPF solutions by circling them on the graph.
(b) Identify all the sets of two defining equations for this problem. For each set, solve (if a solution exists) for the corresponding corner-point solution, and classify it as a CPF solution or corner-point infeasible solution.
(c) Introduce slack variables in order to write the functional constraints in augmented form. Use these slack variables to identify the basic solution that corresponds to each corner-point solution found in part (*b*).
(d) Do the following for *each* set of two defining equations from part (*b*): Identify the indicating variable for each defining equation. Display the set of equations from part (*c*) *after* deleting these two indicating (nonbasic) variables. Then use the latter set of equations to solve for the two remaining variables (the basic variables). Compare the resulting basic solution to the corresponding basic solution obtained in part (*c*).
(e) Without executing the simplex method, use its geometric interpretation (and the objective function) to identify the path (sequence of CPF solutions) it would follow to reach the optimal solution. For each of these CPF solutions in turn, identify the following decisions being made for the next iteration: (*i*) which defining equation is being deleted and which is being added; (*ii*) which indicating variable is being deleted (the entering basic variable) and which is being added (the leaving basic variable).

5.1-2. Repeat Prob. 5.1-1 for the model in Prob. 3.1-5.

5.1-3. Consider the following problem.

Maximize $Z = 2x_1 + 3x_2$,

subject to

$$-3x_1 + x_2 \leq 1$$
$$4x_1 + 2x_2 \leq 20$$
$$4x_1 - x_2 \leq 10$$
$$-x_1 + 2x_2 \leq 5$$

and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

(a) Solve this problem graphically. Identify the CPF solutions by circling them on the graph.
(b) Develop a table giving each of the CPF solutions and the corresponding defining equations, BF solution, and nonbasic variables. Calculate Z for each of these solutions, and use just this information to identify the optimal solution.
(c) Develop the corresponding table for the corner-point infeasible solutions, etc. Also identify the sets of defining equations and nonbasic variables that do not yield a solution.

5.1-4. Consider the following problem.

$$\text{Maximize} \quad Z = 2x_1 - x_2 + x_3,$$

subject to

$$\begin{aligned} 3x_1 + x_2 + x_3 &\le 60 \\ x_1 - x_2 + 2x_3 &\le 10 \\ x_1 + x_2 - x_3 &\le 20 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0, \qquad x_3 \ge 0.$$

After slack variables are introduced and then one complete iteration of the simplex method is performed, the following simplex tableau is obtained.

Iteration	Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side
1	Z	(0)	1	0	−1	3	0	2	0	20
	x_4	(1)	0	0	4	−5	1	−3	0	30
	x_1	(2)	0	1	−1	2	0	1	0	10
	x_6	(3)	0	0	2	−3	0	−1	1	10

(a) Identify the CPF solution obtained at iteration 1.
(b) Identify the constraint boundary equations that define this CPF solution.

5.1-5. Consider the three-variable linear programming problem shown in Fig. 5.2.
(a) Construct a table like Table 5.1, giving the set of defining equations for each CPF solution.
(b) What are the defining equations for the corner-point infeasible solution (6, 0, 5)?
(c) Identify one of the systems of three constraint boundary equations that yields neither a CPF solution nor a corner-point infeasible solution. Explain why this occurs for this system.

5.1-6. Consider the linear programming problem given in Table 6.1 as the dual problem for the Wyndor Glass Co. example.
(a) Identify the 10 sets of defining equations for this problem. For each one, solve (if a solution exists) for the corresponding corner-point solution, and classify it as a CPF solution or corner-point infeasible solution.
(b) For each corner-point solution, give the corresponding basic solution and its set of nonbasic variables. (Compare with Table 6.9.)

5.1-7. Consider the following problem.

$$\text{Minimize} \quad Z = x_1 + 2x_2,$$

subject to

$$\begin{aligned} -x_1 + x_2 &\le 15 \\ 2x_1 + x_2 &\le 90 \\ x_2 &\ge 30 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

(a) Solve this problem graphically.
(b) Develop a table giving each of the CPF solutions and the corresponding defining equations, BF solution, and nonbasic variables.

5.1-8. Reconsider the model in Problem 4.6-3.
(a) Identify the 10 sets of defining equations for this problem. For each one, solve (if a solution exists) for the corresponding corner-point solution, and classify it as a CPF solution or a corner-point infeasible solution.
(b) For each corner-point solution, give the corresponding basic solution and its set of nonbasic variables.

5.1-9. Reconsider the model in Prob. 3.1-4.
(a) Identify the 15 sets of defining equations for this problem. For each one, solve (if a solution exists) for the corresponding corner-point solution, and classify it as a CPF solution or a corner-point infeasible solution.
(b) For each corner-point solution, give the corresponding basic solution and its set of nonbasic variables.

5.1-10. Each of the following statements is true under most circumstances, but not always. In each case, indicate when the statement will not be true and why.
(a) The best CPF solution is an optimal solution.
(b) An optimal solution is a CPF solution.
(c) A CPF solution is the only optimal solution if none of its adjacent CPF solutions are better (as measured by the value of the objective function).

5.1-11. Consider the original form (before augmenting) of a linear programming problem with n decision variables (each with a nonnegativity constraint) and m functional constraints. Label each of the following statements as true or false, and then justify your

answer with specific references (including page citations) to material in the chapter.
(a) If a feasible solution is optimal, it must be a CPF solution.
(b) The number of CPF solutions is at least

$$\frac{(m+n)!}{m!n!}.$$

(c) If a CPF solution has adjacent CPF solutions that are better (as measured by Z), then one of these adjacent CPF solutions must be an optimal solution.

5.1-12. Label each of the following statements about linear programming problems as true or false, and then justify your answer.
(a) If a feasible solution is optimal but not a CPF solution, then infinitely many optimal solutions exist.
(b) If the value of the objective function is equal at two different feasible points $\mathbf{x}^*$ and $\mathbf{x}^{**}$, then all points on the line segment connecting $\mathbf{x}^*$ and $\mathbf{x}^{**}$ are feasible and Z has the same value at all those points.
(c) If the problem has n variables (before augmenting), then the simultaneous solution of any set of n constraint boundary equations is a CPF solution.

5.1-13. Consider the augmented form of linear programming problems that have feasible solutions and a bounded feasible region. Label each of the following statements as true or false, and then justify your answer by referring to specific statements (with page citations) in the chapter.
(a) There must be at least one optimal solution.
(b) An optimal solution must be a BF solution.
(c) The number of BF solutions is finite.

5.1-14.* Reconsider the model in Prob. 4.6-10. Now you are given the information that the basic variables in the optimal solution are x_2 and x_3. Use this information to identify a system of three constraint boundary equations whose simultaneous solution must be this optimal solution. Then solve this system of equations to obtain this solution.

5.1-15. Reconsider Prob. 4.3-7. Now use the given information and the theory of the simplex method to identify a system of three constraint boundary equations (in x_1, x_2, x_3) whose simultaneous solution must be the optimal solution, without applying the simplex method. Solve this system of equations to find the optimal solution.

5.1-16. Reconsider Prob. 4.3-8. Using the given information and the theory of the simplex method, analyze the constraints of the problem in order to identify a system of three constraint boundary equations whose simultaneous solution must be the optimal solution (not augmented). Then solve this system of equations to obtain this solution.

5.1-17. Consider the following problem.

$$\text{Maximize} \quad Z = 2x_1 + 2x_2 + 3x_3,$$

subject to

$$\begin{aligned} 2x_1 + x_2 + 2x_3 &\le 4 \\ x_1 + x_2 + x_3 &\le 3 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0, \qquad x_3 \ge 0.$$

Let x_4 and x_5 be the slack variables for the respective functional constraints. Starting with these two variables as the basic variables for the initial BF solution, you now are given the information that the simplex method proceeds as follows to obtain the optimal solution in two iterations: (1) In iteration 1, the entering basic variable is x_3 and the leaving basic variable is x_4; (2) in iteration 2, the entering basic variable is x_2 and the leaving basic variable is x_5.
(a) Develop a three-dimensional drawing of the feasible region for this problem, and show the path followed by the simplex method.
(b) Give a geometric interpretation of why the simplex method followed this path.
(c) For each of the two edges of the feasible region traversed by the simplex method, give the equation of each of the two constraint boundaries on which it lies, and then give the equation of the additional constraint boundary at each endpoint.
(d) Identify the set of defining equations for each of the three CPF solutions (including the initial one) obtained by the simplex method. Use the defining equations to solve for these solutions.
(e) For each CPF solution obtained in part (d), give the corresponding BF solution and its set of nonbasic variables. Explain how these nonbasic variables identify the defining equations obtained in part (d).

5.1-18. Consider the following problem.

$$\text{Maximize} \quad Z = 3x_1 + 4x_2 + 2x_3,$$

subject to

$$\begin{aligned} x_1 + x_2 + x_3 &\le 20 \\ x_1 + 2x_2 + x_3 &\le 30 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0, \qquad x_3 \ge 0.$$

Let x_4 and x_5 be the slack variables for the respective functional constraints. Starting with these two variables as the basic variables for the initial BF solution, you now are given the information that the simplex method proceeds as follows to obtain the optimal solution in two iterations: (1) In iteration 1, the entering basic vari-

able is x_2 and the leaving basic variable is x_5; (2) in iteration 2, the entering basic variable is x_1 and the leaving basic variable is x_4.
Follow the instructions of Prob. 5.1-17 for this situation.

5.1-19. By inspecting Fig. 5.2, explain why Property 1*b* for CPF solutions holds for this problem if it has the following objective function.
(a) Maximize $Z = x_3$.
(b) Maximize $Z = -x_1 + 2x_3$.

5.1-20. Consider the three-variable linear programming problem shown in Fig. 5.2.
(a) Explain in geometric terms why the set of solutions satisfying any individual constraint is a convex set, as defined in Appendix 2.
(b) Use the conclusion in part (*a*) to explain why the entire feasible region (the set of solutions that simultaneously satisfies every constraint) is a convex set.

5.1-21. Suppose that the three-variable linear programming problem given in Fig. 5.2 has the objective function

$$\text{Maximize} \quad Z = 3x_1 + 4x_2 + 3x_3.$$

Without using the algebra of the simplex method, apply just its geometric reasoning (including choosing the edge giving the maximum rate of increase of Z) to determine and explain the path it would follow in Fig. 5.2 from the origin to the optimal solution.

5.1-22. Consider the three-variable linear programming problem shown in Fig. 5.2.
(a) Construct a table like Table 5.4, giving the indicating variable for each constraint boundary equation and original constraint.
(b) For the CPF solution (2, 4, 3) and its three adjacent CPF solutions (4, 2, 4), (0, 4, 2), and (2, 4, 0), construct a table like Table 5.5, showing the corresponding defining equations, BF solution, and nonbasic variables.
(c) Use the sets of defining equations from part (*b*) to demonstrate that (4, 2, 4), (0, 4, 2), and (2, 4, 0) are indeed adjacent to (2, 4, 3), but that none of these three CPF solutions are adjacent to each other. Then use the sets of nonbasic variables from part (*b*) to demonstrate the same thing.

5.1-23. The formula for the line passing through (2, 4, 3) and (4, 2, 4) in Fig. 5.2 can be written as

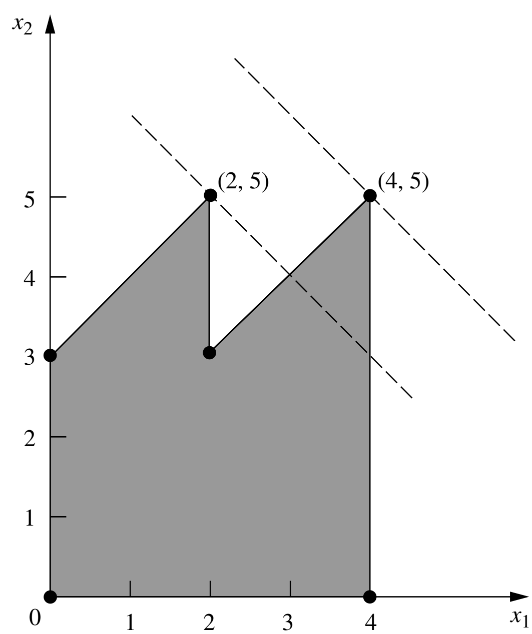
$$(2, 4, 3) + \alpha[(4, 2, 4) - (2, 4, 3)] = (2, 4, 3) + \alpha(2, -2, 1),$$

where $0 \le \alpha \le 1$ for just the line segment between these points. After augmenting with the slack variables x_4, x_5, x_6, x_7 for the respective functional constraints, this formula becomes

$$(2, 4, 3, 2, 0, 0, 0) + \alpha(2, -2, 1, -2, 2, 0, 0).$$

Use this formula directly to answer each of the following questions, and thereby relate the algebra and geometry of the simplex method as it goes through one iteration in moving from (2, 4, 3) to (4, 2, 4). (You are given the information that it is moving along this line segment.)
(a) What is the entering basic variable?
(b) What is the leaving basic variable?
(c) What is the new BF solution?

5.1-24. Consider a two-variable mathematical programming problem that has the feasible region shown on the graph, where the six dots correspond to CPF solutions. The problem has a linear objective function, and the two dashed lines are objective function lines passing through the optimal solution (4, 5) and the second-best CPF solution (2, 5). Note that the nonoptimal solution (2, 5) is better than both of its adjacent CPF solutions, which violates Property 3 in Sec. 5.1 for CPF solutions in linear programming. Demonstrate that this problem *cannot* be a linear programming problem by constructing the feasible region that would result if the six line segments on the boundary were constraint boundaries for linear programming constraints.



5.2-1. Consider the following problem.

$$\text{Maximize} \quad Z = 8x_1 + 4x_2 + 6x_3 + 3x_4 + 9x_5,$$

subject to

$$\begin{aligned}
x_1 + 2x_2 + 3x_3 + 3x_4 &\le 180 \qquad \text{(resource 1)}\\
4x_1 + 3x_2 + 2x_3 + x_4 + x_5 &\le 270 \qquad \text{(resource 2)}\\
x_1 + 3x_2 + x_4 + 3x_5 &\le 180 \qquad \text{(resource 3)}
\end{aligned}$$

and

$$x_j \geq 0, \qquad j = 1, \ldots, 5.$$

You are given the facts that the basic variables in the optimal solution are x_3, x_1, and x_5 and that

$$\begin{bmatrix} 3 & 1 & 0 \\ 2 & 4 & 1 \\ 0 & 1 & 3 \end{bmatrix}^{-1} = \frac{1}{27}\begin{bmatrix} 11 & -3 & 1 \\ -6 & 9 & -3 \\ 2 & -3 & 10 \end{bmatrix}.$$

(a) Use the given information to identify the optimal solution.
(b) Use the given information to identify the shadow prices for the three resources.

I **5.2-2.*** Work through the revised simplex method step by step to solve the following problem.

$$\text{Maximize} \quad Z = 5x_1 + 8x_2 + 7x_3 + 4x_4 + 6x_5,$$

subject to

$$2x_1 + 3x_2 + 3x_3 + 2x_4 + 2x_5 \leq 20$$
$$3x_1 + 5x_2 + 4x_3 + 2x_4 + 4x_5 \leq 30$$

and

$$x_j \geq 0, \qquad j = 1, 2, 3, 4, 5.$$

I **5.2-3.** Work through the revised simplex method step by step to solve the model given in Prob. 4.3-4.

5.2-4. Reconsider Prob. 5.1-1. For the sequence of CPF solutions identified in part (*e*), construct the basis matrix **B** for each of the corresponding BF solutions. For each one, invert **B** manually, use this $\mathbf{B}^{-1}$ to calculate the current solution, and then perform the next iteration (or demonstrate that the current solution is optimal).

I **5.2-5.** Work through the revised simplex method step by step to solve the model given in Prob. 4.1-5.

I **5.2-6.** Work through the revised simplex method step by step to solve the model given in Prob. 4.7-6.

I **5.2-7.** Work through the revised simplex method step by step to solve each of the following models:
(a) Model given in Prob. 3.1-5.
(b) Model given in Prob. 4.7-8.

D **5.3-1.*** Consider the following problem.

$$\text{Maximize} \quad Z = x_1 - x_2 + 2x_3,$$

subject to

$$2x_1 - 2x_2 + 3x_3 \leq 5$$
$$x_1 + x_2 - x_3 \leq 3$$
$$x_1 - x_2 + x_3 \leq 2$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0.$$

Let x_4, x_5, and x_6 denote the slack variables for the respective constraints. After you apply the simplex method, a portion of the final simplex tableau is as follows:

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side
Z	(0)	1				1	1	0	
x_2	(1)	0				1	3	0	
x_6	(2)	0				0	1	1	
x_3	(3)	0				1	2	0	

(a) Use the fundamental insight presented in Sec. 5.3 to identify the missing numbers in the final simplex tableau. Show your calculations.
(b) Identify the defining equations of the CPF solution corresponding to the optimal BF solution in the final simplex tableau.

D **5.3-2.** Consider the following problem.

$$\text{Maximize} \quad Z = 4x_1 + 3x_2 + x_3 + 2x_4,$$

subject to

$$4x_1 + 2x_2 + x_3 + x_4 \leq 5$$
$$3x_1 + x_2 + 2x_3 + x_4 \leq 4$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0, \qquad x_4 \geq 0.$$

Let x_5 and x_6 denote the slack variables for the respective constraints. After you apply the simplex method, a portion of the final simplex tableau is as follows:

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side
Z	(0)	1					1	1	
x_2	(1)	0					1	−1	
x_4	(2)	0					−1	2	

(a) Use the fundamental insight presented in Sec. 5.3 to identify the missing numbers in the final simplex tableau. Show your calculations.
(b) Identify the defining equations of the CPF solution corresponding to the optimal BF solution in the final simplex tableau.

D **5.3-3.** Consider the following problem.

Maximize $Z = 6x_1 + x_2 + 2x_3,$

subject to

$$2x_1 + 2x_2 + \frac{1}{2}x_3 \le 2$$
$$-4x_1 - 2x_2 - \frac{3}{2}x_3 \le 3$$
$$x_1 + 2x_2 + \frac{1}{2}x_3 \le 1$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0, \qquad x_3 \ge 0.$$

Let x_4, x_5, and x_6 denote the slack variables for the respective constraints. After you apply the simplex method, a portion of the final simplex tableau is as follows:

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side
Z	(0)	1				2	0	2	
x_5	(1)	0				1	1	2	
x_3	(2)	0				−2	0	4	
x_1	(3)	0				1	0	−1	

Use the fundamental insight presented in Sec. 5.3 to identify the missing numbers in the final simplex tableau. Show your calculations.

D **5.3-4.** Consider the following problem.

Maximize $Z = x_1 - x_2 + 2x_3,$

subject to

$$x_1 + x_2 + 3x_3 \le 15$$
$$2x_1 - x_2 + x_3 \le 2$$
$$-x_1 + x_2 + x_3 \le 4$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0, \qquad x_3 \ge 0.$$

Let x_4, x_5, and x_6 denote the slack variables for the respective constraints. After the simplex method is applied, a portion of the final simplex tableau is as follows:

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side
Z	(0)	1				0	$\frac{3}{2}$	$\frac{1}{2}$	
x_4	(1)	0				1	−1	−2	
x_3	(2)	0				0	$\frac{1}{2}$	$\frac{1}{2}$	
x_2	(3)	0				0	$-\frac{1}{2}$	$\frac{1}{2}$	

(a) Use the fundamental insight presented in Sec. 5.3 to identify the missing numbers in the final simplex tableau. Show your calculations.

(b) Identify the defining equations of the CPF solution corresponding to the optimal BF solution in the final simplex tableau.

D **5.3-5.** Consider the following problem.

Maximize $Z = 20x_1 + 6x_2 + 8x_3,$

subject to

$$8x_1 + 2x_2 + 3x_3 \le 200$$
$$4x_1 + 3x_2 + 3x_3 \le 100$$
$$2x_1 + 3x_2 + x_3 \le 50$$
$$x_3 \le 20$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0, \qquad x_3 \ge 0.$$

Let x_4, x_5, x_6, and x_7 denote the slack variables for the first through fourth constraints, respectively. Suppose that after some number of iterations of the simplex method, a portion of the current simplex tableau is as follows:

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Right Side
Z	(0)	1				$\frac{9}{4}$	$\frac{1}{2}$	0	0	
x_1	(1)	0				$\frac{3}{16}$	$-\frac{1}{8}$	0	0	
x_2	(2)	0				$-\frac{1}{4}$	$\frac{1}{2}$	0	0	
x_6	(3)	0				$-\frac{3}{8}$	$\frac{1}{4}$	1	0	
x_7	(4)	0				0	0	0	1	

(a) Use the fundamental insight presented in Sec. 5.3 to identify the missing numbers in the current simplex tableau. Show your calculations.
(b) Indicate which of these missing numbers would be generated by the revised simplex method in order to perform the next iteration.
(c) Identify the defining equations of the CPF solution corresponding to the BF solution in the current simplex tableau.

D **5.3-6.** You are using the simplex method to solve the following linear programming problem.

Maximize $Z = 6x_1 + 5x_2 - x_3 + 4x_4,$

subject to

$$3x_1 + 2x_2 - 3x_3 + x_4 \le 120$$
$$3x_1 + 3x_2 + x_3 + 3x_4 \le 180$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0, \quad x_4 \ge 0.$$

You have obtained the following final simplex tableau where x_5 and x_6 are the slack variables for the respective constraints.

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side
Z	(0)	1	0	$\frac{1}{4}$	0	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	Z^*
x_1	(1)	0	1	$\frac{11}{12}$	0	$\frac{5}{6}$	$\frac{1}{12}$	$\frac{1}{4}$	b_1^*
x_3	(2)	0	0	$\frac{1}{4}$	1	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	b_2^*

Use the fundamental insight presented in Sec. 5.3 to identify Z^*, b_1^*, and b_2^*. Show your calculations.

D **5.3-7.** Consider the following problem.

Maximize $Z = c_1x_1 + c_2x_2 + c_3x_3,$

subject to

$$x_1 + 2x_2 + x_3 \le b$$
$$2x_1 + x_2 + 3x_3 \le 2b$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0.$$

Note that values have not been assigned to the coefficients in the objective function (c_1, c_2, c_3), and that the only specification for the right-hand side of the functional constraints is that the second one ($2b$) be twice as large as the first (b).

Now suppose that your boss has inserted her best estimate of the values of c_1, c_2, c_3, and b without informing you and then has run the simplex method. You are given the resulting final simplex tableau below (where x_4 and x_5 are the slack variables for the respective functional constraints), but you are unable to read the value of Z^*.

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	x_5	Right Side
Z	(0)	1	$\frac{7}{10}$	0	0	$\frac{3}{5}$	$\frac{4}{5}$	Z^*
x_2	(1)	0	$\frac{1}{5}$	1	0	$\frac{3}{5}$	$-\frac{1}{5}$	1
x_3	(2)	0	$\frac{3}{5}$	0	1	$-\frac{1}{5}$	$\frac{2}{5}$	3

(a) Use the fundamental insight presented in Sec. 5.3 to identify the value of (c_1, c_2, c_3) that was used.
(b) Use the fundamental insight presented in Sec. 5.3 to identify the value of b that was used.
(c) Calculate the value of Z^* in two ways, where one way uses your results from part (*a*) and the other way uses your result from part (*b*). Show your two methods for finding Z^*.

5.3-8. For iteration 2 of the example in Sec. 5.3, the following expression was shown:

$$\text{Final row } 0 = [-3, \quad -5 \quad 0, \quad 0, \quad 0 \quad 0]$$
$$+ [0, \quad \tfrac{3}{2}, \quad 1] \left[\begin{array}{cc:ccc:c} 1 & 0 & 1 & 0 & 0 & 4 \\ 0 & 2 & 0 & 1 & 0 & 12 \\ 3 & 2 & 0 & 0 & 1 & 18 \end{array}\right].$$

Derive this expression by combining the algebraic operations (in matrix form) for iterations 1 and 2 that affect row 0.

5.3-9. Most of the description of the fundamental insight presented in Sec. 5.3 assumes that the problem is in our standard form. Now consider each of the following other forms, where the additional adjustments in the initialization step are those presented in Sec. 4.6, including the use of artificial variables and the Big M method where appropriate. Describe the resulting adjustments in the fundamental insight.
(a) Equality constraints
(b) Functional constraints in $\ge$ form
(c) Negative right-hand sides
(d) Variables allowed to be negative (with no lower bound)

5.3-10. Reconsider the model in Prob. 4.6-6. Use artificial variables and the Big M method to construct the complete first simplex tableau for the simplex method, and then identify the columns that will contains $\mathbf{S}^*$ for applying the fundamental insight in the final tableau. Explain why these are the appropriate columns.

5.3-11. Consider the following problem.

$$\text{Minimize} \quad Z = 2x_1 + 3x_2 + 2x_3,$$

subject to

$$\begin{aligned} x_1 + 4x_2 + 2x_3 &\geq 8 \\ 3x_1 + 2x_2 + 2x_3 &\geq 6 \end{aligned}$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

Let x_4 and x_6 be the surplus variables for the first and second constraints, respectively. Let $\bar{x}_5$ and $\bar{x}_7$ be the corresponding artificial variables. After you make the adjustments described in Sec. 4.6 for this model form when using the Big M method, the initial simplex tableau ready to apply the simplex method is as follows:

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	x_6	$\bar{x}_7$	Right Side
Z	(0)	-1	$-4M+2$	$-6M+3$	$-2M+2$	M	0	M	0	$-14M$
$\bar{x}_5$	(1)	0	1	4	2	-1	1	0	0	8
$\bar{x}_7$	(2)	0	3	2	0	0	0	-1	1	6

After you apply the simplex method, a portion of the final simplex tableau is as follows:

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	$\bar{x}_5$	x_6	$\bar{x}_7$	Right Side
Z	(0)	-1					$M-0.5$		$M-0.5$	
x_2	(1)	0					0.3		-0.1	
x_1	(2)	0					-0.2		0.4	

(a) Based on the above tableaux, use the fundamental insight presented in Sec. 5.3 to identify the missing numbers in the final simplex tableau. Show your calculations.

(b) Examine the mathematical logic presented in Sec. 5.3 to validate the fundamental insight (see the $\mathbf{T}^* = \mathbf{MT}$ and $\mathbf{t}^* = \mathbf{t} + \mathbf{vT}$ equations and the subsequent derivations of $\mathbf{M}$ and $\mathbf{v}$). This logic assumes that the original model fits our standard form, whereas the current problem does not fit this form. Show how, with minor adjustments, this same logic applies to the current problem when $\mathbf{t}$ is row 0 and $\mathbf{T}$ is rows 1 and 2 in the initial simplex tableau given above. Derive $\mathbf{M}$ and $\mathbf{v}$ for this problem.

(c) When you apply the $\mathbf{t}^* = \mathbf{t} + \mathbf{vT}$ equation, another option is to use $\mathbf{t} = [2, 3, 2, 0, M, 0, M, 0]$, which is the *preliminary* row 0 before the algebraic elimination of the nonzero coefficients of the initial basic variables $\bar{x}_5$ and $\bar{x}_7$. Repeat part (*b*) for this equation with this new $\mathbf{t}$. After you derive the new $\mathbf{v}$, show that this equation yields the same final row 0 for this problem as the equation derived in part (*b*).

(d) Identify the defining equations of the CPF solution corresponding to the optimal BF solution in the final simplex tableau.

5.3-12. Consider the following problem.

$$\text{Maximize} \quad Z = 2x_1 + 4x_2 + 3x_3,$$

subject to

$$\begin{aligned} x_1 + 3x_2 + 2x_3 &= 20 \\ x_1 + 5x_2 \qquad &\geq 10 \end{aligned}$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

Let $\bar{x}_4$ be the artificial variable for the first constraint. Let x_5 and $\bar{x}_6$ be the surplus variable and artificial variable, respectively, for the second constraint.

You are now given the information that a portion of the final simplex tableau is as follows:

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	$\bar{x}_4$	x_5	$\bar{x}_6$	Right Side
Z	(0)	1				$M+2$	0	M	
x_1	(1)	0				1	0	0	
x_5	(2)	0				1	1	-1	

(a) Extend the fundamental insight presented in Sec. 5.3 to identify the missing numbers in the final simplex tableau. Show your calculations.

(b) Identify the defining equations of the CPF solution corresponding to the optimal solution in the final simplex tableau.

5.3-13. Consider the following problem.

$$\text{Maximize} \quad Z = 3x_1 + 7x_2 + 2x_3,$$

subject to

$$\begin{aligned} -2x_1 + 2x_2 + x_3 &\leq 10 \\ 3x_1 + x_2 - x_3 &\leq 20 \end{aligned}$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

You are given the fact that the basic variables in the optimal solution are x_1 and x_3.

(a) Introduce slack variables, and then use the given information to find the optimal solution directly by Gaussian elimination.

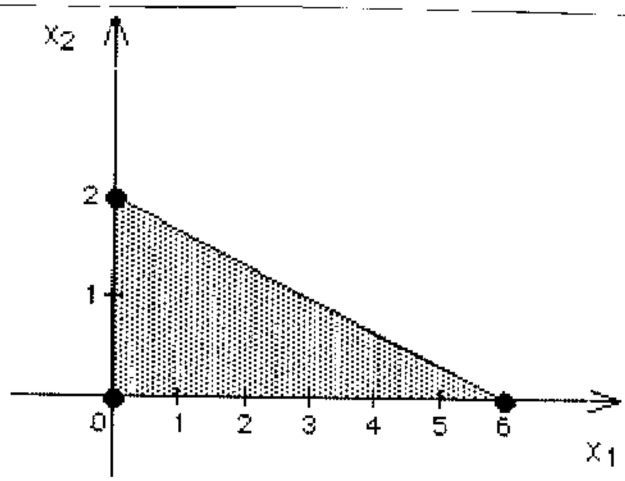
(b) Extend the work in part (*a*) to find the shadow prices.

(c) Use the given information to identify the defining equations of the optimal CPF solution, and then solve these equations to obtain the optimal solution.

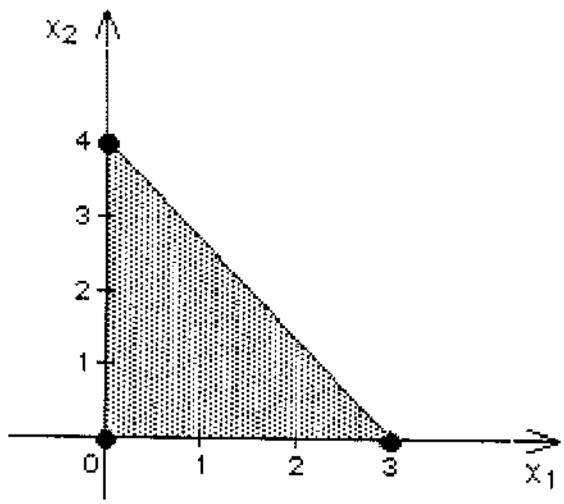
(d) Construct the basis matrix $\mathbf{B}$ for the optimal BF solution, invert $\mathbf{B}$ manually, and then use this $\mathbf{B}^{-1}$ to solve for the optimal solution and the shadow prices $\mathbf{y}^*$. Then apply the optimality test for the revised simplex method to verify that this solution is optimal.

(e) Given $\mathbf{B}^{-1}$ and $\mathbf{y}^*$ from part (*d*), use the fundamental insight presented in Sec. 5.3 to construct the complete final simplex tableau.

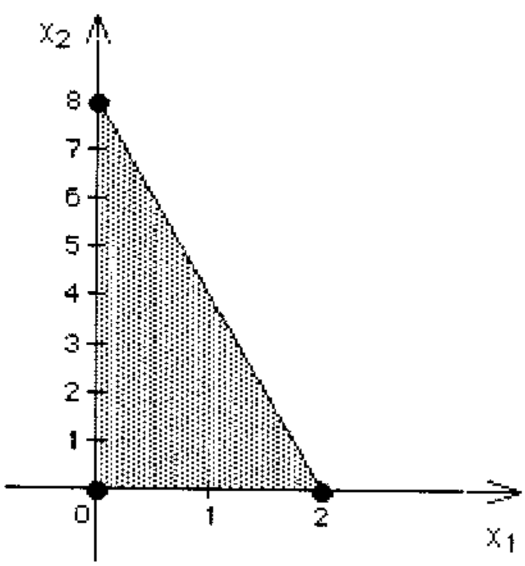
3.1-1 a)



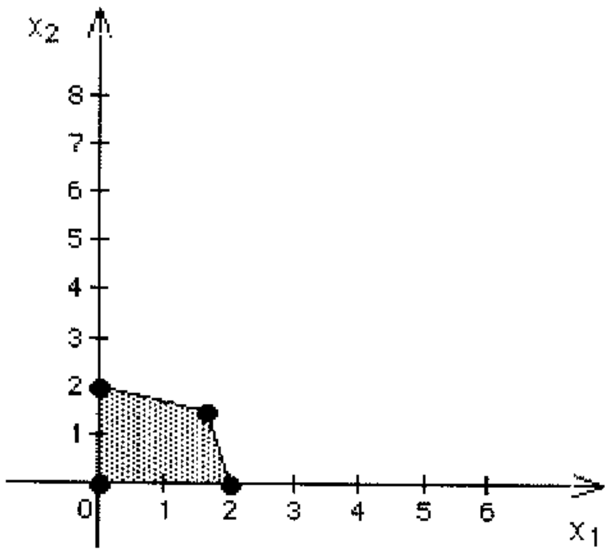
b)



c)

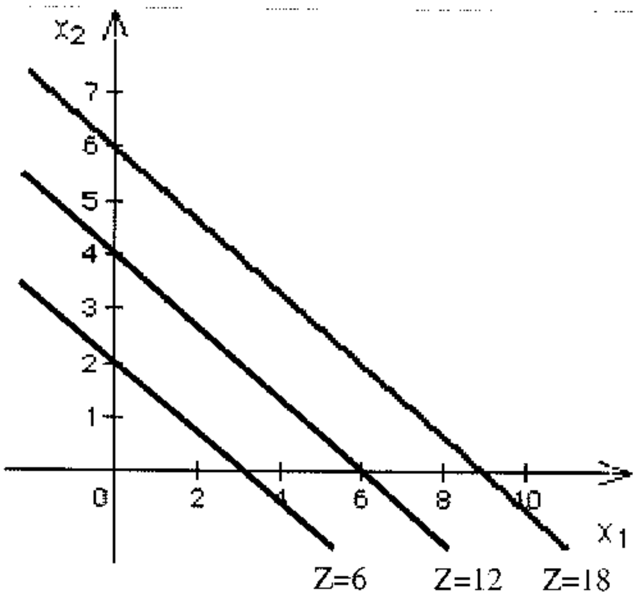


d)



3.1-2

a)



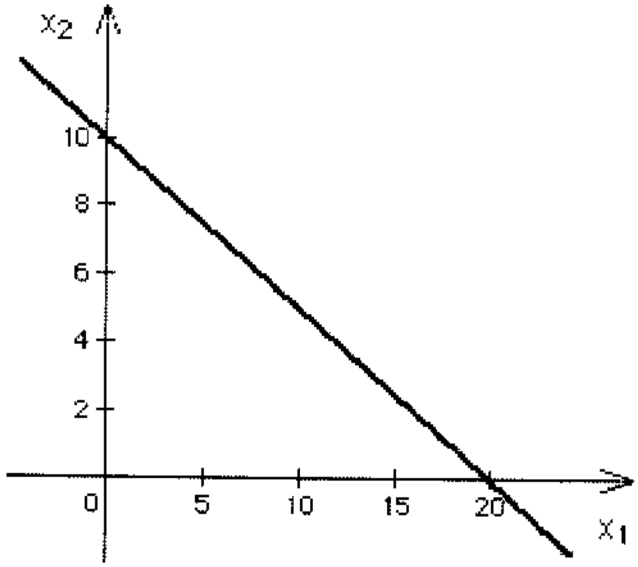
b)

	slope-intercept form	slope	x_2 intercept
Z=6	$x_2 = -\frac{2}{3}x_1 + 2$	$-\frac{2}{3}$	2
Z=12	$x_2 = -\frac{2}{3}x_1 + 4$	$-\frac{2}{3}$	4
Z=18	$x_2 = -\frac{2}{3}x_1 + 6$	$-\frac{2}{3}$	6

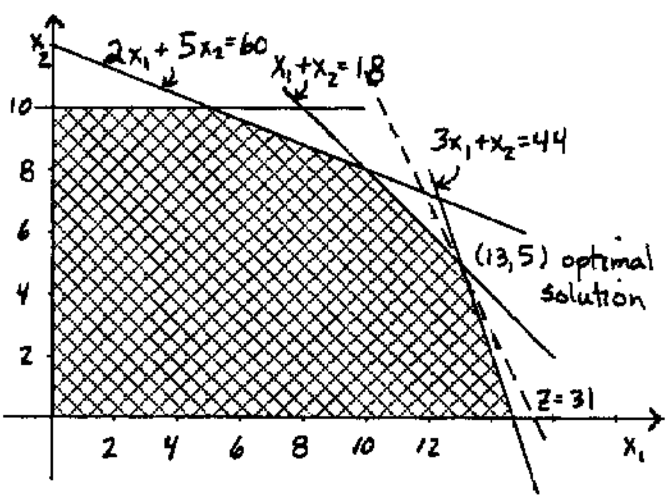
3.1-3 a) $x_2 = -\frac{1}{2}x_1 + 10$

b) slope $= -\frac{1}{2}$ x_2 intercept = 10

c)

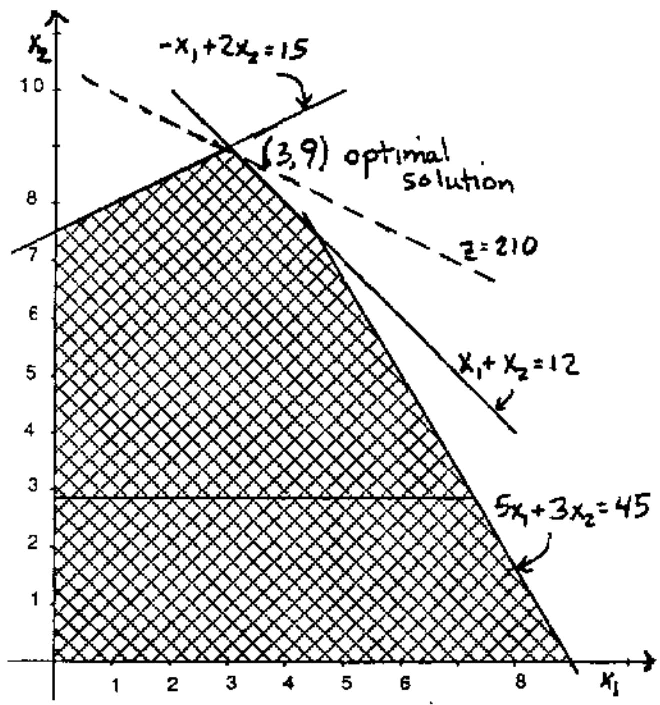


3.1-4



$(x_1, x_2) = (13, 5)$ is optimal with $Z = 31$

3.1-5



$(x_1, x_2) = (3, 9)$ is optimal with $Z = 210$

3.1-6 a) As in the Wyndor Glass Co. problem, we want to find the optimal levels of two activities that compete for limited resources. We want to find the optimal mix of the two activities.

Let W be the number of wood-framed windows to produce.
Let A be the number of aluminum-framed windows to produce.

The following table gives the data for the problem:

Resource	Resource Usage per Unit of Activity wood-framed	aluminum-framed	Amount of Resource Available
glass	6	8	48
aluminum	0	1	4
wood	1	0	6
Unit Profit	$60	$30	

b)

Maximize $P = 60W + 30A$,

subject to $6W + 8A \leq 48$

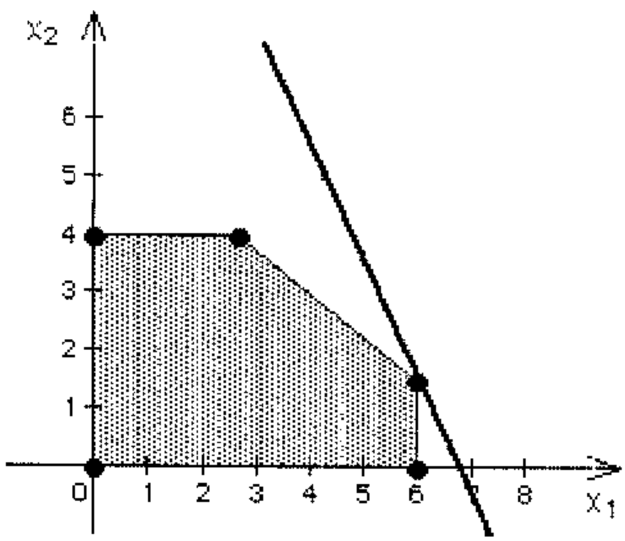
$W \leq 6$

$A \leq 4$

and $W \geq 0,\ A \geq 0$.

c)

Optimal Solution: $(W, A) = (x_1, x_2) = \left(6, 1\frac{1}{2}\right)$ and $P = 405$.



(CONT.)

CONT'D.)

d)

Resource	Resources Used Per Unit Produced wood-framed	aluminum-framed	Totals		Resource Available
glass	6	8	48	≤	48
aluminum	0	1	1.5	≤	4
wood	1	0	6	≤	6
Unit Profit	$40	$30	**$ 285**		
Solution	**6**	**1.5**			

Resource	Resources Used Per Unit Produced wood-framed	aluminum-framed	Totals		Resource Available
glass	6	8	48	≤	48
aluminum	0	1	4	≤	4
wood	1	0	2.66667	≤	6
Unit Profit	$20	$30	**$ 173**		
Solution	**3**	**4**			

e)

Resource	Resources Used Per Unit Produced wood-framed	aluminum-framed	Totals		Resource Available
glass	6	8	48	≤	48
aluminum	0	1	2.25	≤	4
wood	1	0	5	≤	5
Unit Profit	$60	$30	**$ 368**		
Solution	**5**	**2.25**			

31-7

a) Let x_1 = number of 27" TV sets to be produced per month
Let x_2 = number of 20" TV sets to be produced per month
Maximize $P = 120x_1 + 80x_2$,

subject to $20x_1 + 10x_2 \le 500$

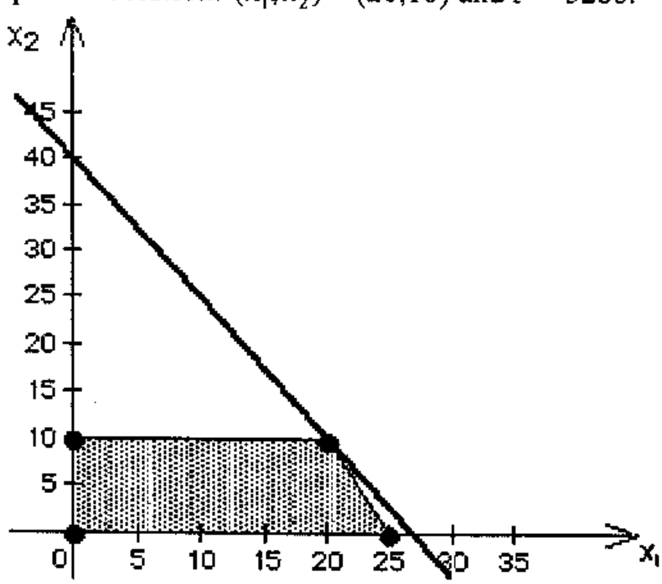
$x_1 \le 40$

$x_2 \le 10$

and $x_1 \ge 0,\ x_2 \ge 0.$

b)

Optimal Solution: $(x_1, x_2) = (20,10)$ and $P = 3200$.



3.1-8

a) Let x_1 = number of units of product 1 to produce
Let x_2 = number of units of product 2 to produce
Maximize $P = x_1 + 2x_2$,

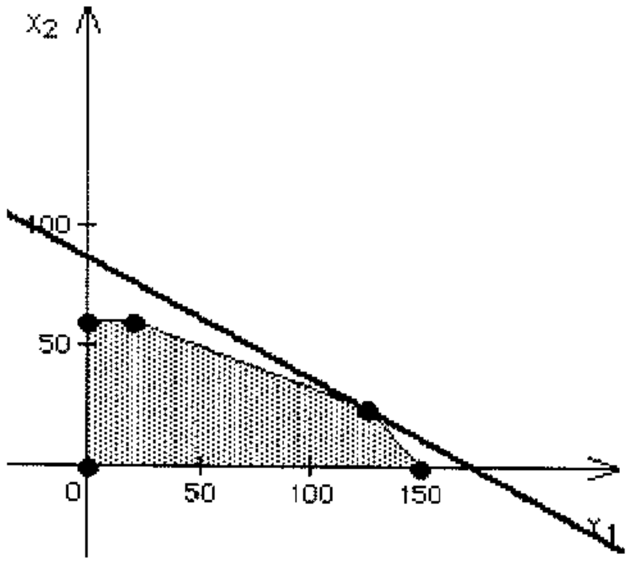
subject to $x_1 + 3x_2 \leq 200$

$2x_1 + 2x_2 \leq 300$

$x_2 \leq 60$

and $x_1 \geq 0,\ x_2 \geq 0.$

b) Optimal Solution: $(x_1, x_2) = (125, 25)$ and $P = 175$.



3.1-9

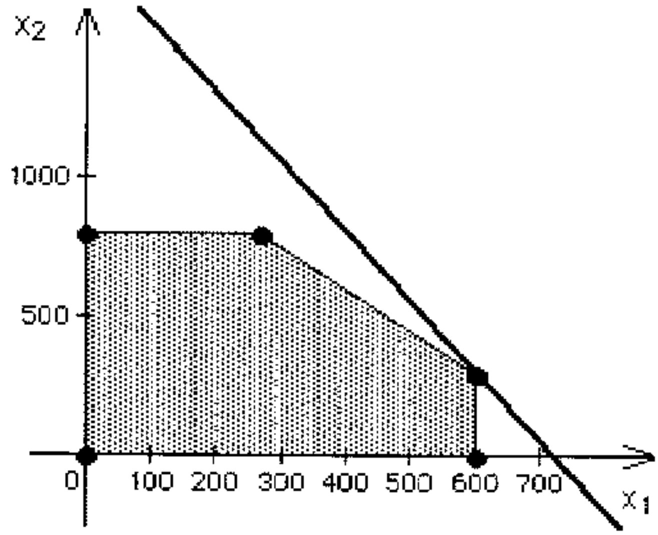
a)

Let X_1 = # units on special risk insurance
X_2 = # " " mortgages

Maximize $Z = 5X_1 + 2X_2$
subject to
$3X_1 + 2X_2 \leq 2400$
$X_2 \leq 800$
$2X_1 \leq 1200$
$X_1 \geq 0,\ X_2 \geq 0$

3.1-9 Optimal Solution: $(S, M) = (x_1, x_2) = (600, 300)$ and $P = 3600$.

b)



c- we have :

$$3x_1 + 2x_2 = 2400$$
$$2x_1 \qquad = 1200$$

so $x_1 = 600$ and $x_2 = \frac{1}{2}(2400 - 3x_1) = 300$

and $Z = 5x_1 + 2x_2 = 5.600 + 2.300 = 3600$

3.1-10

a) Maximize $P = 0.2H + 0.1B$,

subject to $0.1B \le 200$

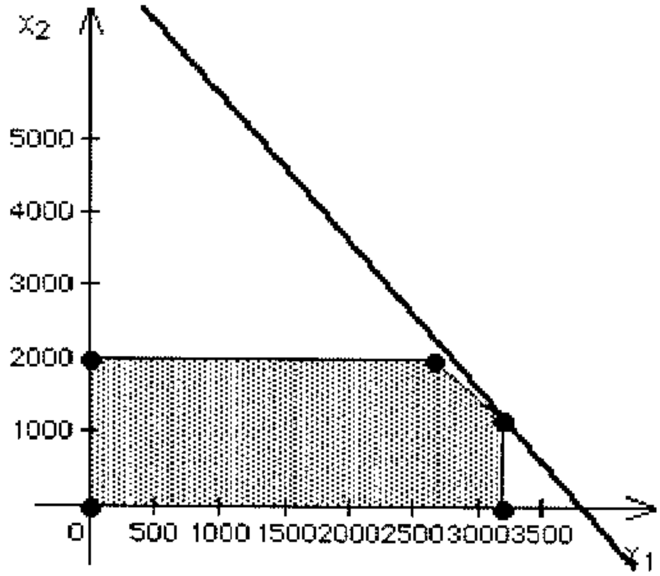
$0.25H \le 800$

$3H + 2B \le 12{,}000$

and $H \ge 0,\ B \ge 0$.

3.1-10

b) Optimal Solution: $(H, B) = (x_1, x_2) = (3200, 1200)$ and $P = 760$.



3.1-11

a)

Let x_1 = number of units of product 1 produced
x_2 = number of units of product 2 produced
x_3 = number of units of product 3 produced.

Maximize $z = 50x_1 + 20x_2 + 25x_3$

subject to

$$9x_1 + 3x_2 + 5x_3 \le 500$$
$$5x_1 + 4x_2 \le 350$$
$$3x_1 + 2x_3 \le 150$$
$$x_3 \le 20$$
$$x_1 \ge 0 \quad x_2 \ge 0 \quad x_3 \ge 0$$

3.1-11

b)

Solve Automatically by the Simplex Method:

Optimal Solution

Value of the
Objective Function: Z = 2904.7619

Variable	Value
X_1	26.1905
X_2	54.7619
X_3	20

Constraint	Slack or Surplus	Shadow Price
1	0	4.7619
2	0	1.42857
3	31.4286	0
4	0	1.19048

Sensitivity Analysis

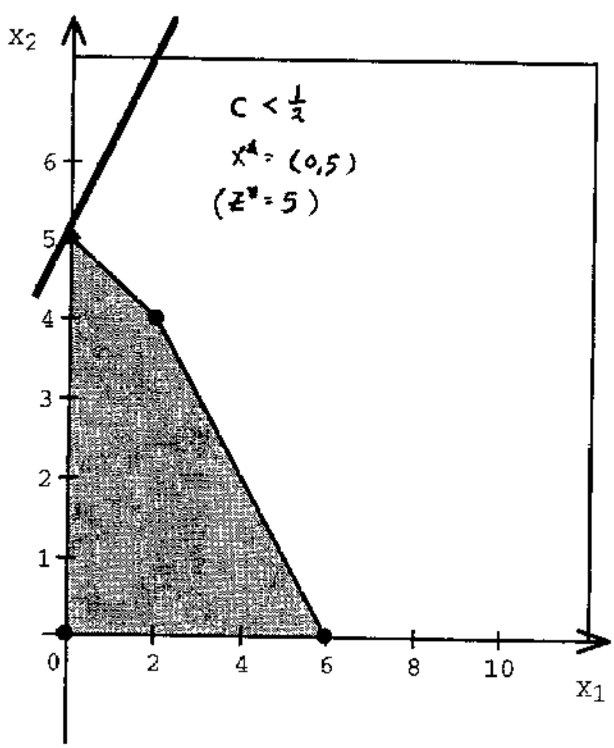
Objective Function Coefficient

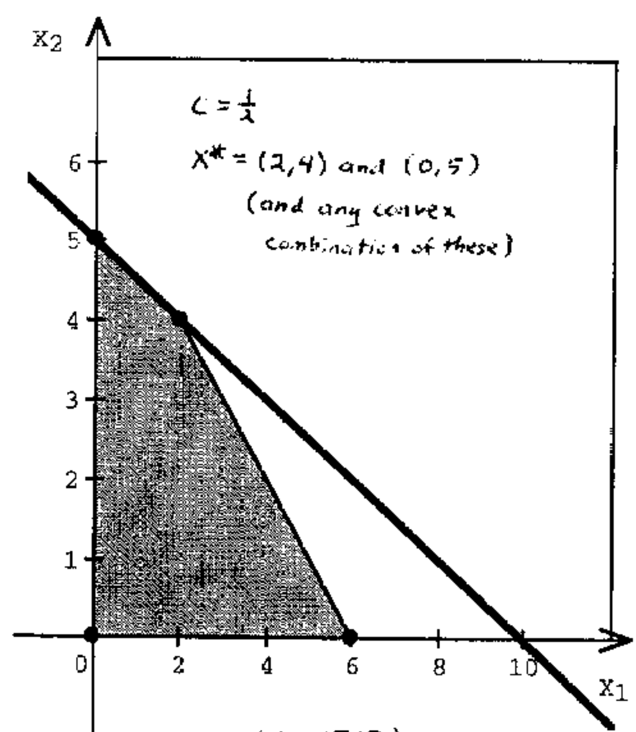
Current Value	Allowable Range To Stay Optimal Minimum	Maximum
50	25	51.25
20	19	40
25	23.8095	$+\infty$

Right Hand Sides

Current Value	Allowable Range To Stay Feasible Minimum	Maximum
500	362.5	555
350	276.667	533.333
150	118.571	$+\infty$
20	0	47.5

3.1-12





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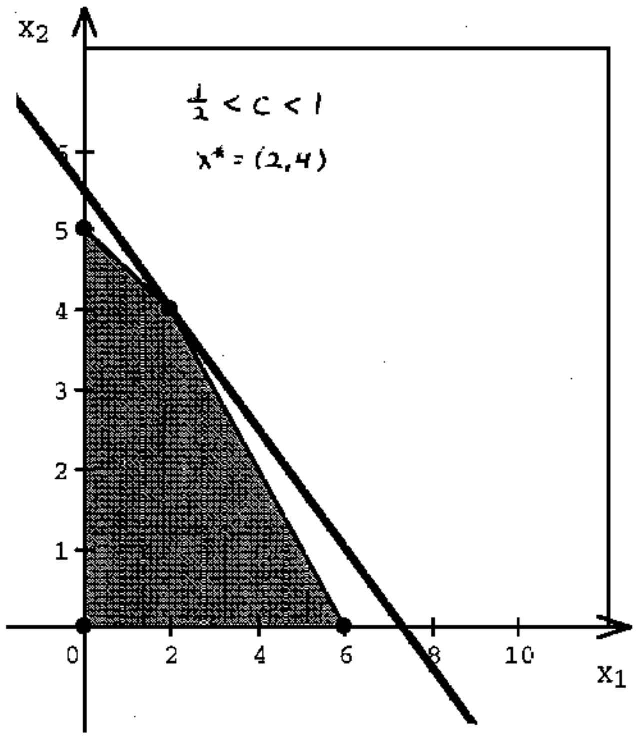
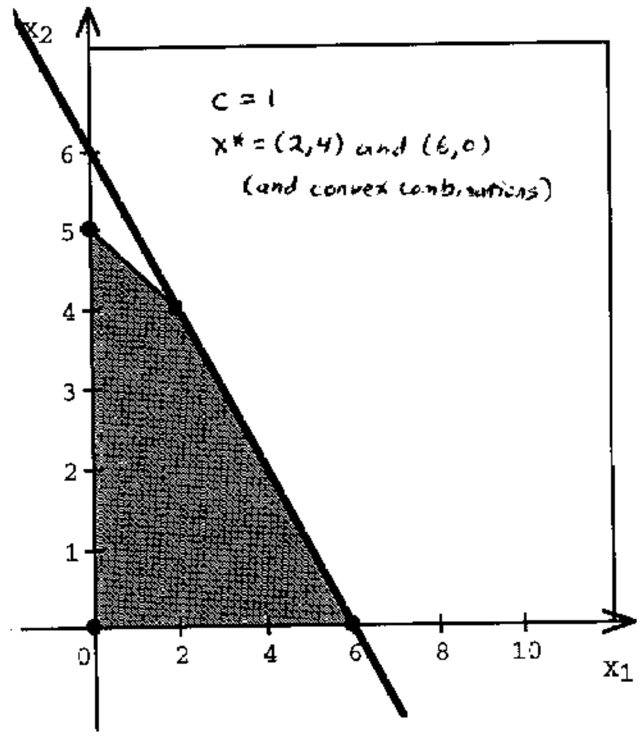
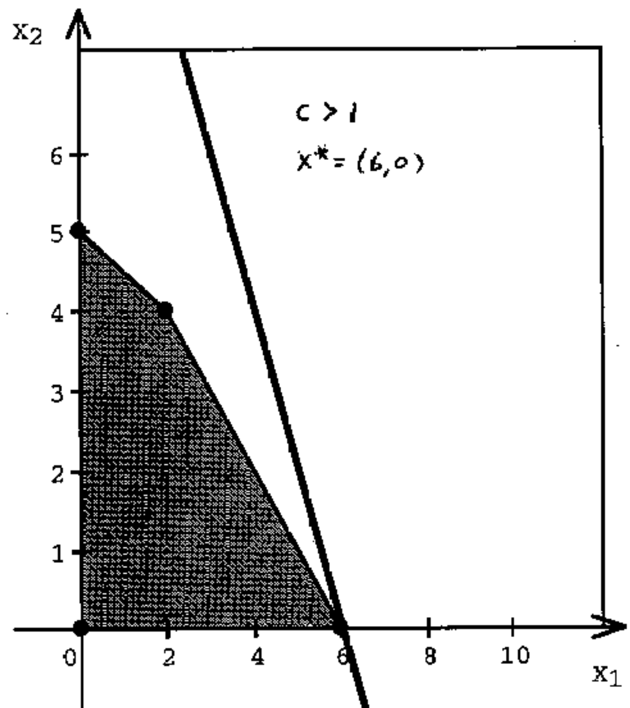
31


$\frac{1}{2} < c < 1$
$x^* = (2,4)$
x_2
x_1


$c = 1$
$x^* = (2,4)$ and $(6,0)$
(and convex combinations)
x_2
x_1


$c > 1$
$x^* = (6,0)$
x_2
x_1

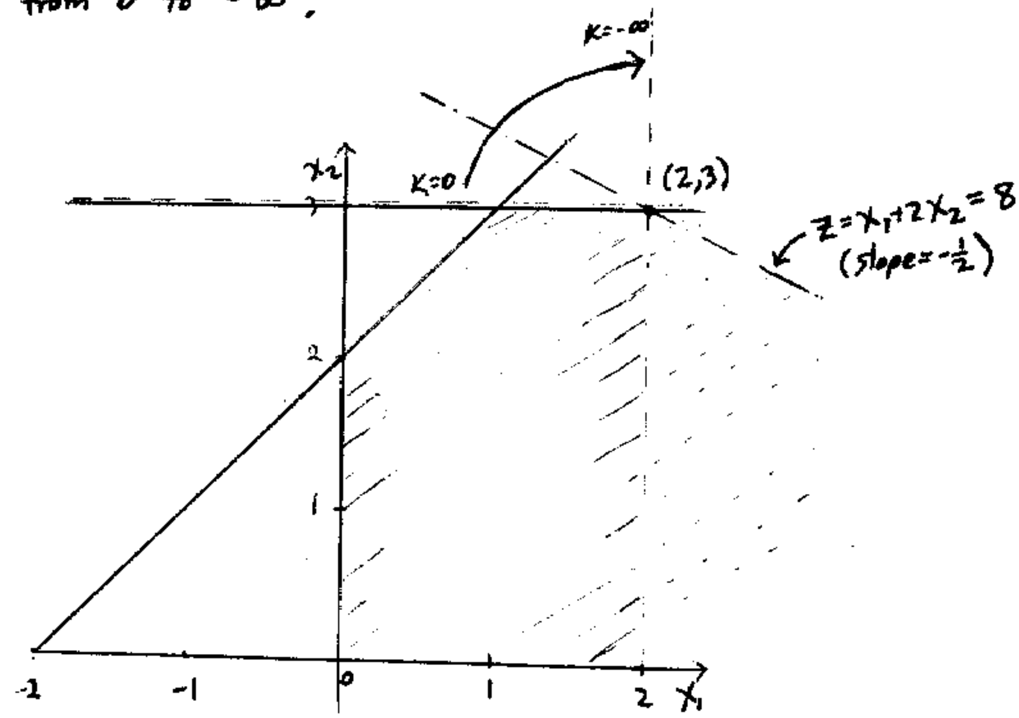
3.1-13

First note that $(2,3)$ always satisfies the 3 constraints. (i.e. $(2,3)$ is always feasible)

In fact since $kx_1 + x_2 = 2k+3$ at $(2,3)$ for any k, the third constraint is always binding.

We only need to check if $(2,3)$ is optimal.

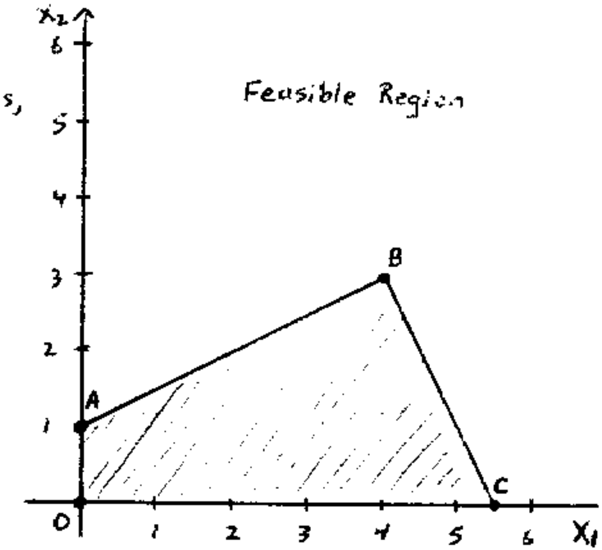
Since the line $kx_1 + x_2 = 2k+3$ always passes through the point $(2,3)$, changing k simply rotates the line. Rewriting: $x_2 = -kx_1 + (2k+3)$, we see that the slope of the line is $-k$, and, therefore, the slope ranges from 0 to $-\infty$.



As we can see, $(2,3)$ is optimal as long as the slope of the 3rd constraint line is less than $-\frac{1}{2}$ (the slope of the objective line). If $k < \frac{1}{2}$, then we can increase the objective by traveling along the 3rd constraint to point $(2+\frac{3}{k}, 0)$ which has an objective value of $2+\frac{3}{k} > 8$ if $k<\frac{1}{2}$.

Therefore, $(2,3)$ is optimal for $k \geq \frac{1}{2}$.

3.1-14



<u>Case 1</u>: $c_2 = 0$ (Objective line is vertical)

If $c_1 > 0$, objective increases as X_1 increases, so $X^* = (\frac{11}{2}, 0)$ (pt. C)

If $c_1 < 0$, opposite is true, so $X^* = (0,0)$ and $(0,1)$ (line $\overline{OA}$)

(Note if $c_1 = 0$, <u>every</u> feasible point gives "optimal" of $0X_1 + 0X_2 = 0$)

<u>Case 2</u>: $c_2 > 0$ (Slope of obj. is $\frac{-c_1}{c_2}$)

If $\frac{-c_1}{c_2} > \frac{1}{2}$ (or equivalently, $\frac{c_1}{c_2} < -\frac{1}{2}$),
$X^* = (0,1)$ (pt. A)

If $\frac{-c_1}{c_2} < -2$,
$X^* = (\frac{11}{2}, 0)$ (pt. C)

If $\frac{1}{2} > \frac{-c_1}{c_2} > -2$,
$X^* = (4,3)$ (pt. B)

(Note, if $\frac{-c_1}{c_2} = \frac{1}{2}$ or -2, X^* is $\overline{AB}$ or $\overline{BC}$, respectively)

<u>Case 3</u>: $c_2 < 0$ (Slope is still $-\frac{c_1}{c_2}$, but the objective increases as the line is shifted <u>down</u>)

If $-\frac{c_1}{c_2} > 0$ (i.e. $c_1 > 0$), $X^* = (\frac{11}{2}, 0)$ (pt. C)

If $-\frac{c_1}{c_2} < 0$ ($c_1 < 0$), $X^* = (0,0)$ (pt. O)

(If $c_1 = 0$, X^* is $\overline{OC}$)

3.2-1

Maximize $P = 3A + 2B$,

subject to $2A + B \le 2$

$A + 2B \le 2$

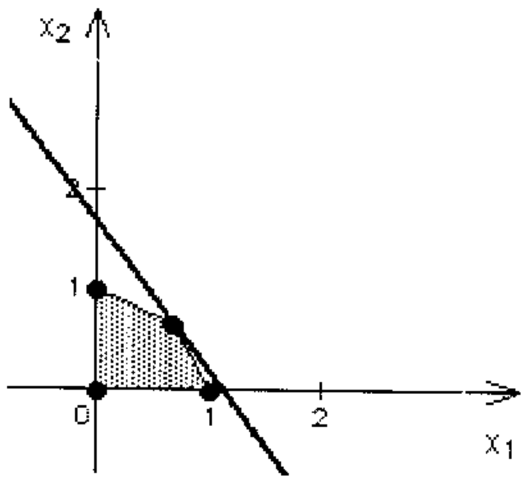
$3A + 3B \le 4$

and $A \ge 0,\ B \ge 0$.

3.2-1

b)

Optimal Solution: $(A, B) = (x_1, x_2) = \left(\frac{2}{3}, \frac{2}{3}\right)$ and $P = 3.33$.



c- We have to solve:

$$2A + B = 2 \qquad (A)$$
$$A + 2B = 2$$

substracting the 2^{nd} equation from the 1^{st} equation:

$$A - B = 0 \quad ; \text{ so } \quad A = B$$

so in (A) we have:

$$2 = 2A + B = 3A \Rightarrow A = 2/3 \Rightarrow B = 2/3$$

3.2-2

a) True (e.g. Max $Z = -x_1 + 4x_2$)

b) True (e.g. Max $Z = -x_1 + 3x_2$)

c) False (e.g. Max $Z = -x_1 - x_2$)

3.2-3

a) As in the Wyndor Glass Co. problem, we want to find the optimal levels of two activities that compete for limited resources. We want to find the optimal mix of the two activities. Let x_1 be the fraction purchased of the partnership in the first friends venture. Let x_2 be the fraction purchased of the partnership in the second friends venture. The following table gives the data for the problem:

Resource	Resource Usage per Unit of Activity 1	2	Amount of Resource Available
Fraction of partnership in first friends venture	1	0	1
Fraction of partnership in second friends venture	0	1	1
Money	\$5000	\$4000	\$6000
Summer Work Hours	400	500	600
Unit Profit	\$4500	\$4500	

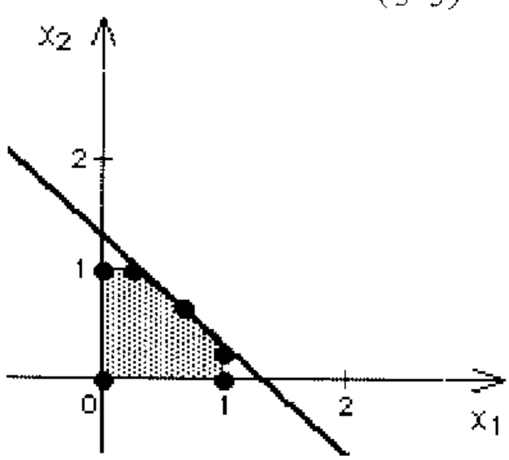
b)

Maximize $P = 4500x_1 + 4500x_2$,

subject to
$$x_1 \leq 1$$
$$x_2 \leq 1$$
$$5000x_1 + 4000x_2 \leq 6000$$
$$400x_1 + 500x_2 \leq 600$$

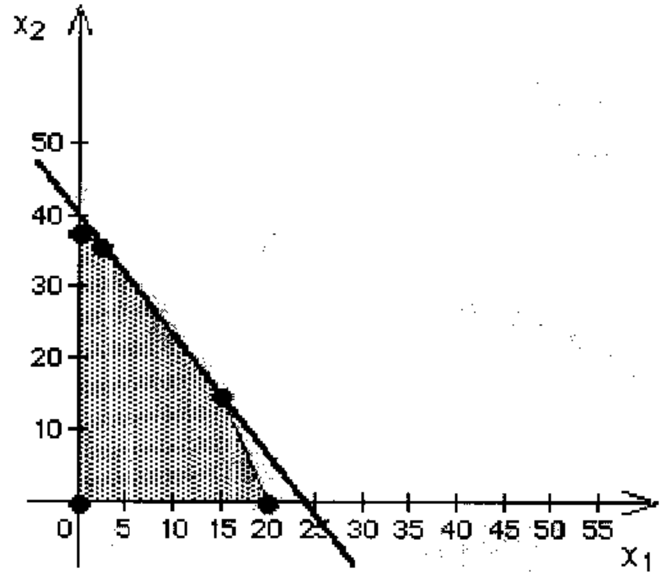
and $x_1 \geq 0,\ x_2 \geq 0.$

c) Optimal Solution: $(x_1, x_2) = \left(\frac{2}{3}, \frac{2}{3}\right)$ and $P = 6000$.

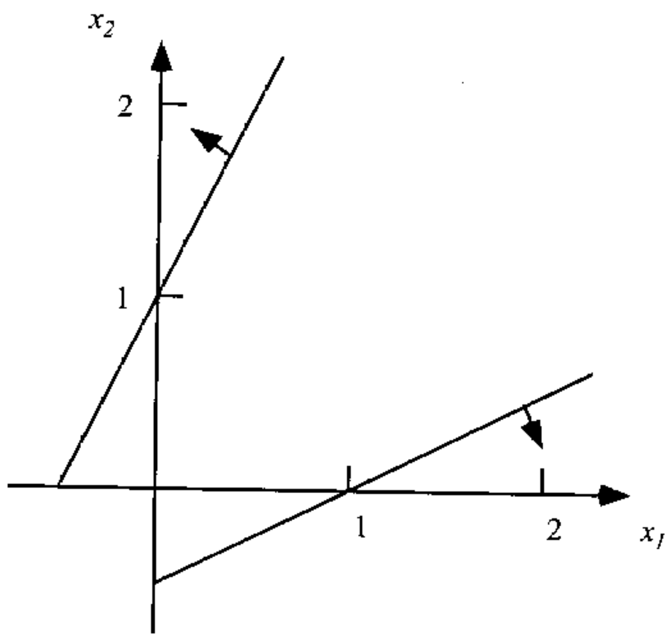


3.2-4

Optimal Solution: $(x_1, x_2) = (15,15)$, $(2.5, 35.833)$ and all points on the connecting line. $Z = 12{,}000$.

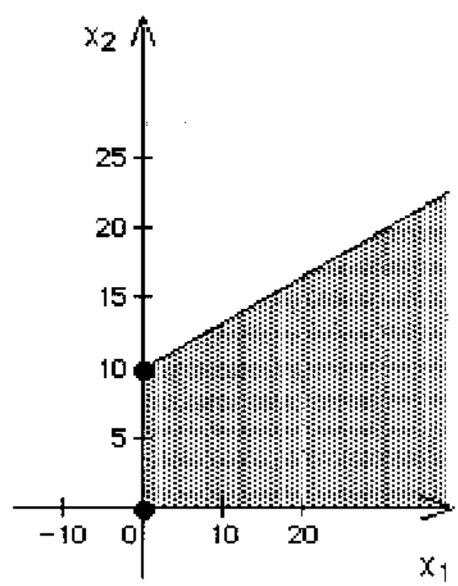


3.2-5



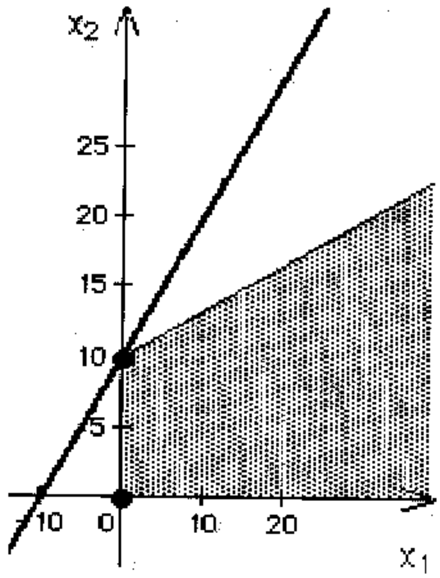
3.2-6

a)

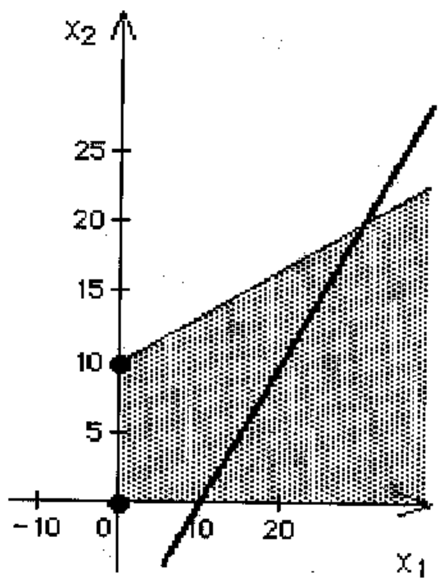


3.2-6

b) Yes. Optimal Solution: $(x_1, x_2) = (0,10)$ and $Z = 10$.



c) No. The objective function value is maximized by sliding the objective function line to the right. This can be done forever, so there is no optimal solution.



d) No, solutions exist that will make Z arbitrarily large. This usually occurs when a constraint is left out of the model.

3.3-1. Proportionality: It is fair to assume the amount of work and money spent and the profit earned are directly proportional to the fraction of partnership purchased in either venture.

Additivity: The profit as well as time and money requirements for one venture should not affect the profit or time and money requirements of the other venture. This assumption is reasonably satisfied.

Divisibility: Because both friends will allow purchase of any fraction of a full partnership, divisibility is a reasonable assumption.

Certainty: Because we don't know how accurate the friends' profit estimates are, this is a more doubtful assumption. We should conduct sensitivity analysis after finding the optimal solutions for the current values of the profits.

3.3-2.

Proportionality: OK, since if either variable is fixed, the objective value grows in proportion to the increase in the other variable.

Additivity: Not OK, since activities interact. For example, the objective value with $(x_1, x_2) = (1,1)$ isn't equal to the objective value with $(x_1, x_2) = (1,0)$ plus the objective value with $(x_1, x_2) = (0,1)$.

Divisibility: Not OK, since activity levels are not allowed to be fractional.

Certainty: OK, since data is given as accurate.

3.4-1

a) Proportionality: OK, since beam effects on tissue types are proportional to beam strength.

Additivity: OK, since it was stated that effects from multiple beams are additive.

Divisibility: OK, since beam strength can be any fractional level.

Certainty: Due to the complicated analysis required to estimate the data on radiation absorbtion in different tissue types, sensitivity analysis should be used.

b)

Proportionality: OK, as long as there is no set up cost associated with planting a crop.

Additivity: OK, as long as crops do not interact.

Divisibility: OK, since acres are divisible.

Certainty: OK, since data can be accurately determined.

c)

Proportionality: OK, since set up costs were considered.

Additivity: OK, since it was stated that there is no interaction.

Divisibility: OK, since methods can be used at fractional levels

Certainty: Data is hard to estimate so it could easily be stochastic. Sensitivity analysis should be used.

3.4-2.*a*) Reclaiming solid wastes

Proportionality: The amalgamation & treatment processes are unlikely to be proportional. There are bound to be set up costs. (Treating 1,000 lbs. of material will not cost the same as treating 10 lbs. of material 100 ties.)

Additivity: Probably O.K., though it is possible that there is some interaction between treatments of materials (e.g., if *A* is treated after *B*, the machines do not need to be cleaned out . . .)

Divisibility: Probably O.K., unless selling/buying materials can only be done in batches (of 100 lbs., say).

Certainty: Selling/buying prices may change; costs of treatment & amalgamation are, most likely, crude estimates and may also change.

3.4-2. *b*) Personnel Scheduling

Proportionality: O.K. It is possible that *some* costs are not proportional to # of agents hired (e.g., benefits, working space, . . .), but for the most part, this assumption is satisfied.

Additivity: O.K. Same, pretty much, as proportionality.

Divisibility: Clearly, one cannot hire a fraction of an agent.

Certainty: The "min. # of agents needed" is suspect. Perhaps 45 agents will suffice instead of 48, for example, for a nominal fee. Also, does an agent in one shift do the same amount of work as one from another (or even the same) shift?

(*c*) Distribution goods through a distribution network

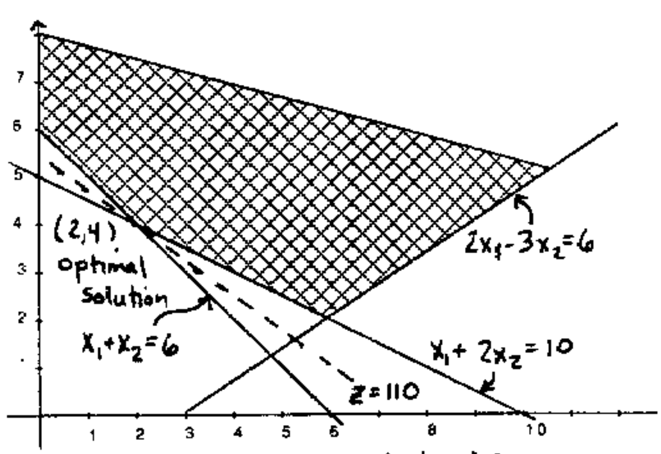
Proportionality: As in (*a*), there is probably a "set-up" cost for delivery (i.e., delivering 50 units one at a time will cost much more than delivering all 50 at the same time).

Additivity: Probably O.K., but it is possible that 2 routes may be "combined" to lower costs (e.g., if $x_{F2\text{-}OC} = x_{OC\text{-}W2} = 50$, the truck *may* be able to deliver 50 units directly from F2 to W2, without stopping at DC, saving some money). Another question is whether F1 and F2 produce equivalent units.

Divisibility: It seems that one cannot deliver a fraction of a unit.

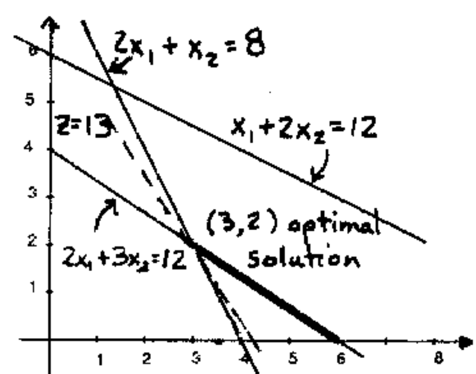
Certainty: Shipping costs are probably approximations and are subject to change, as are amounts produced. Even the capacities may depend on available daily trucking force, weather, etc. As in any problem, sensitivity analysis should be done.

3.4-3.



$(x_1, x_2) = (2,4)$ is optimal with $Z = 110$

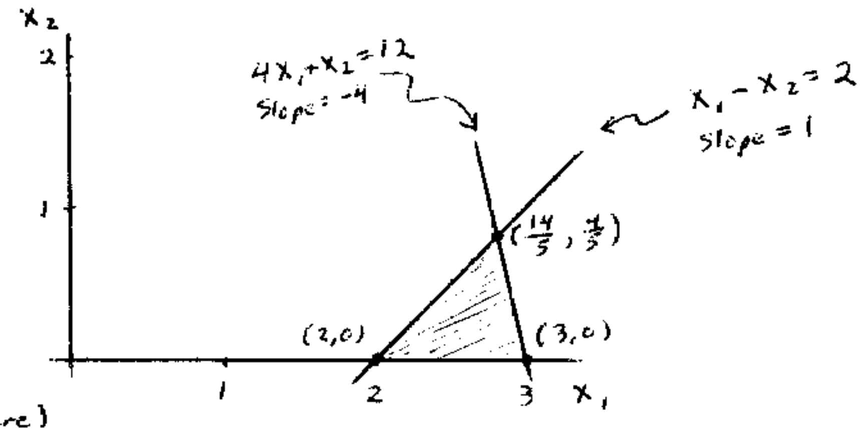
3.4-4.



$(x_1, x_2) = (3,2)$ is optimal with $Z = 13$

3.4-5

Feasible region:



Since $c_2>0$ ($c_2=2$, here)

If slope of objective $= -\frac{c_1}{c_2} > 1$, then $(2,0)$ is optimal

or $c_1<-2 \Rightarrow (x_1^*, x_2^*)=(2,0)$

If $1 > \frac{-c_1}{c_2} > -12$, then $(\frac{14}{5}, \frac{4}{5})$ is optimal

or $-2<c_1<24 \Rightarrow (x_1^*, x_2^*) = (\frac{14}{5}, \frac{4}{5})$

If $\frac{-c_1}{c_2} < -12$ (or $c_1>24$), then $(3,0)$ is optimal

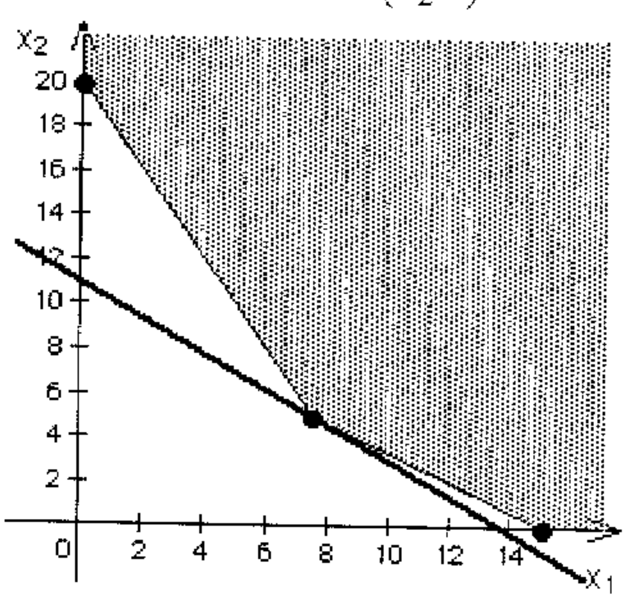
Of course, if $c_1=-2$, then both $(2,0)$ and $(\frac{14}{5},\frac{4}{5})$ are optimal

(as are all convex combinations of these),

and if $c_1=24$, $(\frac{14}{5},\frac{4}{5})$, $(3,0)$, and all convex combinations are optimal.

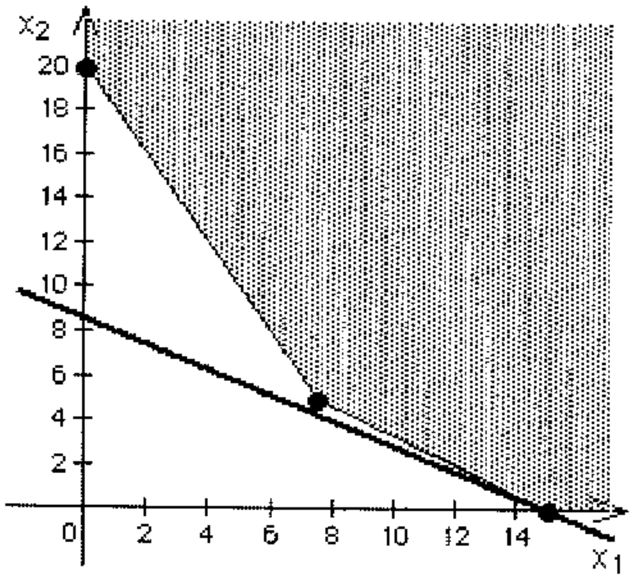
3.4-6

a) Optimal Solution: $(x_1,x_2)=\left(7\frac{1}{2},5\right)$ and $C=550$.

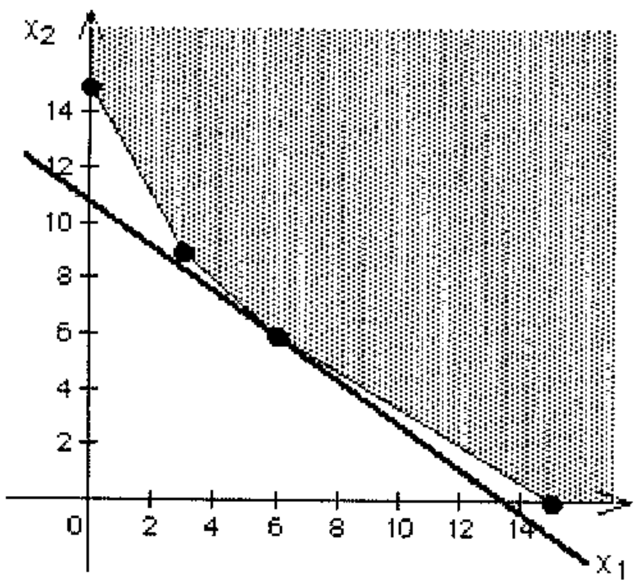


3.4-6

b) Optimal Solution: $(x_1, x_2) = (15, 0)$ and $C = 600$.



c) Optimal Solution: $(x_1, x_2) = (6, 6)$ and $C = 540$.



3.4-7 a)

Minimize $C = 4S + 2P$,

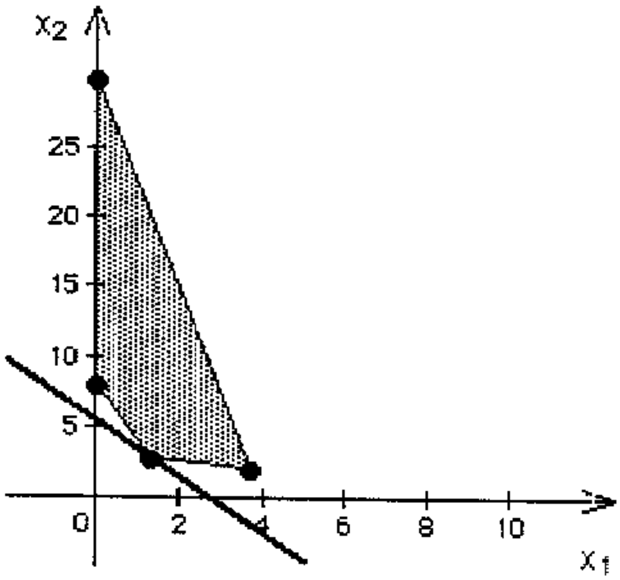
subject to $5S + 15P \geq 50$

$20S + 5P \geq 40$

$15S + 2P \leq 60$

and $S \geq 0, \; P \geq 0.$

b) Optimal Solution: $(S,P)=(x_1,x_2)=(1.3,2.9)$ and $C=10.91$.



c)

	Contribution Per Unit				
	Steak	Potato	Totals		Level
Carbohydrate	5	15	50	≥	50
Protein	20	5	40	≥	40
Fat	15	2	24.91	≤	60
Unit Cost	4	2	**$ 10.91**		
Solution	**1.3**	**2.9**			

3.4-8

a)

Minimize $C=0.4A+0.8B$,

subject to $800A+1000B\geq 8000$

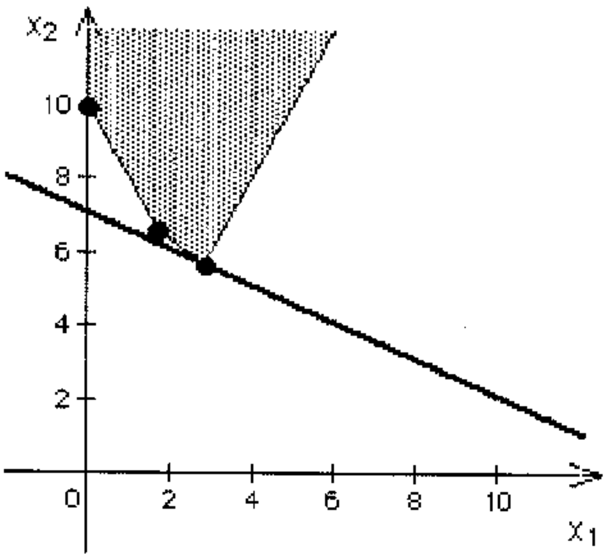
$140A+70B\geq 700$

$\frac{2}{3}A-\frac{1}{3}B\leq 0$

and $A\geq 0,\ B\geq 0$.

3.4-8

b) Optimal Solution: $(A,B)=(x_1,x_2)=(2.85,5.72)$ and $C=5.72$.



3.4-9

a)

Let $x_{ij}=$ the amount of space leased in month i for a period of j months for $i=1,\ldots,5$ and $j=1,\ldots,6-i$.

$$\text{Minimize } C=650(x_{11}+x_{21}+x_{31}+x_{41}+x_{51})+1000(x_{12}+x_{22}+x_{32}+x_{42})$$
$$+1350(x_{13}+x_{23}+x_{33})+1600(x_{14}+x_{24})+1900x_{15}$$

subject to

$$x_{11}+x_{12}+x_{13}+x_{14}+x_{15}\geq 30{,}000$$
$$x_{12}+x_{13}+x_{14}+x_{15}+x_{21}-x_{22}+x_{23}+x_{24}\geq 20{,}000$$
$$x_{13}+x_{14}+x_{15}+x_{22}+x_{23}+x_{24}+x_{31}+x_{32}+x_{33}\geq 40{,}000$$
$$x_{14}+x_{15}+x_{23}+x_{24}+x_{32}+x_{33}+x_{41}+x_{42}\geq 10{,}000$$
$$x_{15}+x_{24}+x_{33}+x_{42}+x_{51}\geq 50{,}000$$

and $x_{ij}\geq 0$, for $i=1,\ldots,5$ and $j=1,\ldots,6-i$.

3.4-9 b)

	A	B	C	D	E	F	G	H	I
1					Contribution Toward Required Amount				
2	Month	1-1	1-2	1-3	1-4	1-5	2-1	2-2	2-3
3	1	1	1	1	1	1			
4	2		1	1	1	1	1	1	1
5	3			1	1	1		1	1
6	4				1	1			1
7	5					1			
8	Unit Cost	$ 650	$ 1,000	$ 1,350	$ 1,600	$ 1,900	$ 650	$ 1,000	$ 1,350
9	Solution	**0**	**0**	**0**	**0**	**30000**	**0**	**0**	**0**

	J	K	L	M	N	O	P	Q	R	S
1			Contribution Toward Required Amount							Resource
2	2-4	3-1	3-2	3-3	4-1	4-2	5-1	Totals		Available
3								30000	≥	30000
4	1							30000	≥	20000
5	1	1	1	1				40000	≥	40000
6	1		1	1	1	1		30000	≥	10000
7	1			1		1	1	50000	≥	50000
8	$ 1,600	$ 650	$ 1,000	$ 1,350	$ 650	$ 1,000	$ 650	**76499994**		
9	**0**	**10000**	**0**	**0**	**0**	**0**	**20000**			

Data cells: B3:P8 and S3:S7
Changing cells: B9:P9
Target cell: Q8
Output cells: Q3:Q7

	Q
3	=SUMPRODUCT(B3:P3,B9:P9)
4	=SUMPRODUCT(B4:P4,B9:P9)
5	=SUMPRODUCT(B5:P5,B9:P9)
6	=SUMPRODUCT(B6:P6,B9:P9)
7	=SUMPRODUCT(B7:P7,B9:P9)
8	**=SUMPRODUCT(B8:P8,B9:P9)**

3.4-10

a) Let f_1 = number of full-time consultants working the morning shift (8 a.m.-4 p.m.),
f_2 = number of full-time consultants working the afternoon shift (12 p.m.-8 p.m.),
f_3 = number of full-time consultants working the evening shift (4 p.m.-midnight),
p_1 = number of part-time consultants working the first shift (8 a.m.-12 p.m.),
p_2 = number of part-time consultants working the second shift (12 p.m.-4 p.m.),
p_3 = number of part-time consultants working the third shift (4 p.m.-8 p.m.),
p_4 = number of part-time consultants working the fourth shift (8 p.m.-midnight).

Minimize $C = (\text{S}14 / \text{hour})(8 \text{ hours})\left[f_1 + f_2 + f_3\right] +$

$$(\$5 / \text{hour})(4 \text{ hours})\left[p_1 + p_2 + p_3 + p_4\right],$$

subject to
$$f_1 + p_1 \geq 4$$
$$f_1 + f_2 + p_2 \geq 8$$
$$f_2 + f_3 + p_3 \geq 10$$
$$f_3 + p_4 \geq 6$$
$$f_1 \geq 2p_1$$
$$f_1 + f_2 \geq 2p_2$$
$$f_2 + f_3 \geq 2p_3$$
$$f_3 \geq 2p_4$$
and
$$f_1 \geq 0,\ f_2 \geq 0,\ f_3 \geq 0,\ p_1 \geq 0,\ p_2 \geq 0,\ p_3 \geq 0,\ p_4 \geq 0.$$

b)

	Benefit Contribution Per Unit of Each Activity									Minimum
Time	FT 1	FT 2	FT 3	PT 1	PT 2	PT 3	PT 4	Totals		Required
8 am-12 noon	1	0	0	1	0	0	0	4	≥	4
	1	0	0	-2	0	0	0	4	≥	0
noon-4 pm	1	1	0	0	1	0	0	8	≥	8
	1	1	0	0	-2	0	0	8	≥	0
4 pm-8 pm	0	1	1	0	0	1	0	10	≥	10
	0	1	1	0	0	-2	0	10	≥	0
8 pm-midnight	0	0	1	0	0	0	1	6	≥	6
	0	0	1	0	0	0	-2	6	≥	0
Unit Cost	14	14	14	12	12	12	12	**$ 196.00**		
Solution	**4**	**4**	**6**	**0**	**0**	**0**	**0**			

3.4-11

a) Let

x_{F1-C1} = number of units shipped from Factory 1 to Customer 1,
x_{F1-C2} = number of units shipped from Factory 1 to Customer 2,
x_{F1-C3} = number of units shipped from Factory 1 to Customer 3,
x_{F2-C1} = number of units shipped from Factory 2 to Customer 1,
x_{F2-C2} = number of units shipped from Factory 2 to Customer 2
x_{F2-C3} = number of units shipped from Factory 2 to Customer 3.

$$\text{Minimize } C = 600x_{F1-C1} + 800x_{F1-C2} + 700x_{F1-C3} + 400x_{F2-C1} + 900x_{F2-C2} + 600x_{F2-C3}$$

subject to
$$x_{F1-C1} + x_{F1-C2} + x_{F1-C3} = 400$$
$$x_{F2-C1} + x_{F2-C2} + x_{F2-C3} = 500$$
$$x_{F1-C1} + x_{F2-C1} = 300$$
$$x_{F1-C2} + x_{F2-C2} = 200$$
$$x_{F1-C3} + x_{F2-C3} = 400$$

and
$$x_{F1-C1} \ge 0,\ x_{F1-C2} \ge 0,\ x_{F1-C3} \ge 0,$$
$$x_{F2-C1} \ge 0,\ x_{F2-C2} \ge 0,\ x_{F2-C3} \ge 0.$$

b)

Contribution Toward Required Amount Per Unit Shipped

Requirement	Shipping Lane F1-C1	F2-C2	F1-C3	F2-C1	F2-C2	F2-C3	Totals		Required Amount
F1 Amount	1	1	1	0	0	0	400	=	400
F2 Amount	0	0	0	1	1	1	500	=	500
C1 Amount	1	0	0	1	0	0	300	=	300
C2 Amount	0	1	0	0	1	0	200	=	200
C3 Amount	0	0	1	0	0	1	400	=	400
Unit Cost	600	800	700	400	200	400	**$ 410,000**		
Solution	**300**	**0**	**100**	**0**	**200**	**300**			

3.4-12 a) Let

$x_{M1\text{-}S1}$ = number of units shipped from Mine 1 to Storage 1,
$x_{M1\text{-}S2}$ = number of units shipped from Mine 1 to Storage 2,
$x_{M2\text{-}S1}$ = number of units shipped from Mine 2 to Storage 1,
$x_{M2\text{-}S2}$ = number of units shipped from Mine 2 to Storage 2,
$x_{S1\text{-}P}$ = number of units shipped from Storage 1 to the Plant,
$x_{S2\text{-}P}$ = number of units shipped from Storage 2 to the Plant.

$$\text{Minimize } C = 2000x_{M1-S1} + 1700x_{M1-S2} + 1600x_{M2-S1} + 1100x_{M2-S2} + 400x_{S1-P} + 800x_{S2-P}$$

subject to
$$x_{M1-S1} + x_{M1-S2} = 40$$
$$x_{M2-S1} + x_{M2-S2} = 60$$
$$x_{M1-S1} + x_{M2-S1} - x_{S1-P} = 0$$
$$x_{M1-S2} + x_{M2-S2} - x_{S2-P} = 0$$
$$x_{S1-P} + x_{S2-P} = 100$$
$$x_{M1-S1} \leq 30,\ x_{M1-S2} \leq 30$$
$$x_{M2-S1} \leq 50,\ x_{M2-S2} \leq 50$$
$$x_{S1-P} \leq 70,\ x_{S2-P} \leq 70$$
and
$$x_{M1-S1} \geq 0,\ x_{M1-S2} \geq 0,\ x_{M2-S1} \geq 0,$$
$$x_{M2-S2} \geq 0,\ x_{S1-P} \geq 0,\ x_{S2-P} \geq 0.$$

b)

	Contribution Toward Required Amount Per Unit Shipped								Required
Requirement	M1-S1	M1-S2	M2-S1	M2-S2	S1-P	S2-P	Totals		Amount
M1 Amount	1	1	0	0	0	0	40	=	40
M2 Amount	0	0	1	1	0	0	60	=	60
S1 Amount	1	0	1	0	-1	0	0	=	0
S2 Amount	0	1	0	1	0	-1	0	=	0
P Amount	0	0	0	0	1	1	100	=	100
Capacity	30	30	50	50	70	70			
Unit Cost	2000	1700	1600	1100	400	800	**$ 194,000**		
Solution	**10**	**30**	**10**	**50**	**70**	**30**			

3.4-13 a)

$$A_1 + B_1 + R_1 = 60{,}000$$
$$A_2 + B_2 + C_2 + R_2 = R_1$$
$$A_3 + B_3 + R_3 = R_2 + 1.40A_1$$
$$A_4 + R_4 = R_3 + 1.40A_2 + 1.70B_1$$
$$D_5 + R_5 = R_4 + 1.40A_3 + 1.70B_2$$

b)

Let A_t = amount invested in investment A at the beginning of year t

B_t = amount invested in investment A at the beginning of year t

C_t = amount invested in investment A at the beginning of year t

D_t = amount invested in investment A at the beginning of year t

R_t = amount not invested at the beginning of year t.

Maximize $P = 1.40A_4 + 1.70B_3 + 1.90C_2 + 1.30D_5 + R_5$

subject to

$$A_1 + B_1 + R_1 = 60{,}000$$
$$A_2 + B_2 + C_2 - R_1 + R_2 = 0$$
$$-1.40A_1 + A_3 + B_3 - R_2 + R_3 = 0$$
$$-1.40A_2 + A_4 - 1.70B_1 - R_3 + R_4 = 0$$
$$-1.40A_3 - 1.70B_2 + D_5 - R_4 + R_5 = 0$$

and $A_t \geq 0,\ B_t \geq 0,\ C_t \geq 0,\ D_t \geq 0,\ R_t \geq 0.$

c)

Contribution Toward Required Amount Per Unit

Year	A1	A2	A3	A4	B1	B2	B3	C2	D5	R1	R2	R3	R4	R5	Totals		Required Amount
	Investment									Remainder							
1	1	0	0	0	1	0	0	0	0	1	0	0	0	0	60000	=	60000
2	0	1	0	0	0	1	0	1	0	-1	1	0	0	0	0	=	0
3	-1.4	0	1	0	0	0	1	0	0	0	-1	1	0	0	0	=	0
4	0	-1.4	0	1	-1.7	0	0	0	0	0	0	-1	1	0	-1.33577E-12	=	0
5	0	0	-1.4	0	0	-1.7	0	0	1	0	0	0	-1	1	1.45519E-11	=	0
Unit Profit	0	0	0	1.4	0	0	1.7	1.9	1.3	0	0	0	0	1	**$ 152,880**		
Solution	**60000**	**0**	**84000**	**0**	**0**	**0**	**0**	**0**	**117600**	**0**	**0**	**0**	**0**	**0**			

3.4-14 a)

Let x_1 = amount of Alloy 1 used,

x_2 = amount of Alloy 2 used,

x_3 = amount of Alloy 3 used,

x_4 = amount of Alloy 4 used,

x_5 = amount of Alloy 5 used.

Minimize $C = 22x_1 + 20x_2 + 25x_3 + 24x_4 + 27x_5$

subject to

$$60x_1 + 25x_2 + 45x_3 + 20x_4 + 50x_5 = 40$$
$$10x_1 + 15x_2 + 45x_3 + 50x_4 + 40x_5 = 35$$
$$30x_1 + 60x_2 + 10x_3 + 30x_4 + 10x_5 = 25$$
$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$

and $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0.$

3.4-14

b)

	Contribution Toward Required Amount							Required
Requirement	Alloy 1	Alloy 2	Alloy 3	Alloy 4	Alloy 5	Totals		Amount
% tin	60	25	45	20	50	40	=	40
% zinc	10	15	45	50	45	35	=	35
% lead	30	60	10	30	10	25	=	25
% total	1	1	1	1	1	1	=	1
Unit Cost	22	20	25	24	27	**$ 23.46**		
Solution	**0.0435**	**0.2826**	**0.6739**	**0**	**3E-15**			

3.4-15

a) Let

x_{P1L} = number of large units produced at Plant 1,

x_{P1M} = number of medium units produced at Plant 1.

x_{P1S} = number of small units produced at Plant 1,

x_{P2L} = number of large units produced at Plant 2,

x_{P2M} = number of medium units produced at Plant 2.

x_{P2S} = number of small units produced at Plant 2,

x_{P3L} = number of large units produced at Plant 3,

x_{P3M} = number of medium units produced at Plant 3.

x_{P3S} = number of small units produced at Plant 3.

Maximize $P = 420x_{P1L} + 360x_{P1M} + 300x_{P1S} + 420x_{P2L} + 360x_{P2M} + 300x_{P2S} + 420x_{P3L} + 360x_{P3M} + 300x_{P3S}$

subject to

$$x_{P1L} + x_{P1M} + x_{P1S} \leq 750$$
$$x_{P2L} + x_{P2M} + x_{P2S} \leq 900$$
$$x_{P3L} + x_{P3M} + x_{P3S} \leq 450$$
$$20x_{P1L} + 15x_{P1M} + 12x_{P1S} \leq 13000$$
$$20x_{P2L} + 15x_{P2M} + 12x_{P2S} \leq 12000$$
$$20x_{P3L} + 15x_{P3M} + 12x_{P3S} \leq 5000$$
$$x_{P1L} + x_{P2L} + x_{P3L} \leq 900$$
$$x_{P1M} + x_{P2M} + x_{P3M} \leq 1200$$
$$x_{P1S} + x_{P2S} + x_{P3S} \leq 750$$
$$\frac{1}{750}x_{P1L} + \frac{1}{750}x_{P1M} + \frac{1}{750}x_{P1S} - \frac{1}{900}x_{P2L} - \frac{1}{900}x_{P2M} - \frac{1}{900}x_{P2S} = 0$$
$$\frac{1}{750}x_{P1L} + \frac{1}{750}x_{P1M} + \frac{1}{750}x_{P1S} - \frac{1}{450}x_{P3L} - \frac{1}{450}x_{P3M} - \frac{1}{450}x_{P3S} = 0$$

and

$$x_{P1L} \geq 0,\ x_{P1M} \geq 0,\ x_{P1S} \geq 0,\ x_{P2L} \geq 0,\ x_{P2M} \geq 0,$$
$$x_{P2S} \geq 0,\ x_{P3L} \geq 0,\ x_{P3M} \geq 0,\ x_{P3S} \geq 0.$$

3.4-15

b)

Resource	Resource Usage Per Unit of Each Activity											Resource
	P1-L	P1-M	P1-S	P2-L	P2-M	P2-S	P3-L	P3-M	P3-S	Totals		Available
Capacity P1	1	1	1	0	0	0	0	0	0	694.444	≤	750
Capacity P2	0	0	0	1	1	1	0	0	0	833.333	≤	900
Capacity P3	0	0	0	0	0	0	1	1	1	416.667	≤	450
Space P1	20	15	12	0	0	0	0	0	0	13000	≤	13000
Space P2	0	0	0	20	15	12	0	0	0	12000	≤	12000
Space P3	0	0	0	0	0	0	20	15	12	5000	≤	5000
Sales P1	1	0	0	1	0	0	1	0	0	516.667	≤	900
Sales P2	0	1	0	0	1	0	0	1	0	844.444	≤	1200
Sales P3	0	0	1	0	0	1	0	0	1	583.333	≤	750

Requirement	Contribution Toward Required Amount Activity											Required
	P1-L	P1-M	P1-S	P2-L	P2-M	P2-S	P3-L	P3-M	P3-S	Totals		Amount
%P1=%P2	0.00133	0.00133	0.00133	-0.0011	-0.0011	-0.0011	0	0	0	0	=	0
%P1=%P3	0.00133	0.00133	0.00133	0	0	0	-0.0022	-0.0022	-0.0022	0	=	0
Unit Profit	420	360	300	420	360	300	420	360	300	**$ 696,000**		
Solution	**516.667**	**177.778**	**0**	**0**	**666.667**	**166.667**	**0**	**0**	**416.667**			

3.4-16 a)

Let

x_{1F} = number of tons of cargo type 1 stowed in the front compartment,
x_{1C} = number of tons of cargo type 1 stowed in the center compartment
x_{1B} = number of tons of cargo type 1 stowed in the back compartment,
x_{2F} = number of tons of cargo type 2 stowed in the front compartment,
x_{2C} = number of tons of cargo type 2 stowed in the center compartment
x_{2B} = number of tons of cargo type 2 stowed in the back compartment,
x_{3F} = number of tons of cargo type 3 stowed in the front compartment,
x_{3C} = number of tons of cargo type 3 stowed in the center compartment
x_{3B} = number of tons of cargo type 3 stowed in the back compartment,
x_{4F} = number of tons of cargo type 4 stowed in the front compartment,
x_{4C} = number of tons of cargo type 4 stowed in the center compartment
x_{4B} = number of tons of cargo type 4 stowed in the back compartment.

Maximize $P = 320x_{1F} + 320x_{1C} + 320x_{1B} + 400x_{2F} + 400x_{2C} + 400x_{2B} +$
$360x_{3F} + 360x_{3C} + 360x_{3B} + 290x_{4F} + 290x_{4C} + 400x_{4B}$

subject to

$$x_{1F} + x_{2F} + x_{3F} + x_{4F} \le 12$$
$$x_{1C} + x_{2C} + x_{3C} + x_{4C} \le 18$$
$$x_{1B} + x_{2B} + x_{3B} + x_{4B} \le 10$$
$$x_{1F} + x_{1C} + x_{1B} \le 20$$
$$x_{2F} + x_{2C} + x_{2B} \le 16$$
$$x_{3F} + x_{3C} + x_{3B} \le 25$$
$$x_{4F} + x_{4C} + x_{4B} \le 13$$
$$500x_{1F} + 700x_{2F} + 600x_{3F} + 400x_{4F} \le 7000$$
$$500x_{1C} + 700x_{2C} + 600x_{3C} + 400x_{4C} \le 9000$$
$$500x_{1B} + 700x_{2B} + 600x_{3B} + 400x_{4B} \le 5000$$
$$\frac{1}{12}x_{1F} + \frac{1}{12}x_{2F} + \frac{1}{12}x_{3F} + \frac{1}{12}x_{4F} - \frac{1}{18}x_{1C} - \frac{1}{18}x_{2C} - \frac{1}{18}x_{3C} - \frac{1}{18}x_{4C} = 0$$
$$\frac{1}{12}x_{1F} + \frac{1}{12}x_{2F} + \frac{1}{12}x_{3F} + \frac{1}{12}x_{4F} - \frac{1}{10}x_{1B} - \frac{1}{10}x_{2B} - \frac{1}{10}x_{3B} - \frac{1}{10}x_{4B} = 0$$

and

$$x_{1F} \ge 0,\ x_{1C} \ge 0,\ x_{1B} \ge 0,\ x_{2F} \ge 0,\ x_{2C} \ge 0,\ x_{2B} \ge 0,$$
$$x_{3F} \ge 0,\ x_{3C} \ge 0,\ x_{3B} \ge 0,\ x_{4F} \ge 0,\ x_{4C} \ge 0,\ x_{4B} \ge 0.$$

3.4-16

b)

Resource	1F	1C	1B	2F	2C	2B	3F	3C	3B	4F	4C	4B	Totals		Resource Available
	Resource Usage Per Unit of Each Activity														
Front Wt.	1	0	0	1	0	0	1	0	0	1	0	0	12	≤	12
Center Wt.	0	1	0	0	1	0	0	1	0	0	1	0	18	≤	18
Back Wt.	0	0	1	0	0	1	0	0	1	0	0	1	10	≤	10
Cargo 1 Wt.	1	1	1	0	0	0	0	0	0	0	0	0	15	≤	20
Cargo 2 Wt.	0	0	0	1	1	1	0	0	0	0	0	0	12	≤	16
Cargo 3 Wt.	0	0	0	0	0	0	1	1	1	0	0	0	0	≤	25
Cargo 4 Wt.	0	0	0	0	0	0	0	0	0	1	1	1	13	≤	13
Space Front	500	0	0	700	0	0	600	0	0	400	0	0	7000	≤	7000
Space Center	0	500	0	0	700	0	0	600	0	0	400	0	9000	≤	9000
Space Back	0	0	500	0	0	700	0	0	600	0	0	400	5000	≤	5000

Requirement	1F	1C	1B	2F	2C	2B	3F	3C	3B	4F	4C	4B	Totals		Required Amount
	Contribution Toward Required Amount														
%F=%C	0.0833	-0.0556	0	0.0833	-0.0556	0	0.0833	-0.0556	0	0.08333	-0.0556	0	0	=	0
%F=%B	0.0833	0	-0.1	0.0833	0	-0.1	0.0833	0	-0.1	0.0833	0	-0.1	0	=	0
Unit Profit	320	320	320	400	400	400	360	360	360	290	290	290	**$ 13,330**		
Solution	**0**	**5**	**10**	**7.33333**	**4.167**	**0.000**	**0**	**0**	**0**	**4.66667**	**8.333**	**0.000**			

3.4-17 a)

Let M = number of men's gloves to produce per week,
W = number of women's gloves to produce per week,
C = number of children's gloves to produce per week,
F = number of full-time workers to employ,
PT = number of part-time workers to employ.

Maximize $P = 8M + 10W + 6C - 13(40)F - 10(20)PT$

subject to
$$2M + 1.5W + C \leq 5000$$
$$30M + 45W + 40C \leq 40(60)F + 20(60)PT$$
$$F \geq 20$$
$$F \geq 2PT$$

and $M \geq 0,\ W \geq 0,\ C \geq 0,\ F \geq 0,\ PT \geq 0.$

b)

Resource	Men's	Women's	Children's	Full-Time	Part-Time	Totals		Resource Available
	Resource Usage Per Unit of Each Activity							
Leather	2	1.5	1	0	0	5000	≤	5000
Labor	30	45	40	-2400	-1200	0	≤	0
Full-Time Employees	0	0	0	1	0	25	≥	20
Employee Ratio	0	0	0	1	-2	0	≥	0
Unit Profit	8	10	6	-520	-200	**4500**		
Solution	**2500**	**0**	**0**	**25**	**12.5**			

3.4-18

a) Let x_{ij} = # hours operator i is assigned to work of day j

$(i = KC, OH, HB, SC, KS, NK \;;\; j = M, T, W, Th, F)$

$$\text{Minimize } Z = 10(x_{KC,M} + x_{KC,W} + x_{KC,F}) + 10.1(x_{OH,T} + x_{OH,Th})$$
$$+ 9.9(x_{HB,M} + x_{HB,T} + x_{HB,W} + x_{HB,F})$$
$$+ 9.8(x_{SC,M} + x_{SC,T} + x_{SC,W} + x_{SC,F})$$
$$+ 10.8(x_{KS,M} + x_{KS,W} + x_{KS,Th}) + 11.3(x_{NK,Th} + x_{NK,F})$$

subject to

$x_{KC,M} \le 6$	$x_{OH,M} \le 6$	$x_{HB,M} \le 4$
$x_{KC,W} \le 6$	$x_{OH,Th} \le 6$	$x_{HB,T} \le 8$
$x_{KC,F} \le 6$		$x_{HB,W} \le 4$
		$x_{HB,F} \le 4$
$x_{SC,M} \le 5$	$x_{KS,M} \le 3$	$x_{NK,Th} \le 6$
$x_{SC,T} \le 5$	$x_{KS,W} \le 3$	$x_{NK,F} \le 2$
$x_{SC,W} \le 5$	$x_{KS,Th} \le 8$	
$x_{SC,F} \le 5$		

$$x_{KC,M} + x_{KC,W} + x_{KC,F} \ge 8$$
$$x_{OH,T} + x_{OH,Th} \ge 8$$
$$x_{HB,M} + x_{HB,T} + x_{HB,W} + x_{HB,F} \ge 8$$
$$x_{SC,M} + x_{SC,T} + x_{SC,W} + x_{SC,F} \ge 8$$
$$x_{KS,M} + x_{KS,W} + x_{KS,Th} \ge 7$$
$$x_{NK,Th} + x_{NK,F} \ge 7$$
$$x_{KC,M} + x_{HB,M} + x_{SC,M} + x_{KS,M} = 14$$
$$x_{OH,T} + x_{HB,T} + x_{SC,T} = 14$$
$$x_{KC,W} + x_{HB,W} + x_{SC,W} + x_{KS,W} = 14$$
$$x_{OH,Th} + x_{KS,Th} + x_{NK,Th} = 14$$
$$x_{KC,F} + x_{HB,F} + x_{SC,F} + x_{NK,F} = 14$$
$$x_{ij} \ge 0 \quad \forall i, j$$

3.4-18 b)

Resource	KC,M	KC,W	KC,F	DH,Tu	DH,Th	HB,M	HB,Tu	HB,W	HB,F	SC,M	SC,Tu	SC,W	SC,F	KS,M	KS,W	KS,Th	NK,Th	NK,F	Totals		Resource Available
	Resource Usage Per Unit of Each Activity																				
KC Knowledge	1	1	1																9	≥	8
DH Knowledge				1	1														8	≥	8
HB Knowledge						1	1	1	1										19	≥	8
SC Knowledge										1	1	1	1						20	≥	8
KS Knowledge														1	1	1			7	≥	7
NK Knowledge																	1	1	7	≥	7
Mon Hours	1					1				1				1					14	≥	14
Tues Hours				1			1				1								14	≥	14
Wed Hours		1						1				1			1				14	≥	14
Thurs Hours					1											1	1		14	≥	14
Fri Hours			1						1				1					1	14	≥	14
Availability KC,M	1																		4	≤	6
Availability KC,W		1																	2	≤	6
Availability DH,Tu				1															2	≤	6
Availability DH,TH					1														6	≤	6
Availability HB,M						1													4	≤	4
Availability HB,Tu							1												7	≤	8
Availability HB,W								1											4	≤	4
Availability HB,F									1										4	≤	4
Availability SC,M										1									5	≤	5
Availability SC,Tu											1								5	≤	5
Availability SC,W												1							5	≤	5
Availability SC,F													1						6	≤	5
Availability KS,M														1					1	≤	3
Availability KS,W															1				3	≤	3
Availability KS,Th																1			3	≤	8
Availability NK,Th																	1		5	≤	6
Availability NK,F																		1	2	≤	2
Unit Cost	10	10	10	10.1	10.1	9.9	9.9	9.9	9.9	9.8	9.8	9.8	9.8	10.8	10.8	10.8	11.3	11.3	**710**		
Solution	**4**	**2**	**3**	**2**	**6**	**4**	**7**	**4**	**4**	**5**	**5**	**5**	**5**	**1**	**3**	**3**	**5**	**2**			

3.4-19

a) Let S = Tablespoons of strawberry flavoring,
CR = Tablespoons of cream,
V = Tablespoons of vitamin supplement,
A = Tablespoons of artificial sweetener,
T = Tablespoons of thickening agent,

Minimize $C = 10S + 8CR + 25V + 15A + 6T$

subject to

$$50S + 100CR + 120A + 80T \geq 380$$
$$50S + 100CR + 120A + 80T \leq 420$$
$$S + 75CR + 30T \leq 0.2(50S + 100C + 120A + 80T)$$
$$20S + 50V + 2T \geq 50$$
$$S \geq 2A$$
$$3S + 8CR + V + 2A + 25T = 15$$

and $S \geq 0,\ CR \geq 0,\ V \geq 0,\ A \geq 0,\ T \geq 0.$

Resource	Strawberry	Cream	Vitamin	Sweetener	Thickener	Totals		Available
	Resource Usage Per Unit							
Min Calories	50	100	0	120	80	380	≥	380
Max Calories	50	100	0	120	80	380	≤	420
Fat	-9	55	0	-24	14	-73	≤	0
Vitamin	20	0	50	0	2	128	≥	50
Taste	1	0	0	-2	0	0	≥	0
Thickness	3	8	1	2	25	15	=	15
Unit Cost	10	8	25	15	6	**90**		
Solution	**3.455**	**0**	**1**	**2**	**0**			

3.4-20 a) Let B = slices of bread,
P= Tablespoons of peanut butter,
S = Tablespoons of strawberry jelly,
G = graham crackers,
M = cups of milk,
J = cups of juice.

Minimize $C = 5B + 4P + 7S + 8G + 15M + 35J$

subject to

$$70B + 100P + 50S + 60G + 150M + 100J \geq 400$$
$$70B + 100P + 50S + 60G + 150M + 100J \leq 600$$
$$10B + 75P + 20G + 70M \leq .3(70B + 100P + 50S + 60G + 150M + 100J)$$
$$3S + 2M + 120J \geq 60$$
$$3B + 4P + G + 8M + J \geq 12$$
$$B = 2$$
$$P \geq 2S$$
$$M + J \geq 1$$

and $B \geq 0,\ P \geq 0,\ S \geq 0,\ G \geq 0,\ M \geq 0,\ J \geq 0.$

b)

Resource	Bread	PB	Jelly	Crackers	Milk	Juice	Totals		Resource Available
	Resource Usage Per Unit of Each Activity								
Min Calories	70	100	50	60	150	100	600	≥	400
Max Calories	70	100	50	60	150	100	600	≤	600
Fat	-11	45	-15	2	25	-30	-145	≤	0
Vitamin C	0	0	3	0	2	120	514	≥	60
Protein	3	4	0	1	8	1	12	≥	12
Bread	1	0	0	0	0	0	2	=	2
PB&J	0	1	-2	0	0	0	0	≥	0
Liquid	0	0	0	0	1	1	4	≥	1
Unit Cost	5	4	7	8	15	35	**163**		
Solution	**2**	**0**	**0**	**0**	**0**	**4**			

3. 5-1. a) The two factors which often hinder the use of optimization models by managers are cultural differences and response time. Cultural differences cause managers and model developers to often have a hard time understanding each other. Response time is often slow due to the time to translate, formulate and solve the manager's problem using optimization systems.

b) As stated in the article, "[b]ased on unit profits, the company in the past emphasized the manufacture of thicker plywoods (TYPEA), but the optimization procedure showed that in fact thinner plywoods (TYPEB) were more profitable. This product mix change has increased the overall profitability of the company by 20%."

c) The chapter before "Conclusions" describes this: "With the success of this application, management is now eager to use optimization for other problems, too. Since Ponderosa uses timber for products other than plywood, they intend to explore the optimum allocation of this raw material between different products. Raw material and inventory management is another potential area of application. Also, the conversion of the optimization model into the financial planning language will now facilitate the integration of financial models with production models."

d) The Xerox of the appendices:

APPENDIX A. MATHEMATICAL FORMULATION OF THE PROBLEM

X_{it} = Amount of product i in period t $(i = 1, \ldots, n)$, $(t = 1, \ldots, T)$
C_{it} = Contribution margin of product i in period t
Y_{jt} = Amount of veneer sheet type j produced in period t $(j = 1, \ldots, n_1)$
Z_{kt} = Amount of green veneer type k produced in period t $(k = 1, \ldots, n_2)$
C_{pt} = Consumption of log type p in period t $(p = 1, \ldots, n_3)$
F_{pt} = Final inventory of log type p in period t
S_{pt} = Supply of log type p in period t
P_i = Pressing hours required per unit of product i
G_i = Polishing hours required per unit of product i
A_t = Total polishing hours available in period t
B_t = Total pressing hours available in period t
D_{ij} = Amount of type j veneer sheet required per unit of product i
E_{pT} = Final inventory requirement coefficients for log type p in period T
Q_{kp} = Green veneer type k yield per unit of log type p
H_{kj} = Green veneer type k required per unit of veneer sheet of type j
U_{it} = Upper limit on market demand for product i in period t
L_{it} = Lower limit on market demand for product i in period t

Objective

$$\text{Maximize } \sum_{i,t} C_{it} X_{it}.$$

Constraints

Polishing capacity limited:

$$\sum_i G_i X_{it} \leqslant A_t, \quad t = 1, \ldots, T$$

Pressing capacity limited:

$$\sum_i P_i X_{it} \leqslant B_t, \quad t = 1, \ldots, T$$

Veneer sheet required:

$$\sum_i D_{ij} X_{it} = Y_{jt}, \quad j = 1, \ldots, n_1, \quad t = 1, \ldots, T$$

Material balance on log:

$$C_{pt} + F_{pt} - F_{pt-1} = S_{pt}, \quad p = 1, \ldots, n_3, \quad t = 1, \ldots, T$$

Final inventory composition requirement:

$$\sum_p E_{pT} F_{pT} = 0$$

Green veneer yield from logs:

$$\sum_p Q_{kp} C_{pt} = Z_{kt}, \quad k = 1, \ldots, n_2, \quad t = 1, \ldots, T$$

Green veneers required:

$$\sum_j H_{kj} Y_{jt} = Z_{kt}, \quad k = 1, \ldots, n_2, \quad t = 1, \ldots, T$$

Market constraints:

$$L_{it} \leqslant X_{it} \leqslant U_{it}.$$

35-1. d) (cont)

APPENDIX B. STRUCTURE OF THE LP MODEL

Vector Rows	Products by Grade and Thickness	Veneer Sheet Production	Green Veneer Production	Log Consumption	Final Log Inventory	Right Hand Side
Market Constraints	I					≤Limits Forecast by Sales
Veneer Requirements	X1	−I				=0
Pressing & Polishing Constraints	X2					≤ Production Capacity
Green Veneer Requirement		X3	−I +I			=0
Green Veneer Generated			−I	X4		=0
Material Balance				I	I	= Initial Inventory + Log Supply
Final Inventory Composition Requirements					X5	=0
Objective Function	Unit Contribution Margin					= Maximize Contribution Margin

3.5-2. a) The shift schedules at airports and reservations offices were done by hand prior to this study. (see paragraph 2, col 2, p. 42)

b) The project requirements (see bottom of col. 1, p. 42) were:
- i) to determine the needs for increased manpower,
- ii) to identify excess manpower for reallocation,
- iii) to reduce the time required for preparing schedules,
- iv) to make manpower allocation more day- and time-sensitve, and
- v) to quantify the cost associated with scheduling.

c) United Airlines dealt with the integrality problem by using "[a] heuristic rounding algorithm similar to that described in Henderson and Berry [1979, see article for reference] ... incorporated in the Report Module ... and serv[ing] to covert the Schduling Module's fractionated LP solution into a workable shift schedule." (see p. 45)

d) Flexibility was necessary to "[satisfy] the group culture at each office [which was] essential in garnering field support. As a result, office-specified input variables, such as the number of start times, the preferred shift lengths, the length of breaks, preferred days off combinations, and so forth, became an intergral part of SMPS. This versatility gave office managers the luxury of evaluating schedules incorporating different input parameters but identical manpower requirements." (p. 47)

e) Benefits included: (p.48)
- i) significant labor cost savings,
- ii) improved customer service,
- iii) improved emplyee schedules,
- iv) quantified manpower planning and evaluation,

(pp. 48-49 describes these in more detail)

3.5-3. a) During the years preceding this OR study, "the price of crude oil increased tenfold ... as the short-term interest rates more than tripled ... This meant that the cost associated with financing the working capital employed in the refining and marketing industry increased more than 30 fold during that time ... Con-sequently, it [became] vastly more important to maintain tight control over required working capital." (p. 2)

b) "Citgo's distribution network of pipelines, tankers, and barges span[ned] the eastern two-third of the United States. The addition of the Southland 7-Eleven store increase[d] their marketing and distribution network to all of the 48 contiguous United States." (p. 4)

c) An 11-week planning horizon, partitioned into 6 one-week periods and 1 five-week period, was used. Also, "the model [was] partitioned into time zones, and replications of the model [were] employed to represent the multiple time periods." (p. 6)

d) Citgo used "a medium-sized computer, an IBM 4381 and mod." Typical run times for "model generation, solution and reports [were] respectively two minutes, half a minute, and seven minutes."

3.5-3. e) The four types of model users were the product managers, the pricing manager, the product traders, and the budget manager. "Product manager compare[d] the model recommendations to the actual operational decisions to determine the existence and cause of discrepancies. They also use[d] the model's what-if capabilities to generate economically viable alternatives to current and forecasted operations ... The pricing manager use[d] the model in several ways. He use[d] one set of reports to set ranges for terminal prices for each product ... In addition, the pricing manager use[d] the wholesale report to help set prices and reommended volumes for bulk sales made to reduce excess inventories ... [Product] traders use[d] the Volume Summary Report and the Infeasibility Report to determine which side of the trading board they should be on for each product ... They [could] also use the model's what-if capabilities to determine the sensitivity of spot prices to the required purchases or sales volumes as prices fluctuate[d] during the week ... [T] he budget manager use[d] the Financial Summary Report to generate various components of the monthly and quarterly budgets." (p. 9)

f) The major reports generated by the SDM system are listed on page 8 of the article. They are:

i) Infeasibility report,
ii) In-transit, Terminal, Exchange, Inventory reports,
iii) Spot Recommendation report,
iv) Purchases, Sales, Trades reports,
v) Wholesale report,
vi) Volume Summary report,
vii) Financial Summary report.

g) "The education of the users was a challenge because both organizational responsibilities and people within the organizations flucutuated." (p. 10) "Another major implementation challenge concerned the collection, validation and correction of input data for the model." (p. 11) "A third implementation challenge concerned the forecasting sales volumes and wholesales prices. Citgo forcasted only total volumes for monthly and quarterly budgets. The SDM systems, on the other hand, required volume and price forecasts to be detailed by terminal, by product, by line of business, and by week." (p. 11)

h) "The major benefit realized from the SDM Model was the reduction in Citgo's product inventory with no drop in service levels ... Another direct benefit of implementing the model was the improvement in operational decision making: improvements in coordination, pricing and purchasing decisions, as well as the incorporation of the 'new concepts' into the decision process itself ... In addition to the direct benefits, several indirect benefits result[ed] from the modeling process and implementation ... One such indirect benefit [was] the establishment of a corporate data base which provide[d] common, up-to-date, on-line operational information for current decision support. Another such benefit [was] the utilization of a single forcast throughout the different departments, thus keeping the entire organization focused and traveling down a single path. A third benefit [was] the closed-loop planning process fostered by the continual feedback provided by the product manager when comparing actual decision to model-recommended decision ... A fourth benefit [was] the increased interdepartmental communication the model's use has fostered at Citgo ... A final benefit [was] the insight gained from the modeling process itself." (pp. 15-16)

3.6-1

a)

Maximize $P = 20x_1 + 30x_2$,

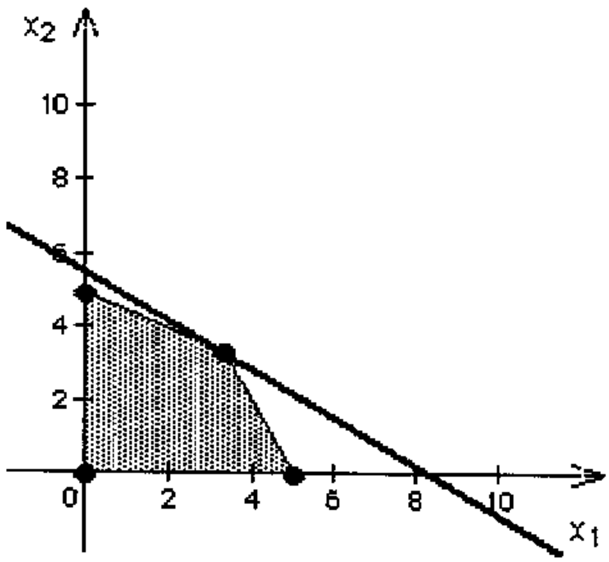
subject to $2x_1 + x_2 \le 10$

$3x_1 + 3x_2 \le 20$

$2x_1 + 4x_2 \le 20$

and $x_1 \ge 0,\ x_2 \ge 0.$

b) Optimal Solution: $(x_1, x_2) = \left(3\frac{1}{3}, 3\frac{1}{3}\right)$ and $P = 166.67$.



c,e)

Resource	Resource Usage Per Unit of Each Activity: Activity 1	Activity 2	Totals		Resource Available
1	2	1	10	≤	10
2	3	3	20	≤	20
3	2	4	20	≤	20
Unit Profit	20	30	**$ 166.67**		
Solution	**3.333**	**3.333**			

d)

(x_1, x_2)	Feasible?	P	
(2,2)	Yes	$100	
(3,3)	Yes	$150	
(2,4)	Yes	$160	Best
(4,2)	Yes	$140	
(3,4)	No		
(4,3)	No		

3.6-2

a)

Maximize $P = 50A + 40B + 30C$,

subject to $.02A + .03B + .05C \le 40$

$.05A + .02B + .04C \le 40$

and $A \ge 0,\ B \ge 0,\ C \ge 0.$

b & d)

Resource	Resource Usage Per Unit of Each Activity Part A	Part B	Part C	Totals		Resource Available
Machine 1	0.02	0.03	0.05	40	≤	40
Machine 2	0.05	0.02	0.04	40	≤	40
Unit Profit	50	40	30	**$ 61,818.18**		
Solution	**363.636**	**1090.909**	**0**			

c)

(x_1, x_2, x_3)	Feasible?	P	
(500,500,300)	No		
(350,1000,0)	Yes	$57,500	
(400,1000,0)	Yes	$60,000	Best

Many answers are possible.

3.6-3

a)

Minimize $C = 60x_1 + 50x_2$,

subject to $5x_1 + 3x_2 \ge 60$

$2x_1 + 2x_2 \ge 30$

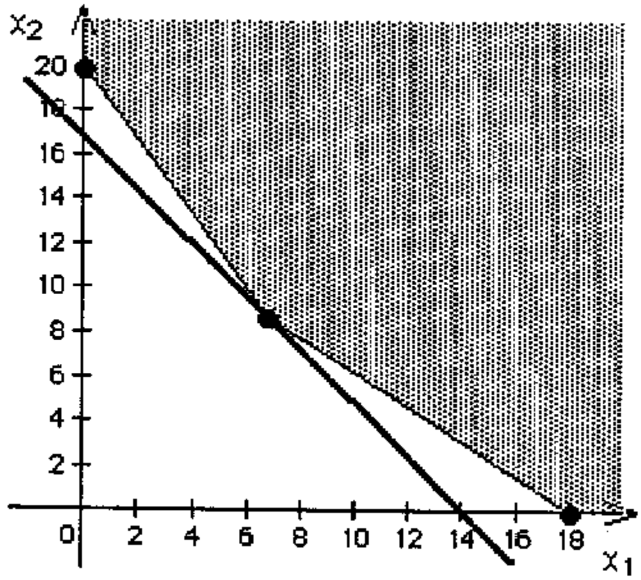
$7x_1 + 9x_2 \ge 126$

and $x_1 \ge 0,\ x_2 \ge 0.$

3.6-3

b)

Optimal Solution: $(x_1, x_2) = (6.75, 8.75)$ and $C = 842.50$.



c &e)

Benefit	Benefit Contribution Per Unit of Each Activity: Activity 1	Activity 2	Totals		Minimum Level
1	5	3	60	≥	60
2	2	2	31	≥	30
3	7	9	126	≥	126
Unit Cost	60	50	**$ 842.50**		
Solution	**6.75**	**8.75**			

d)

(x_1, x_2)	Feasible?	C	
(7,7)	No		
(7,8)	No		
(8,7)	No		
(8,8)	Yes	$880	Best
(8,9)	Yes	$930	
(9,8)	Yes	$940	

3.6-4

a)

Minimize $C = 84C + 72T + 60A$,

subject to $90C + 20T + 40A \geq 200$

$30C + 80T + 60A \geq 180$

$10C + 20T + 60A \geq 150$

and $C \geq 0,\ T \geq 0,\ A \geq 0.$

b & e)

Nutritional Ingredient	Corn	Kilogram of Tankage	Alfalfa	Totals		Minimum Level
Carbohydrates	90	20	40	200	≥	200
Proteins	30	80	60	180	≥	180
Vitamins	10	20	60	157	≥	150
Unit Cost	84	72	60	**$ 242**		
Solution	**1**	**0**	**2**			

c) $(x_1, x_2, x_3) = (1,2,2)$ is a feasible solution with a daily cost of $348.00. This diet will provide 210 kg of carbohydrates, 310 kg of protein, and 170 kg of vitamins daily.

d) Answers will vary.

3.6-5

a)

Minimize $C = x_1 + x_2 + x_3$,

subject to $2x_1 + x_2 + .5x_3 \geq 400$

$.5x_1 + .5x_2 + x_3 \geq 100$

$1.5x_2 + 2x_3 \geq 300$

and $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$

b & e)

Benefit Contribution Per Unit of Each Asset

Year	Asset 1	Asset 2	Asset 3	Totals		Minimum Cash Flow Required
5	2	1	0.5	400	≥	400
10	0.5	0.5	1	150	≥	100
20	0	1.5	2	300	≥	300
Unit Cost	1	1	1	**$ 300**		
Solution	**100**	**200**	**0**			

c) $(x_1, x_2, x_3) = (100,100,200)$ is a feasible solution. This would generate $400 million in 5 years, $300 million in 10 years, and $550 million in 20 years. The total invested will be $400 million.

d) Answers will vary.

3.7-1

a-

Notation

i: products

j: months

k: plants

l: processes

m: regions

X_{ijklm}: amount produced, of product i, in month j, in plant k, using process l, to be sold in region m.

D_{ijm}: Demand of product i in month j in region m

C_{ikl}: unit production cost of product i in plant k using process l

R_{ikl}: production rate of product i in plant k using process l.

S_{im} amount stored to be sold in March, of product i in region m.

P_i: Selling price of product i

$T_{i,k,m}$: transportation cost of product i, produced in plant k, to be sold in region m.

A_j: days available for production in month j.

L: storage limit

M_i: storage cost (per unit) of product i.

(CONT'D)

3.7-1 a) (CONT'D)

The objective is to:

MAX Profit = Revenue - Production Cost
- Inventory Cost
- Transportation Cost

So

$$\text{Profit} = \sum_i P_i \Big(\sum_{j,k,l,m} x_{ijklm} \Big) - \sum_{i,k,l} C_{ikl} \Big(\sum_{j,m} x_{ijklm} \Big)$$

$$- \sum_i M_i \Big(\sum_m S_{im} \Big) - \sum_{i,k,m} T_{ikm} \Big(\sum_{j,l} x_{ijklm} \Big)$$

Subject to the constraints:

$$\sum_{k,l} x_{ijklm} - S_{im} \le D_{ijm} \; ; \text{ For } j = \text{Feb.}, \; i = 1,2, \; m = 1,2$$

$$\sum_{k,l} x_{ijklm} + S_{im} \le D_{ijm} \, , \text{ For } j = \text{March}, \; i = 1,2, \; m = 1,2$$

$$\sum_i S_{im} \le L \quad \text{for } m = 1,2$$

$$\sum_{i,l} \frac{1}{R_{ikl}} \Big(\sum_m x_{ijklm} \Big) \le A_j \quad \text{for } j = \text{Feb, March and } k = 1,2.$$

$$x_{i,j,k,l,m} \ge 0 \quad \text{for } i = 1,2 \, ; j = F, M \, ; k = 1,2 \, ; l = 1,2 \, ; m = 1,2.$$

3.7-1 b)

Quantity produced to be sold in the same region with process 1

Product	Plant 1 February	Plant 1 March	Plant2 February	Plant2 March
1	0	0	0	0
2	2400	2760	3200	3680

Quantity produced to be sold in the other region with process 1

Product	Plant 1 February	Plant 1 March	Plant2 February	Plant2 March
1	0	0	0	0
2	0	0	0	0

Quantity produced to be sold in the same region with process 2

Product	Plant 1 February	Plant 1 March	Plant2 February	Plant2 March
1	0	0	0	0
2	0	0	0	0

Quantity produced to be sold in the other region with process 2

Product	Plant 1 February	Plant 1 March	Plant2 February	Plant2 March
1	0	0	0	0
2	0	0	0	0

Days available

February	20
March	23

Storage Limit

1000

		Total Profit
Revenue	1348480	$333,680
Prod Cost	1014800	
Transp Cost	0	
Storage Cost	0	

3-47

Demand

Product	Region1 February	Region1 March	Region2 February	Region2 March
1	3600	6300	4900	4200
2	4500	5400	5100	6000

Production costs

Product	Plant 1 Process 1	Plant 1 Process 2	Plant2 Process 1	Plant2 Process 2
1	$ 62	$ 59	$ 61	$ 65
2	$ 78	$ 85	$ 89	$ 86

Production rates

Product	Plant 1 Process 1	Plant 1 Process 2	Plant2 Process 1	Plant2 Process 2
1	100	140	130	110
2	120	150	160	130

Transportation Costs

Region	Product 1: 1	Product 1: 2	Product 2: 1	Product 2: 2
1	0	9	0	7
2	9	0	7	0

Product	Revenue
1	$83
2	$112

Product	Storage cost
1	$ 3
2	$ 4

Stored quantity

Region	Product 1	Product 2
1	0	0
2	0	0

Demand satisfied

Product	Plant 1 February	Plant 1 March	Plant2 February	Plant2 March
1	0	0	0	0
2	2400	2760	3200	3680

Capacity used

	Plant1	Plant2
Feb	20	20
Mar	23	23

Amount stored

Plant1	0
Plant2	0

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c-

```
{    Problem 3.7-1 in MPL    }

TITLE
    ManufacturingProblem;

INDEX
    product = (pr1,pr2);
    month = (feb,mar);
    plant = (pl1,pl2);
    process = (ps1,ps2);
    region = (r1,r2);

DATA
    demand[product,month,region] := (3600,4900,
                       6300,4200,
                       4500,5100,
                       5400,6000);
    days[month] := (20,23);
    storagecost[product] := (3,4);
    prodcost[product,plant,process] := (62,59,
                           61,65,
                           78,85,
                           89,86);
    rate[product,plant,process] := (100,140,
                     130,110,
                     120,150,
                     160,130);
    price[product] := (83,112);
    transpcost[product,plant,region] := (0,9,
                           9,0,
                           0,7,
                           7,0);

DECISION VARIABLES
    Volume[product,month,plant,process,region];
    Store[product,region];

MACRO

    Revenues := SUM(product,month,plant,process,region: price*Volume);
    ProductionCost := SUM(product,plant,process,month,region: prodcost*Volume);
    TransportationCost := SUM(product,plant,region,month,process: transpcost*Volume);
    StorageCost := SUM(product,region: storagecost*Store);

MODEL

   MAX  TotalProfit = Revenues - ProductionCost - TransportationCost - StorageCost;

SUBJECT TO
    SalesFeb[product,region,month] where(month=feb) : SUM(plant,process: Volume - Store) <= demand;
    SalesMar[product,region,month] where(month=mar) : SUM(plant,process: Volume + Store) <= demand;
    StorageLimit[region] : SUM(product: Store) <= 1000;
    Capacity[plant,month] : SUM(product,process,region: Volume/rate) <= days;

END
```

(CONT'D)

3.7-1 c) CONT'D)

SOLUTION RESULT

Optimal solution found

MAX TotalPro = 333680.0000

MACROS

Macro Name	Values
Revenues	1348480.0000
ProductionCost	1014800.0000
TransportationCost	0.0000
StorageCost	0.0000

DECISION VARIABLES

VARIABLE Volume[product,month,plant,process,region] :

product	month	plant	process	region	Activity	Reduced Cost
pr1	feb	pl1	ps1	r1	0.0000	-19.8000
pr1	feb	pl1	ps1	r2	0.0000	-28.8000
pr1	feb	pl1	ps2	r1	0.0000	-5.1429
pr1	feb	pl1	ps2	r2	0.0000	-14.1429
pr1	feb	pl2	ps1	r1	0.0000	-15.3077
pr1	feb	pl2	ps1	r2	0.0000	-6.3077
pr1	feb	pl2	ps2	r1	0.0000	-24.4545
pr1	feb	pl2	ps2	r2	0.0000	-15.4545
pr1	mar	pl1	ps1	r1	0.0000	-19.8000
pr1	mar	pl1	ps1	r2	0.0000	-28.8000
pr1	mar	pl1	ps2	r1	0.0000	-5.1429
pr1	mar	pl1	ps2	r2	0.0000	-14.1429
pr1	mar	pl2	ps1	r1	0.0000	-15.3077
pr1	mar	pl2	ps1	r2	0.0000	-6.3077
pr1	mar	pl2	ps2	r1	0.0000	-24.4545
pr1	mar	pl2	ps2	r2	0.0000	-15.4545
pr2	feb	pl1	ps1	r1	2400.0000	0.0000
pr2	feb	pl1	ps1	r2	0.0000	-7.0000
pr2	feb	pl1	ps2	r1	0.0000	-0.2000
pr2	feb	pl1	ps2	r2	0.0000	-7.2000
pr2	feb	pl2	ps1	r1	0.0000	-7.0000
pr2	feb	pl2	ps1	r2	3200.0000	0.0000
pr2	feb	pl2	ps2	r1	0.0000	-9.3077
pr2	feb	pl2	ps2	r2	0.0000	-2.3077
pr2	mar	pl1	ps1	r1	2760.0000	0.0000
pr2	mar	pl1	ps1	r2	0.0000	-7.0000
pr2	mar	pl1	ps2	r1	0.0000	-0.2000
pr2	mar	pl1	ps2	r2	0.0000	-7.2000
pr2	mar	pl2	ps1	r1	0.0000	-7.0000
pr2	mar	pl2	ps1	r2	3680.0000	0.0000
pr2	mar	pl2	ps2	r1	0.0000	-9.3077
pr2	mar	pl2	ps2	r2	0.0000	-2.3077

(CONT'D)

3.7-1 c) CONT'D]

VARIABLE Store[product,region] :

product	region	Activity	Reduced Cost
pr1	r1	0.0000	-3.0000
pr1	r2	0.0000	-3.0000
pr2	r1	0.0000	-4.0000
pr2	r2	0.0000	-4.0000

d)

```
MODEL:
: Problem 3.7-1 - Lingo Version;
SETS:
PRODUCT/PR1 PR2/: PRICE, STORAGECOST;
MONTH/FEB MAR/: DAYS;
PLANT/PL1 PL2/;
PROCESS/PS1 PS2/;
REGION/R1 R2/;
LINK1(PRODUCT,MONTH,PLANT,PROCESS,REGION): VAR;
LINK2(PRODUCT,MONTH,REGION): DEMAND;
LINK3(PRODUCT,PLANT,PROCESS): PRODCOST;
LINK4(PRODUCT,PLANT,PROCESS): RATE;
LINK5(PRODUCT,REGION): STORE;
LINK6(PRODUCT,PLANT,REGION): TRANSPCOST;
ENDSETS

!OBJECTIVE FUNCTION;
MAX = @SUM(PRODUCT(I): PRICE(I)*@SUM(MONTH(J): @SUM(PLANT(K): @SUM(PROCESS(L):
@SUM(REGION(M): VAR(I,J,K,L,M)))))) - @SUM(LINK3(I,K,L): PRODCOST(I,K,L)*@SUM(MONTH(J):
@SUM(REGION(M): VAR(I,J,K,L,M)))) - @SUM(PRODUCT(I): STORAGECOST(I)*@SUM(REGION(M):
STORE(I,M))) - @SUM(LINK6(I,K,M): TRANSPCOST(I,K,M)*@SUM(MONTH(J): @SUM(PROCESS(L):
VAR(I,J,K,L,M))));

!CONSTRAINTS;
@FOR(PRODUCT(I): @FOR(REGION(M): @SUM(PLANT(K): @SUM(PROCESS(L): VAR(I,FEB,K,L,M))) -
STORE(I,M) <= DEMAND(I,FEB,M)));
@FOR(PRODUCT(I): @FOR(REGION(M): @SUM(PLANT(K): @SUM(PROCESS(L): VAR(I,MAR,K,L,M))) +
STORE(I,M) <= DEMAND(I,MAR,M)));
@FOR(REGION(M): @SUM(PRODUCT(I): STORE(I,M))<=1000);
@FOR(PLANT(K): @FOR(MONTH(J): @SUM(PRODUCT(I): @SUM(PROCESS(L):
(1/RATE(I,K,L))*@SUM(REGION(M): VAR(I,J,K,L,M)))) <= DAYS(J)));

!DATA PART;
DATA:
DEMAND = 3600 4900
         6300 4200
         4500 5100
         5400 6000;
DAYS = 20 23;
STORAGECOST = 3 4;
PRODCOST = 62 59
           61 65
           78 85
           89 86;
RATE = 100 140
       130 110
       120 150
       160 130;
PRICE = 83 112;
TRANSPCOST = 0 9
             9 0
             0 7
             7 0;
ENDDATA
END
```

3.7-1 d) CONT'D)

```
Global optimal solution found at step:              8
Objective value:                             333680.0

                    Variable              Value

      VAR( P1, FEB, P1, P1, R1)        0.0000000
      VAR( P1, FEB, P1, P1, R2)        0.0000000
      VAR( P1, FEB, P1, P2, R1)        0.0000000
      VAR( P1, FEB, P1, P2, R2)        0.0000000
      VAR( P1, FEB, P2, P1, R1)        0.0000000
      VAR( P1, FEB, P2, P1, R2)        0.0000000
      VAR( P1, FEB, P2, P2, R1)        0.0000000
      VAR( P1, FEB, P2, P2, R2)        0.0000000
      VAR( P1, MAR, P1, P1, R1)        0.0000000
      VAR( P1, MAR, P1, P1, R2)        0.0000000
      VAR( P1, MAR, P1, P2, R1)        0.0000000
      VAR( P1, MAR, P1, P2, R2)        0.0000000
      VAR( P1, MAR, P2, P1, R1)        0.0000000
      VAR( P1, MAR, P2, P1, R2)        0.0000000
      VAR( P1, MAR, P2, P2, R1)        0.0000000
      VAR( P1, MAR, P2, P2, R2)        0.0000000
      VAR( P2, FEB, P1, P1, R1)         2400.000
      VAR( P2, FEB, P1, P1, R2)        0.0000000
      VAR( P2, FEB, P1, P2, R1)        0.0000000
      VAR( P2, FEB, P1, P2, R2)        0.0000000
      VAR( P2, FEB, P2, P1, R1)        0.0000000
      VAR( P2, FEB, P2, P1, R2)         3200.000
      VAR( P2, FEB, P2, P2, R1)        0.0000000
      VAR( P2, FEB, P2, P2, R2)        0.0000000
      VAR( P2, MAR, P1, P1, R1)         2760.000
      VAR( P2, MAR, P1, P1, R2)        0.0000000
      VAR( P2, MAR, P1, P2, R1)        0.0000000
      VAR( P2, MAR, P1, P2, R2)        0.0000000
      VAR( P2, MAR, P2, P1, R1)        0.0000000
      VAR( P2, MAR, P2, P1, R2)         3680.000
      VAR( P2, MAR, P2, P2, R1)        0.0000000
      VAR( P2, MAR, P2, P2, R2)        0.0000000

                    STORE( P1, R1)     0.0000000
                    STORE( P1, R2)     0.0000000
                    STORE( P2, R1)     0.0000000
                    STORE( P2, R2)     0.0000000
```

3.7-2

a)

```
MAX

    50x1+20x2+25x3 ;

SUBJECT TO

    9x1+3x2+5x3<=500 ;
    5x1+4x2<=350;
    3x1+2x3<=150;
    x3<=20;

END
```

Variable Name	Activity
x1	26.1905
x2	54.7619
x3	20.0000

b)

```
max = 50*x1+20*x2+25*x3;
9*x1+3*x2+5*x3<=500;
5*x1+4*x2<=350;
3*x1+2*x3<=150;
x3<=20;
x1>=0; x2>=0; x3>=0;
```

```
Global optimal solution found at step:          4
Objective value:                         2904.762

                    Variable           Value
                          X1        26.19048
                          X2        54.76190
                          X3        20.00000
```

3.7-3

a)

```
TITLE
    TransportationProblem;

INDEX
    supply = (Wh1,Wh2);
    dest   = (C1,C2,C3);

DATA
    MaxCapacity[supply]  := (400,500);
    Required[dest]       := (300,200,400);

    ShippingCost[supply,dest] := (600,800,700,
                   400,900,600);


DECISION VARIABLES
    VolumeShipped[supply,dest]  ->  ""


MODEL

    MIN  TotalCost = SUM(supply,dest: ShippingCost * VolumeShipped);

SUBJECT TO

    Capacity[supply] : SUM(dest: VolumeShipped)    =  MaxCapacity ;
    Demand[dest]     : SUM(supply: VolumeShipped)  =  Required ;

END
```

5)

```
MODEL:
! Problem 3.7-3 - Lingo Version;
SETS:
      FACTORIES /F1 F2/: CAPACITY;
      CUSTOMERS /C1 C2 C3/: DEMAND;
      LINKS(FACTORIES, CUSTOMERS): COST, VOLUME;
ENDSETS
[OBJECTIVE] MIN = @SUM(LINKS(I,J):COST(I,J)*VOLUME(I,J));
!DEMAND CONSTRAINTS;
@FOR(CUSTOMERS(J): @SUM(FACTORIES(I): VOLUME(I,J))=DEMAND(J));
!SUPPLY CONSTRAINTS;
@FOR(FACTORIES(I): @SUM(CUSTOMERS(J):VOLUME(I,J))=CAPACITY(I));


!HERE IS THE DATA;
DATA:
CAPACITY = 400 500;
DEMAND = 300 200 400;
COST = 600 800 700
       400 200 400;
ENDDATA
END
```

```
Global optimal solution found at step:          2
Objective value:                         410000.0

                Variable           Value

          VOLUME( F1, C1)       300.0000
          VOLUME( F1, C2)      0.0000000
          VOLUME( F1, C3)       100.0000
          VOLUME( F2, C1)      0.0000000
          VOLUME( F2, C2)       200.0000
          VOLUME( F2, C3)       300.0000
```

3.1-4
a)

```
TITLE
    TransportationProblem;

INDEX
    Factories = (F1,F2,F3);
    Models    = (L,M,S);

DATA
    Capacity[Factories]  := (750,900,450);
    Space[Factories]     := (13000,12000,5000);
    Sales[Models]      := (900,1200,750);
    Size[Models]       := (20,15,12);
    Profit[Factories,Models] := (420,360,300,
                420,360,300,
                420,360,300);


DECISION VARIABLES

    Volume[Factories,Models]  ->  ""

MODEL

    MAX  TotalProfit = SUM(Factories,Models: Profit * Volume);

SUBJECT TO

    Cap[Factories] : SUM(Models: Volume)    <=  Capacity ;
    Sal[Models]    : SUM(Factories: Volume)  <=  Sales ;
    Spa[Factories] : SUM(Models: Size*Volume)  <= Space;
    Ratio1 : SUM(Models: Volume[F1]/Capacity[F1]-Volume[F2]/Capacity[F2])  = 0;
    Ratio2 : SUM(Models: Volume[F1]/Capacity[F1]-Volume[F3]/Capacity[F3])  = 0;

END
```

```
SOLUTION RESULT

  Optimal solution found

    MAX TotalPro =      696000.0000



DECISION VARIABLES

VARIABLE Volume[Factories,Models] :
```

Factories	Models	Activity
F1	L	516.6667
F1	M	177.7778
F1	S	0.0000
F2	L	0.0000
F2	M	666.6667
F2	S	166.6667
F3	L	0.0000
F3	M	0.0000
F3	S	416.6667

3.7-4

5)

```
MODEL:
! Problem 3.7-4 - Lingo Version;
SETS:
      FACTORIES /F1 F2 F3/: CAPACITY, SPACE, SALES;
      MODELS /L M S/: DEMAND, SIZE;
      LINKS(FACTORIES, MODELS): PROFIT, VOLUME;
ENDSETS

[OBJECTIVE] MAX = @SUM(LINKS(I,J):PROFIT(I,J)*VOLUME(I,J));

!CAPACITY CONSTRAINTS;
@FOR(FACTORIES(I): @SUM(MODELS(J):VOLUME(I,J))<=CAPACITY(I));

!SPACE CONSTRAINTS;
@FOR(FACTORIES(I): @SUM(MODELS(J):SIZE(J)*VOLUME(I,J))<=SPACE(I));

!SALES CONSTRAINTS;
@FOR(MODELS(J): @SUM(FACTORIES(I):VOLUME(I,J))<=SALES(J));

!PROPORTIONALITY CONSTRAINTS;
@SUM(MODELS(J):VOLUME(1,J)/CAPACITY(1)-VOLUME(2,J)/CAPACITY(2))=0;
@SUM(MODELS(J):VOLUME(1,J)/CAPACITY(1)-VOLUME(3,J)/CAPACITY(3))=0;


!HERE IS THE DATA;
DATA:
CAPACITY = 750 900 450;
SPACE = 13000 12000 5000;
SALES = 900 1200 750;
DEMAND = 900 1200 750;
SIZE = 20 15 12;
PROFIT = 420 360 300
         420 360 300
         420 360 300;
ENDDATA
END
```

```
        Global optimal solution found at step:            16
        Objective value:                            696000.0

              VOLUME( F1, L)          516.6667
              VOLUME( F1, M)          177.7778
              VOLUME( F1, S)         0.0000000
              VOLUME( F2, L)         0.0000000
              VOLUME( F2, M)          666.6667
              VOLUME( F2, S)          166.6667
              VOLUME( F3, L)         0.0000000
              VOLUME( F3, M)         0.0000000
              VOLUME( F3, S)          416.6667
```

4.7-5

a)

```
TITLE
    TransportationProblem;

INDEX
    student  = (KC,OH,HB,SC,KS,NK);
    day = (M,TU,W,TH,F);

DATA

    Wage[student]        :=(10,10.1,9.9,9.8,10.8,11.3);
    Gender[student]          := (0,0,0,0,1,1);
    Available[student,day] := (6,0,6,0,6,
                   0,6,0,6,0
                   4,8,4,0,4
                   5,5,5,0,5
                   3,0,3,8,0
                   0,0,0,6,2);


DECISION VARIABLES

    Work[student,day]   ->   ""

MODEL

    MIN  TotalCost = SUM(student,day: Wage * Work);

SUBJECT TO

    TimeConstraint[student,day] : Work <= Available ;
    MinimumWork0[student] where(Gender=0) : SUM(day: Work) >=8 ;
    MinimumWork1[student] where(Gender=1) : SUM(day: Work) >=7 ;
    AlwaysOpen[day] : SUM(student: Work) = 14 ;
END
```

MIN TotalCos =   709.6000

VARIABLE Work[student,day] :

student	day	Activity
KC	M	4.0000
KC	TU	0.0000
KC	W	2.0000
KC	TH	0.0000
KC	F	3.0000
OH	M	0.0000
OH	TU	2.0000
OH	W	0.0000
OH	TH	6.0000
OH	F	0.0000
HB	M	4.0000
HB	TU	7.0000
HB	W	4.0000
HB	TH	0.0000
HB	F	4.0000
SC	M	5.0000
SC	TU	5.0000
SC	W	5.0000
SC	TH	0.0000
SC	F	5.0000
KS	M	1.0000
KS	TU	0.0000
KS	W	3.0000
KS	TH	3.0000
KS	F	0.0000
NK	M	0.0000
NK	TU	0.0000
NK	W	0.0000
NK	TH	5.0000
NK	F	2.0000

4.7-5

b)

```
MODEL:
! Problem 3.7-5 - Lingo Version;
SETS:
      STUDENTS /KC OH HB SC KS NK/: WAGE, GENDER;
      DAYS /M TU W TH F/;
      LINKS(STUDENTS, DAYS): AVAILABLE, WORK;
ENDSETS

[OBJECTIVE] MIN = @SUM(LINKS(I,J):WAGE(I)*WORK(I,J));

!TIME CONSTRAINTS;
@FOR(LINKS(I,J): WORK(I,J)<=AVAILABLE(I,J));

!MINIMUM WORK CONSTRAINTS;
@FOR(STUDENTS(I)|GENDER(I) #EQ# 0: @SUM(LINKS(I,J):WORK(I,J))>=8);
@FOR(STUDENTS(I)|GENDER(I) #EQ# 1: @SUM(LINKS(I,J):WORK(I,J))>=7);

!ALWAYS OPEN CONSTRAINTS;
@FOR(DAYS(J): @SUM(LINKS(I,J): WORK(I,J))=14);

!HERE IS THE DATA;
DATA:
WAGE = 10 10.1 9.9 9.8 10.8 11.3;
GENDER = 0 0 0 0 1 1;
AVAILABLE=6 0 6 0 6
          0 6 0 6 0
          4 8 4 0 4
          5 5 5 0 5
          3 0 3 8 0
          0 0 0 6 2;
ENDDATA
END
```

WORK(KC, M)	2.000000	WORK(SC, M)	5.000000
WORK(KC, TU)	0.0000000	WORK(SC, TU)	5.000000
WORK(KC, W)	3.000000	WORK(SC, W)	5.000000
WORK(KC, TH)	0.0000000	WORK(SC, TH)	0.0000000
WORK(KC, F)	4.000000	WORK(SC, F)	5.000000
WORK(OH, M)	0.0000000	WORK(KS, M)	3.000000
WORK(OH, TU)	2.000000	WORK(KS, TU)	0.0000000
WORK(OH, W)	0.0000000	WORK(KS, W)	2.000000
WORK(OH, TH)	6.000000	WORK(KS, TH)	2.000000
WORK(OH, F)	0.0000000	WORK(KS, F)	0.0000000
WORK(HB, M)	4.000000	WORK(NK, M)	0.0000000
WORK(HB, TU)	7.000000	WORK(NK, TU)	0.0000000
WORK(HB, W)	4.000000	WORK(NK, W)	0.0000000
WORK(HB, TH)	0.0000000	WORK(NK, TH)	6.000000
WORK(HB, F)	4.000000	WORK(NK, F)	1.000000

3.7-6 a)

```
MODEL

    MIN  84c+72t+60a;

SUBJECT TO

90c+20t+40a>=200;
30c+80t+60a>=180;
10c+20t+60a>=150;

END
```

```
SOLUTION RESULT

  Optimal solution found

    MIN Z          =         241.7143


DECISION VARIABLES

PLAIN VARIABLES

   Variable Name                 Activity
  ----------------------------------------
   c                               1.1429
   t                               0.0000
   a                               2.4286
  ----------------------------------------
```

b)

```
! Problem 3.7-6 - Lingo Version;

[OBJECTIVE] MIN = 84*C+72*T+60*A;

!CONSTRAINTS;

90*C+20*T+40*A>=200;
30*C+80*T+60*A>=180;
10*C+20*T+60*A>=150;
```

```
Global optimal solution found at step:            8
Objective value:                           241.7143

                 Variable           Value
                        C        1.142857
                        T       0.0000000
                        A        2.428571
```

3.7-7 a)

```
MODEL

    MIN  x1+x2+x3;

SUBJECT TO

2x1+x2+0.5x3>=400;
0.5x1+0.5x2+x3>=100;
1.5x2+2x3>=300;

END
```

```
SOLUTION RESULT

  Optimal solution found

   MIN Z          =         300.0000

   Variable Name              Activity
  -------------------------------------
   x1                         100.0000
   x2                         200.0000
   x3                           0.0000
  -------------------------------------
```

3.7-7 b)

```
! Problem 3.7-7 - Lingo Version;

[OBJECTIVE] MIN = X+Y+Z;

!CONSTRAINTS;

2*X+Y+0.5*Z>=400;
0.5*X+0.5*Y+Z>=100;
1.5*Y+2*Z>=300;
```

```
Global optimal solution found at step:            8
Objective value:                           300.0000

                    Variable           Value
                           X        100.0000
                           Y        200.0000
                           Z       0.0000000
```

```
Global optimal solution found at step:           21
Objective value:                           709.6000
```

3.7-8

a) let

i : 1,...,10 denote mills

j : 1,...,1000 denote customers

k : 1,...,5 type of paper

l : 1,2,3 machines

m : 1,...,4 materials

Now define:

$x_{i,j,k,l}$: amount of paper k to be made at papermill i on machine type l shipped to customer j.

$P_{i,k,l}$: Production cost per unit made in mill i using paper k and machine l

$T_{i,j,k}$: Transportation cost (per unit) for paper k from mill i to customer j

$D_{j,k}$: Demand of paper k by customer j

$F_{k,l,m}$: number of units of material m needed to produce 1 unit of paper type k on machine l.

$\bar{C}_{i,m}$: units of material m available in mill i

$\hat{F}_{k,l}$: capacity units of machine type l that will produce 1 unit of paper type K.

$\hat{C}_{i,l}$: capacity units of machine type l available in paper mill i

(CONTD)

(CONT'D)

with these definitions our LP becomes:

$$\text{minimize} \sum_{i,k,l} P_{i,k,l} \left(\sum_j x_{i,j,k,l} \right) + \sum_{i,j,k} T_{i,j,k} \left(\sum_l x_{i,j,k,l} \right)$$

subject to:

Demand: $\sum_{i,l} x_{i,j,k,l} \geq D_{j,k}$ $\quad j: 1, \ldots, 1000$, $k: 1, \ldots, 5$

Raw Material:

$$\sum_{k,l} \bar{r}_{k,l,m} \left(\sum_j x_{i,j,k,l} \right) \leq \bar{c}_{i,m} \qquad i: 1, \ldots, 10 \quad m: 1, \ldots, 4$$

Capacity:

$$\sum_k \hat{r}_{k,l} \left(\sum_j x_{i,j,k,l} \right) \leq \hat{c}_{i,l} \qquad i: 1, \ldots, 10 \quad l: 1, 2, 3$$

$$x_{i,j,k,l} \geq 0 \quad \forall i, j, k, l.$$

Notice that

$\sum_l x_{i,j,k,l}$: amount of paper k shipped to customer j from mill i.

and $\sum_j x_{i,j,k,l}$: amount of paper k made with machine l in mill i

b) There are 5,070 functional constraints and 150,000 variables.

c)

```
{   Problem 3.7-8   }

TITLE
    PaperManufacturing;

INDEX
    mill = 1..10;
    customer = 1..1000;
    machine = 1..3;
    material = 1..4;
    paper = 1..5;

DATA
    Required[customer,paper] = DATAFILE(Required.dat);
    Rate1[paper,machine,material] = DATAFILE(Rate1.dat);
    RawMaterial[mill,material] = DATAFILE(RawMaterial.dat);
    Rate2[paper,machine] = DATAFILE(Rate2.dat);
    MaxCapacity[mill,machine] = DATAFILE(MaxCapacity.dat);
    ProdCost[mill,paper,machine] = DATAFILE(ProdCost);
    TranspCost[mill,customer,paper] = DATAFILE(TranspCost);

DECISION VARIABLES
    Quantity[mill,customer,machine,paper]  ->  ""

MODEL

    MIN  TotalCost = SUM(mill,customer,machine,paper: ProdCost * Quantity)
        + SUM(mill,customer,machine,paper: TranspCost * Quantity);

SUBJECT TO

    Demand[customer,paper] : SUM(mill,machine: Quantity) >= Required ;
    Supply[mill,material] : SUM(customer,paper,machine: Rate1 * Quantity) <= RawMaterial;
    Capacity[mill,machine] : SUM(customer,paper: Rate2 * Quantity) < MaxCapacity ;
END
```

d)

```
MODEL:
! Problem 3.7-8 - Lingo Version;
SETS:
MILLS /1..10/;
CUSTOMERS /1..1000/;
MACHINES /1..3/;
MATERIALS /1..4/;
PAPER /1..5/;
LINK1(CUSTOMERS,PAPER): DEMAND;
LINK2(PAPER, MACHINES, MATERIALS): RATE1;
LINK3(MILLS, MATERIALS): CAPACITY1;
LINK4(PAPER, MACHINES): RATE2;
LINK5(MILLS, MACHINES): CAPACITY2;
LINK6(MILLS, PAPER, MACHINES): PROD_COST;
LINK7(MILLS, CUSTOMERS, PAPER): TRANSP_COST;
LINK8(MILLS, CUSTOMERS, PAPER, MACHINES): QUANTITY;
ENDSETS

!OBJECTIVE IS TO MINIMIZE PRODUCTION COST + TRANSPORTATION COST;
MIN = @SUM(LINK6(I,K,L):PROD_COST(I,K,L) * @SUM(CUSTOMERS(J): QUANTITY(I,J,K,L))) +
      @SUM(LINK7(I,J,K):TRANSP_COST * @SUM(MACHINES(L): QUANTITY(I,J,K,L)));

!DEMAND CONSTRAINTS;
@FOR(LINK1(J,K): @SUM(MILLS(I): @SUM(MACHINES(L): QUANTITY(I,J,K,L))) >= DEMAND(J,K));

!RAW MATERIALS SUPPLY CONSTRAINTS;
@FOR(LINK3(I,M): @SUM(PAPER(K): @SUM(MACHINES(L): RATE1(K,L,M)*@SUM(CUSTOMERS(J):
QUANTITY(I,J,K,L)))) <= CAPACITY1(I,M));

!CAPACITY SUPPLY CONSTRAINTS;
@FOR(LINK5(I,L): @SUM(PAPER(K): RATE2(K,L) * @SUM(CUSTOMERS(J): QUANTITY(I,J,K,L))) <=
CAPACITY2(I,L));

!READ DATA FROM AN EXCEL FILE;
DATA:
DEMAND, RATE1, CAPACITY1, RATE2, CAPACITY2, PROD_COST, TRANSP_COST =
@WKX('C:\LINGO\DATA.WK4','DEMAND','RATE1','CAPACITY1','RATE2','CAPACITY2','PROD_COST','TRA
NSP_COST');
ENDDATA
END
```

Cases

3-1 a) In this case, we have two decision variables: one variable to determine the number of Family Thrillseekers we should assemble and one variable to determine the number of Classy Cruisers we should assemble. We also have the following three constraints:

1. The plant has a maximum of 48,000 labor hours. Each Thrillseeker requires six labor hours, and each Cruiser requires 10.5 labor hours. The sum of the total number of labor hours required to assemble all Thrillseekers and all Cruisers must be less than or equal to 48,000 hours.

2. The plant has a maximum of 20,000 doors available. Each Thrillseeker requires four doors, and each Cruiser requires two doors. The sum of the total number of doors required to assemble all Thrillseekers and all Cruisers must be less than or equal to 20,000 doors.

3. Because the demand for Cruisers is limited to 3,500 cars, the decision variable for the number of Cruisers we should assemble must be less than or equal to 3,500.

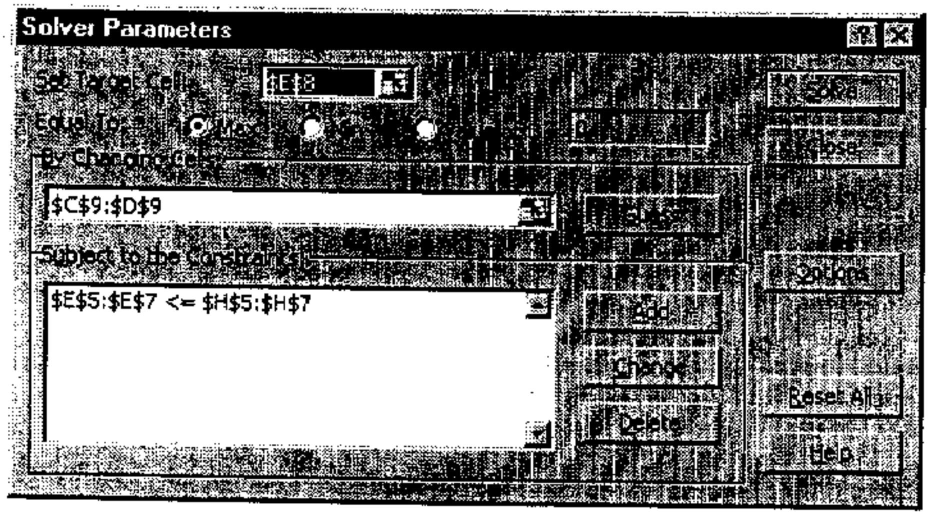
The formulas used in the problem formulation follow.

	A	B	C	D	E	F	G	H
3			Thrillseeker	Cruiser	Totals			Right-Hand
4		Constraint						Side
5		Labor Hours	6	10.5	=SUMPRODUCT(C5:D5,C9:D9)	<	=	48000
6		Doors	4	2	=SUMPRODUCT(C6:D6,C9:D9)	<	=	20000
7		Cruiser Demand	0	1	=SUMPRODUCT(C7:D7,C9:D9)	<	=	3500
8		Profit ($thousands)	3.6	5.4	**=SUMPRODUCT(C8:D8,C9:D9)**			
9		Solution	**3800**	**2400**				

The values used in the problem formulation follow.

	A	B	C	D	E	F	G	H
3			Thrillseeker	Cruiser	Totals			Right-Hand
4		Constraint						Side
5		Labor Hours	6	10.5	48000	<	=	48000
6		Doors	4	2	20000	<	=	20000
7		Cruiser Demand	0	1	2400	<	=	3500
8		Profit ($thousands)	3.6	5.4	**26640**			
9		Solution	**3800**	**2400**				

We specify the following Solver settings.



Finally, throughout this case we use the following solver options.

Rachel's plant should assemble 3,800 Thrillseekers and 2,400 Cruisers to obtain a maximum profit of $26,640,000.

b) In part (a) above, we observed that the Cruiser demand constraint was not binding. Therefore, raising the demand for the Cruiser will not change the optimal solution. The marketing campaign should not be undertaken.

c) The new value of the right-hand side of the labor constraint becomes 48,000 * 1.25 = 60,000 labor hours. All formulas and Solver settings used in part (a) remain the same. The values for the problem formulation follow.

	A	B	C	D	E	F	G	H
3			Thrillseeker	Cruiser	Totals			Right-Hand
4		Constraint						Side
5		Labor Hours	6	10.5	56250	<	=	60000
6		Doors	4	2	20000	<	=	20000
7		Cruiser Demand	0	1	3500	<	=	3500
8		Profit ($thousands)	3.6	5.4	**30600**			
9		Solution	**3250**	**3500**				

Rachel's plant should now assemble 3,250 Thrillseekers and 3,500 Cruisers to achieve a maximum profit of $30,600,000.

d) Using over time labor increases the profit by $30,600,000 – $26,640,000 = $3,960,000. Rachel should therefore be willing to pay at most $3,960,000 extra for overtime labor beyond regular time rates.

e) The value of the right-hand side of the Cruiser demand constraint is 3,500 * 1.20 = 4,200 cars. The value of the right-hand side of the labor hour constraint is 48,000 * 1.25 = 60,000 hours. All formulas and Solver settings used in part (a) remain the same. Ignoring the costs of the advertising campaign and overtime labor, the values for the problem formulation follow.

	A	B	C	D	E	F	G	H
3			Thrillseeker	Cruiser	Totals			Right-Hand
4		Constraint						Side
5		Labor Hours	6	10.5	60000	<	=	60000
6		Doors	4	2	20000	<	=	20000
7		Cruiser Demand	0	1	4000	<	=	4200
8		Profit ($thousands)	3.6	5.4	**32400**			
9		Solution	**3000**	**4000**				

Rachel's plant should produce 3,000 Thrillseekers and 4,000 Cruisers for a maximum profit of $32,400,000. This profit excludes the costs of advertising and using overtime labor.

f) The advertising campaign costs $500,000. In the solution to part (e) above, we used the maximum overtime labor available, and the maximum use of overtime labor costs $1,600,000. Thus, our solution in part (e) required an extra $500,000 + $1,600,000 = $2,100,000. We perform the following cost/benefit analysis:

Profit in part (e):	$32,400,000
− Advertising and overtime costs:	<u>$ 2,100,000</u>
	$30,300,000

We compare the $30,300,000 profit with the $26,640,000 profit obtained in part (a) and conclude that the decision to run the advertising campaign and use overtime labor is a very wise, profitable decision.

g) Because we consider this question independently, the values of the right-hand sides for the Cruiser demand constraint and the labor hour constraint are the same as those in part (a). We now change the profit for the Thrillseeker from 3.6 to 2.8 in the problem formulation. All formulas and Solver settings used in part (a) remain the same. The values for the problem formulation follow.

	A	B	C	D	E	F	G	H
3			Thrillseeker	Cruiser	Totals			Right-Hand
4		Constraint						Side
5		Labor Hours	6	10.5	48000	<	=	48000
6		Doors	4	2	14500	<	=	20000
7		Cruiser Demand	0	1	3500	<	=	3500
8		Profit ($thousands)	2.8	5.4	**24150**			
9		Solution	**1875**	**3500**				

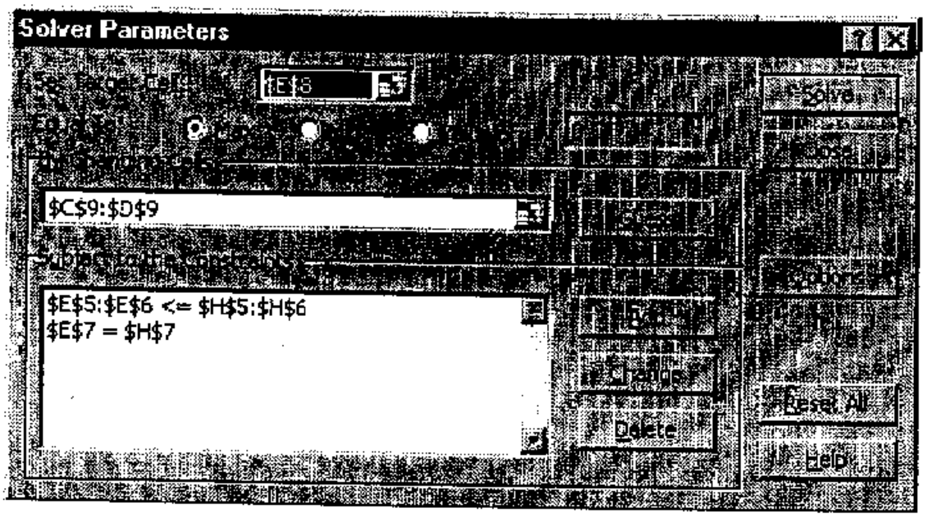
Rachel's plant should assemble 1,875 Thrillseekers and 3,500 Cruisers to obtain a maximum profit of $24,150,000.

h) Because we consider this question independently, the profit for the Thrillseeker remains the same as the profit specified in part (a). The labor hour constraint changes. Each Thrillseeker now requires 7.5 hours for assembly. All formulas and Solver settings used in part (a) remain the same. The values for the new problem formulation follow.

	A	B	C	D	E	F	G	H
3			Thrillseeker	Cruiser	Totals			Right-Hand
4		Constraint						Side
5		Labor Hours	7.5	10.5	48000	<	=	48000
6		Doors	4	2	13000	<	=	20000
7		Cruiser Demand	0	1	3500	<	=	3500
8		Profit ($thousands)	3.6	5.4	**24300**			
9		Solution	**1500**	**3500**				

Rachel's plant should assemble 1,500 Thrillseekers and 3,500 Cruisers for a maximum profit of $24,300,000.

i) Because we consider this question independently, we use the problem formulation used in part (a). In this problem, however, the number of Cruisers assembled has to be strictly equal to the total demand. We use the following new Solver settings:



The formulas used in the problem formulation remain the same as those used in part (a). The values used in the problem follow.

	A	B	C	D	E	F	G	H
3			Thrillseeker	Cruiser	Totals			Right-Hand
4		Constraint						Side
5		Labor Hours	6	10.5	48000	<	=	48000
6		Doors	4	2	14500	<	=	20000
7		Cruiser Demand	0	1	3500		=	3500
8		Profit (Sthousands)	3.6	5.4	**25650**			
9		Solution	**1875**	**3500**				

The new profit is $25,650,000, which is $26,640,000 – $25,650,000 = $990,000 less than the profit obtained in part (a). This decrease in profit is less than $2,000,000, so Rachel should meet the full demand for the Cruiser.

j) We now combine the new considerations described in parts (f), (g), and (h). In part (f), we decided to use both the advertising campaign and the overtime labor. The advertising campaign raises the demand for the Cruiser to 4,200 sedans, and the overtime labor increases the labor hour capacity of the plant to 60,000 labor hours. In part (g), we decreased the profit generated by a Thrillseeker to $2,800. In part (h), we increased the time to assemble a Thrillseeker to 7.5 hours. Including the increased demand for Cruisers, the increased plant capacity, the decreased unit profit for a Thrillseeker, and the increased time to assemble a Thrillseeker, the new problem is formulated as follows:

	A	B	C	D	E	F	G	H
3			Thrillseeker	Cruiser	Totals			Right-Hand
4		Constraint						Side
5		Labor Hours	7.5	10.5	60000	<	=	60000
6		Doors	4	2	16880	<	=	20000
7		Cruiser Demand	0	1	4200	<	=	4200
8		Profit ($thousands)	2.8	5.4	**28616**			
9		Solution	**2120**	**4200**				

The formulas and Solver settings used for this problem are the same as those used in part (a). Rachel's plant should assemble 2,120 Thrillseekers and 4,200 Cruisers for a maximum profit of $28,616,000 – $2,100,000 = $26,516,000.

3-2 a) We want to determine the amount of potatoes and green beans Maria should purchase to minimize ingredient costs. We have two decision variables: one variable to represent the amount (in pounds) of potatoes Maria should purchase and one variable to represent the amount (in pounds) of green beans Maria should purchase. We also have constraints on nutrition, taste, and weight.

Nutrition Constraints

1. We first need to ensure that the dish has 180 grams of protein. We are told that 100 grams of potatoes have 1.5 grams of protein and 10 ounces of green beans have 5.67 grams of protein. Since we have decided to measure our decision variables in pounds, however, we need to determine the grams of protein in one pound of each ingredient.

We perform the following conversion for potatoes:

$$100\text{g of potatoes}\left(\frac{1\text{ lb}}{453.6\text{g}}\right) = 0.220459\text{ lb of potatoes}$$

$$\frac{1.5\text{g of protein}}{0.22046\text{ lb of potatoes}} = \frac{6.804\text{g of protein}}{1\text{ lb of potatoes}}$$

We perform the following conversion for green beans:

$$10\text{ oz of green beans}\left(\frac{28.35\text{ g}}{1\text{ oz}}\right) = 283.5\text{ g of green beans}$$

$$283.5\text{ g of green beans}\left(\frac{1\text{ lb}}{453.6\text{ g}}\right) = 0.625\text{ lb of green beans}$$

$$\frac{5.67\text{ g of protein}}{0.625\text{ lb of green beans}} = \frac{9.072\text{ g of protein}}{1\text{ lb of green beans}}$$

The total grams of protein in the potatoes and green beans Maria purchases for the casserole must be greater than or equal to 180 grams.

casserole must be greater than or equal to [illegible] milligrams

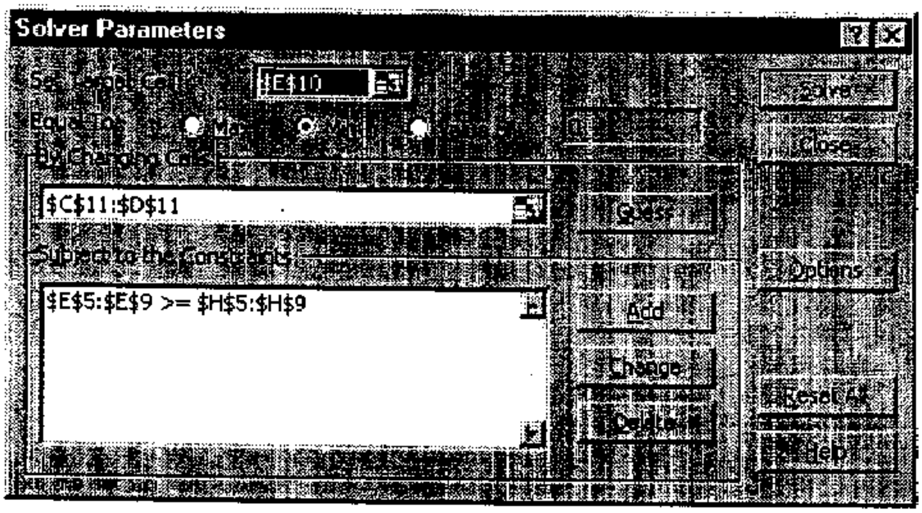
3. [illegible] milligrams or [illegible]

	A	B	C	D	E	F	G	H
3			Potatoes	Green Beans	Totals			Right-Hand
4		Constraint						Side
5		Protein (g)	6.804	9.072	=SUMPRODUCT(C5:D5,C11:D11)	>	=	180
6		Iron (mg)	1.3608	5.4432	=SUMPRODUCT(C6:D6,C11:D11)	>	=	80
7		Vitamin C (mg	54.432	45.36	=SUMPRODUCT(C7:D7,C11:D11)	>	=	1050
8		Taste	5	-6	=SUMPRODUCT(C8:D8,C11:D11)	>	=	0
9		Amount (lb)	1	1	=SUMPRODUCT(C9:D9,C11:D11)	>	=	22.046
10		Cost (per lb)	0.4	1	**=SUMPRODUCT(C10:D10,C11:D11)**			
11		Solution (lb)	**13.5667**	**11.3056**				

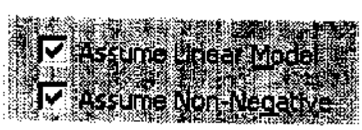
The values for the problem and solution follow.

	A	B	C	D	E	F	G	H
3			Potatoes	Green Beans	Totals			Right-Hand
4		Constraint						Side
5		Protein (g)	6.804	9.072	194.8717949	>	=	180
6		Iron (mg)	1.3608	5.4432	80	>	=	80
7		Vitamin C (mg	54.432	45.36	1251.282051	>	=	1050
8		Taste	5	-6	0	>	=	0
9		Amount (lb)	1	1	24.87224709	>	=	22.046
10		Cost (per lb)	0.4	1	**16.73223895**			
11		Solution (lb)	**13.567**	**11.306**				

The Solver settings used to solve the problem follow.



Finally, throughout this case we use the following Solver options.



Maria should purchase 13.567 lb of potatoes and 11.306 lb of green beans to obtain a minimum cost of $16.73.

b) The taste constrint changes. The new constraint is now.

$$\frac{\text{pounds of potatoes}}{\text{pounds of green beans}} > \frac{1}{2}$$

2 (pounds of potatoes) > 1 (pounds of green beans)

The formulas and Solver settings used to solve the problem remain the same as in part (a). The values for the problem and solution follow.

	A	B	C	D	E	F	G	H
3			Potatoes	Green Beans	Totals			Right-Hand
4		Constraint						Side
5		Protein (g)	6.804	9.072	180	>	=	180
6		Iron (mg)	1.3608	5.4432	80	>	=	80
7		Vitamin C (mg	54.432	45.36	1110	>	=	1050
8		Taste	2	-1	8.450911229	>	=	0
9		Amount (lb)	1	1	22.41328683	>	=	22.046
10		Cost (per lb)	0.4	1	**16.2404468**			
11		Solution (lb)	**10.288**	**12.125**				

Maria should purchase 10.288 lb of potatoes and 12.125 lb of green beans to obtain a minimum cost of $16.24.

c) The right-hand side of the iron constraint changes from 80 mg to 65 mg. The formulas and Solver settings used in the problem remain the same as in part (a). The values for the new problem formulation and solution follow.

	A	B	C	D	E	F	G	H
3			Potatoes	Green Beans	Totals			Right-Hand
4		Constraint						Side
5		Protein (g)	6.804	9.072	180	>	=	180
6		Iron (mg)	1.3608	5.4432	65	>	=	65
7		Vitamin C (mg	54.432	45.36	1222.5	>	=	1050
8		Taste	5	-6	31.04791299	>	=	0
9		Amount (lb)	1	1	23.79115226	>	=	22.046
10		Cost (per lb)	0.4	1	**14.31143445**			
11		Solution (lb)	**15.800**	**7.992**				

Maria should purchase 15.8 lb of potatoes and 7.992 lb of green beans to obtain a minimum cost of $14.31.

d) The iron requirement remains 65 mg. We need to change the price per pound of green beans from $1.00 per pound to $0.50 per pound. The formulas and Solver settings used in the problem remain the same as in part (a). The values for the new problem formulation and solution follow.

	A	B	C	D	E	F	G	H
3			Potatoes	Green Beans	Totals			Right-Hand
4		Constraint						Side
5		Protein (g)	6.804	9.072	180	>	=	180
6		Iron (mg)	1.3608	5.4432	73.89473684	>	=	65
7		Vitamin C (mg	54.432	45.36	1155.789474	>	=	1050
8		Taste	5	-6	0	>	=	0
9		Amount (lb)	1	1	22.97410192	>	=	22.046
10		Cost (per lb)	0.4	0.5	**10.2339181**	3		
11		Solution (lb)	**12.531**	**10.443**				

Maria should purchase 12.531 lb of potatoes and 10.443 lb of green beans to obtain a minimum cost of $10.23.

e) We still have two decision variables: one variable to represent the amount (in pounds) of potatoes Maria should purchase and one variable to represent the amount (in pounds) of lima beans Maria should purchase. To determine the grams of protein in one pound of lima beans, we perform the following conversion:

$$\frac{22.68 \text{ g of protein}}{0.625 \text{ lb of lima beans}} = \frac{36.288 \text{ g of protein}}{1 \text{ lb of lima beans}}$$

To determine the milligrams of iron in one pound of lima beans, we perform the following conversion:

$$\frac{6.804 \text{ mg of iron}}{0.625 \text{ lb of lima beans}} = \frac{10.8864 \text{ mg of iron}}{1 \text{ lb of lima beans}}$$

Lima beans contain no vitamin C, so we do not have to perform a measurement conversion for vitamin C.

We change the decision variable from green beans to lima beans and insert the new parameters for protein, iron, vitamin C, and cost. The formulas and Solver settings used in the problem remain the same as in part (a). The values for the new problem formulation and solution follows.

	A	B	C	D	E	F	G	H
3			Potatoes	Lima Beans	Totals			Right-Hand
4		Constraint						Side
5		Protein (g)	6.804	36.288	260.4166667	>	=	180
6		Iron (mg)	1.3608	10.8864	65	>	=	65
7		Vitamin C (mg)	54.432	0	1050	>	=	1050
8		Taste	5	-6	75.094	>	=	0
9		Amount (lb)	1	1	22.84961052	>	=	22.046
10		Cost (per lb)	0.4	0.6	**9.85174162**	3		
11		Solution (lb)	**19.290**	**3.559**				

Maria should purchase 19.29 lb of potatoes and 3.559 lb of lima beans to obtain a minimum cost of $9.85.

f) Edson takes pride in the taste of his casserole, and the optimal solution from above does not seem to preserve the taste of the casserole. First, Maria forces Edson to use lima beans instead of green beans, and lima beans are not an ingredient in Edson's original recipe. Second, although Edson places no upper limit on the ratio of potatoes to beans, the above recipe uses an over five to one ratio of potatoes to beans. This ratio seems unreasonable since such a large amount of potatoes will overpower the taste of beans in the recipe.

g) We only need to change the values on the right-hand side of the iron and vitamin C constraints. The formulas and Solver settings used in the problem remain the same as in part (a). The values used in the new problem formulation and solution follow.

	A	B	C	D	E	F	G	H
3			Potatoes	Lima Beans	Totals			Right-Hand
4		Constraint						Side
5		Protein (g)	6.804	36.288	428.5718034	>	=	180
6		Iron (mg)	1.3608	10.8864	120	>	=	120
7		Vitamin C (mg	54.432	0	685.7232823	>	=	500
8		Taste	5	-6	6.300	>	=	0
9		Amount (lb)	1	1	22.046	>	=	22.046
10		Cost (per lb)	0.4	0.6	**10.7080406**	1		
11		Solution (lb)	**12.598**	**9.448**				

Maria should purchase 12.598 lb of potatoes and 9.448 lb of lima beans to obtain a minimum cost of $10.71.

3-3 a) The number of operators that the hospital needs to staff the call center during each two-hour shift can be found in the following table:

	A	B	C	D	E	F
1	work	average number of	number of calls from	number of calls from	number of operators	number of operators
2	shift	calls per hour	English speakers	Spanish speakers	speaking English	speaking Spanish
3	7am to 9am	40	32	8	6	2
4	9am to 11am	85	68	17	12	3
5	11am to 1pm	70	56	14	10	3
6	1pm to 3pm	95	76	19	13	4
7	3pm to 5pm	80	64	16	11	3
8	5pm to 7pm	35	28	7	5	2
9	7pm to 9pm	10	8	2	2	1

For example, the average number of phone calls per hour during the shift from 7am to 9am equals 40. Since, on average, 80% of all phone calls are from English speakers, there is an average number of 32 phone calls per hour from English speakers during that shift. Since one operator takes, on average, 6 phone calls per hour, the hospital needs 32/6 = 5.333 English-speaking operators during that shift. The hospital cannot employ fractions of an operator and so needs 6 English-speaking operators for the shift from 7am to 9am.

b) The problems of determining how many Spanish-speaking operators and English-speaking operators Lenny needs to hire to begin each shift are independent. Therefore we can formulate two smaller linear programming models instead of one large model. We are going to have one model for the scheduling of the Spanish-speaking operators and another one for the scheduling of the English-speaking operators.

Lenny wants to minimize the operating costs while answering all phone calls. For the given scheduling problem we make the assumption that the only operating costs are the wages of the employees for the hours that they answer phone calls. The wages for the hours during which they perform paperwork are paid by other cost centers. Moreover, it does not matter for the callers whether an operator starts his or her work day with phone calls or with paperwork. For example, we do not need to distinguish between operators who start their day answering phone calls at 9am and operators who start their day with paperwork at 7am, because both groups of operators will be answering phone calls at the same time. And only this time matters for the analysis of Lenny's problem.

We define the decision variables according to the time when the employees have their first shift of answering phone calls. For the scheduling problem of the English-speaking operators we have 7 decision variables. First, we have 5 decision variables for full-time employees.

The number of operators having their first shift on the phone from 7am to 9am.
The number of operators having their first shift on the phone from 9am to 11am.
The number of operators having their first shift on the phone from 11am to 1pm.
The number of operators having their first shift on the phone from 1pm to 3pm.
The number of operators having their first shift on the phone from 3pm to 5pm.

In addition, we define 2 decision variables for part-time employees.

The number of part-time operators having their first shift from 3pm to 5pm.
The number of part-time operators having their first shift from 5pm to 7pm.

The unit cost coefficients in the objective function are the wages operators earn while they answer phone calls. All operators who have their first shift on the phone from 7am to 9am, 9am to 11am, or 11am to 1pm finish their work on the phone before 5pm. They earn 4*$10 = $40 during their time answering phone calls. All operators who have their first shift on the phone from 1pm to 3pm or 3pm to 5pm have one shift on the phone before 5pm and another one after 5pm. They earn 2*$10+2*$12 = $44 during their time answering phone calls. The second group of part-time operators, those having their first shift from 5pm to 7pm, earn 4*$12 = $48 during their time answering phone calls.

There are 7 constraints, one for each two-hour shift during which phone calls need to be answered. The right-hand sides for these constraints are the number of operators needed to ensure that all phone calls get answered in a timely manner. On the left-hand side we determine the number of operators on the phone during any given shift. For example, during the 11am to 1pm shift the total number of operators answering phone calls equals the sum of the number of operators who started answering calls at 7am and are currently in their second shift of the day and the number of operators who started answering calls at 11am.

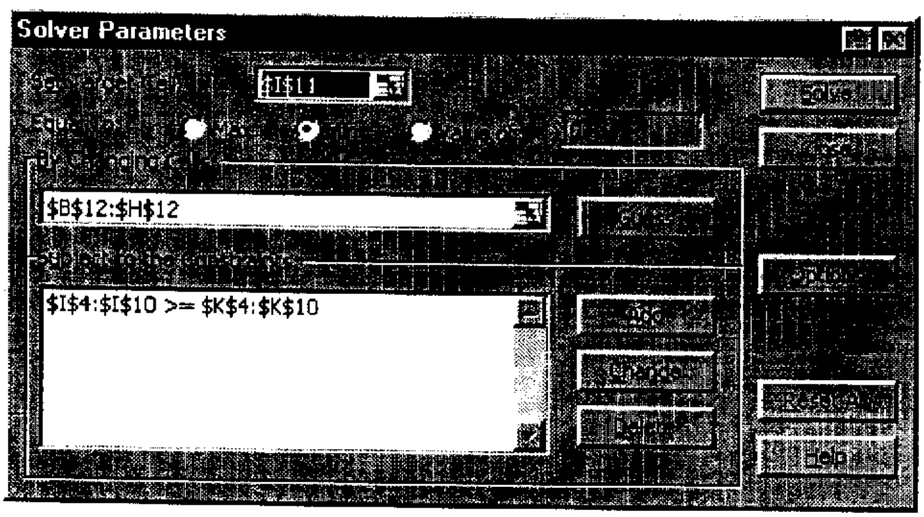
The following spreadsheet describes the entire problem formulation for the English-speaking employees:

	A	B	C	D	E	F	G	H	I	J	K
1											
2	Shifts of	Number of operators whose first shift of answering phone calls in English is from									Required number
3	phone operators	7am to 9am	9am to 11am	11am to 1 pm	1pm to 3pm	3pm to 5pm	3pm to 5pm (P)	5pm to 7pm (P)	Totals		of operators
4	7am to 9am	1	0	0	0	0	0	0	6	>=	6
5	9am to 11am	0	1	0	0	0	0	0	13	>=	12
6	11am to 1 pm	1	0	1	0	0	0	0	10	>=	10
7	1pm to 3pm	0	1	0	1	0	0	0	13	>=	13
8	3pm to 5pm	0	0	1	0	1	1	0	11	>=	11
9	5pm to 7pm	0	0	0	1	0	1	1	5	>=	5
10	7pm to 9pm	0	0	0	0	1	0	1	2	>=	2
11	Unit cost	40	40	40	44	44	44	48	**1228**	=	Total cost
12	Solution	**6**	**13**	**4**	**0**	**2**	**5**	**0**			

The following formulas are used in the problem formulation:

	I
1	
2	
3	Totals
4	=SUMPRODUCT(B4:H4,B12:H12)
5	=SUMPRODUCT(B5:H5,B12:H12)
6	=SUMPRODUCT(B6:H6,B12:H12)
7	=SUMPRODUCT(B7:H7,B12:H12)
8	=SUMPRODUCT(B8:H8,B12:H12)
9	=SUMPRODUCT(B9:H9,B12:H12)
10	=SUMPRODUCT(B10:H10,B12:H12)
11	**=SUMPRODUCT(B11:H11,B12:H12)**
12	

The solver appears as follows:

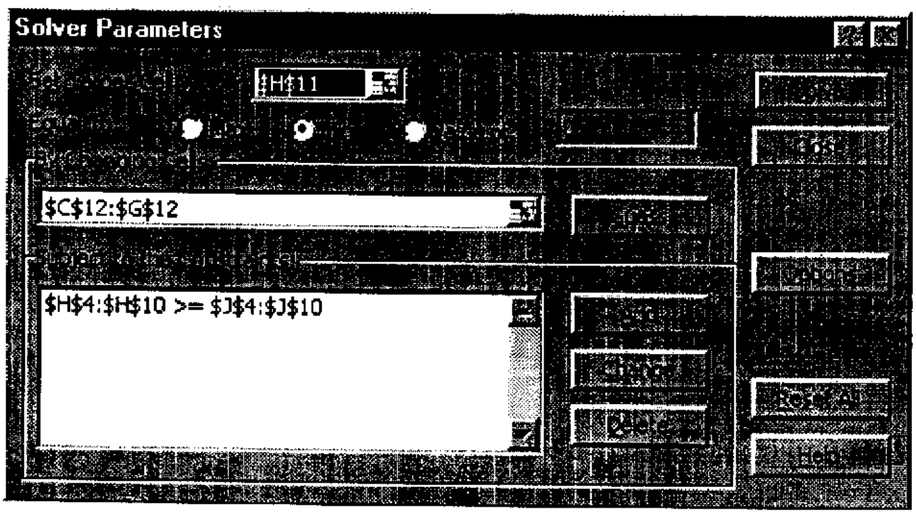


Throughout this analysis we use the following solver options:

The linear programming model for the Spanish-speaking employees can be developed in a similar fashion.

	A	B	C	D	E	F	G	H	I	J
1										
2		Shifts of	Number of operators whose first shift of answering phone calls in Spanish is from							Required number
3		phone operators	7am to 9am	9am to 11am	11am to 1 pm	1pm to 3pm	3pm to 5pm	Totals		of operators
4		7am to 9am	1	0	0	0	0	2	>=	2
5		9am to 11am	0	1	0	0	0	3	>=	3
6		11am to 1 pm	1	0	1	0	0	4	>=	3
7		1pm to 3pm	0	1	0	1	0	5	>=	4
8		3pm to 5pm	0	0	1	0	1	3	>=	3
9		5pm to 7pm	0	0	0	1	0	2	>=	2
10		7pm to 9pm	0	0	0	0	1	1	>=	1
11		Unit cost	40	40	40	44	44	412	=	Total cost
12		Solution	2	3	2	2	1			

	H
1	
2	
3	Totals
4	=SUMPRODUCT(C4:G4,C12:G12)
5	=SUMPRODUCT(C5:G5,C12:G12)
6	=SUMPRODUCT(C6:G6,C12:G12)
7	=SUMPRODUCT(C7:G7,C12:G12)
8	=SUMPRODUCT(C8:G8,C12:G12)
9	=SUMPRODUCT(C9:G9,C12:G12)
10	=SUMPRODUCT(C10:G10,C12:G12)
11	**=SUMPRODUCT(C11:G11,C12:G12)**
12	

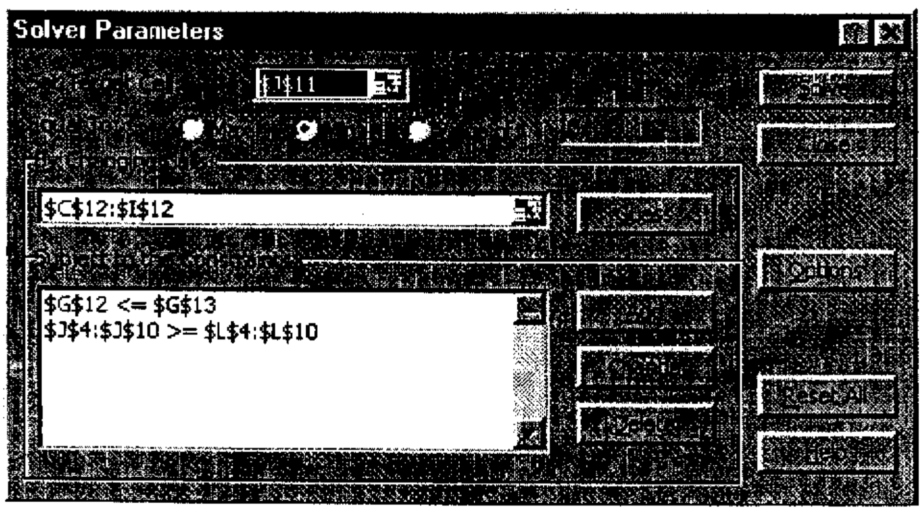


c) Lenny should hire 25 full-time English-speaking operators. Of these operators, 6 have their first phone shift from 7am to 9am, 13 from 9am to 11am, 4 from 11am to 1pm, and 2 from 3pm to 5pm. Lenny should also hire 5 part-time operators who start their work at 3pm. In addition, Lenny should hire 10 Spanish-speaking operators. Of these operators, 2 have their first shift on the phone from 7am to 9am, 3 from 9am to 11am, 2 from 11am to 1pm and 1pm to 3pm, and 1 from 3pm to 5pm. The total (wage) cost of running the calling center equals $1640 per day.

d) The restriction that Lenny can find only one English-speaking operator who wants to start work at 1pm affects only the linear programming model for English-speaking operators. This restriction does not put a bound on the number of operators who start their first phone shift at 1pm because those operators can start work at 11am with paperwork. However, this restriction does put an upper bound on the number of operators having their first phone shift from 3pm to 5pm. The new worksheet appears as follows.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		Shifts of	Number of operators whose first shift of answering phone calls in English is from									Required number
3		phone operators	7 am to 9am	9am to 11am	11am to 1 pm	1pm to 3pm	3pm to 5pm	3pm to 5pm (P)	5 pm to 7pm (P)	Totals		of operators
4		7am to 9am	1	0	0	0	0	0	0	6	>=	6
5		9am to 11am	0	1	0	0	0	0	0	13	>=	12
6		11am to 1 pm	1	0	1	0	0	0	0	12	>=	10
7		1pm to 3pm	0	1	0	1	0	0	0	13	>=	13
8		3pm to 5pm	0	0	1	0	1	1	0	11	>=	11
9		5pm to 7pm	0	0	0	1	0	1	1	5	>=	5
10		7pm to 9pm	0	0	0	0	1	0	1	2	>=	2
11		Unit cost	40	40	40	44	44	44	48	**1268**	=	Total cost
12		Solution	**6**	**13**	**6**	**0**	**1**	**4**	**1**			
13		Upper bounds					1					

The Solver dialogue box displays the additional constraint.



Lenny should hire 26 full-time English-speaking operators. Of these operators, 6 have their first phone shift from 7am to 9am, 13 from 9am to 11am, 6 from 11am to 1pm, and 1 from 3pm to 5pm. Lenny should also hire 4 part-time operators who start their work at 3pm and 1 part-time operator starting work at 5pm. The hiring of Spanish-speaking operators is unaffected. The new total (wage) costs equal $1680 per day.

e) For each hour, we need to divide the average number of calls per hour by the average processing speed, which is 6 calls per hour. The number of bilingual operators that the hospital needs to staff the call center during each two-hour shift can be found in the following table:

	A	B	C
1	work	average number of	number of operators
2	shift	calls per hour	speaking English
3	7am to 9am	40	7
4	9am to 11am	85	15
5	11am to 1pm	70	12
6	1pm to 3pm	95	16
7	3pm to 5pm	80	14
8	5pm to 7pm	35	6
9	7pm to 9pm	10	2

f) The linear programming model for Lenny's scheduling problem can be found in the same way as before, only that now all operators are bilingual.

	A	B	C	D	E	F	G	H	I	J	K
1											
2	Shifts of	Number of operators whose first shift of answering phone calls in both languages is from									Required number
3	phone operators	7am to 9am	9am to 11am	11am to 1pm	1pm to 3pm	3pm to 5pm	3pm to 5pm (P)	5pm to 7pm (P)	Totals		of operators
4	7am to 9am	1	0	0	0	0	0	0	7	>=	7
5	9am to 11am	0	1	0	0	0	0	0	16	>=	15
6	11am to 1pm	1	0	1	0	0	0	0	13	>=	12
7	1pm to 3pm	0	1	0	1	0	0	0	16	>=	16
8	3pm to 5pm	0	0	1	0	1	1	0	14	>=	14
9	5pm to 7pm	0	0	0	1	0	1	1	6	>=	6
10	7pm to 9pm	0	0	0	0	1	0	1	2	>=	2
11	Unit cost	40	40	40	44	44	44	48	**1512**	=	Total cost
12	Solution	**7**	**16**	**6**	**0**	**2**	**6**	**0**			

(The formulas and the solver dialogue box are identical to those in part (b).)

Lenny should hire 31 full-time bilingual operators. Of these operators, 7 have their first phone shift from 7am to 9am, 16 from 9am to 11am, 6 from 11am to 1pm, and 2 from 3pm to 5pm. Lenny should also hire 6 part-time operators who start their work at 3pm. The total (wage) cost of running the calling center equals \$1512 per day.

g) The total cost of part (f) is \$1512 per day; the total cost of part (b) is \$1640. Lenny could pay an additional \$1640-\$1512 = \$128 in total wages to the bilingual operators without increasing the total operating cost beyond those for the scenario with only monolingual operators. The increase of \$128 represents a percentage increase of 128/1512 = 8.466%.

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h) Creative Chaos Consultants has made the assumption that the number of phone calls is independent of the day of the week. But maybe the number of phone calls is very different on a Monday than it is on a Friday. So instead of using the same number of average phone calls for every day of the week, it might be more appropriate to determine whether the day of the week affects the demand for phone operators. As a result Lenny might need to hire more part-time employees for some days with an increased calling volume.

   Similarly, Lenny might want to take a closer look at the length of the shifts he has scheduled. Using shorter shift periods would allow him to "fine tune" his calling centers and make it more responsive to demand fluctuations.

   Lenny should investigate why operators are able to answer only 6 phone calls per hour. Maybe additional training of the operators could enable them to answer phone calls quicker and so increase the number of phone calls they are able to answer in an hour.

   Finally, Lenny should investigate whether it is possible to have employees switching back and forth between paperwork and answering phone calls. During slow times phone operators could do some paperwork while they are sitting next to a phone, while in times of sudden large call volumes employees who are scheduled to do paperwork could quickly switch to answering phone calls.

   Lenny might also want to think about the installation of an automated answering system that gives callers a menu of selections. Depending upon the caller's selection, the call is routed to an operator who specializes in answering questions about that selection.

4.7-8 c) (CONT'D)

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$3	Variables	11	0	5	1E+30	0.363636
\$D\$3	Variables	0	-0.33333333	4	0.33333333	1E+30
\$E\$3	Variables	3	0	-1	2.66666667	1.333333
\$F\$3	Variables	0	-0.66666667	3	0.66666667	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$H\$5	Constraints	24	0.66666667	24	12	132
\$H\$6		36	1	36	1E+30	12

4.7-9. a)

Resource	Resource Usage Per Unit of Each Activity: Activity 1	Activity 2	Totals		Resource Available
A	5	4	16.25	≤	20
B	6	9	30	≤	30
C	2	5	15	≤	15
Unit Profit	20	30	**\$ 100.00**		
Solution	**1.25**	**2.5**			

Adjustable Cells

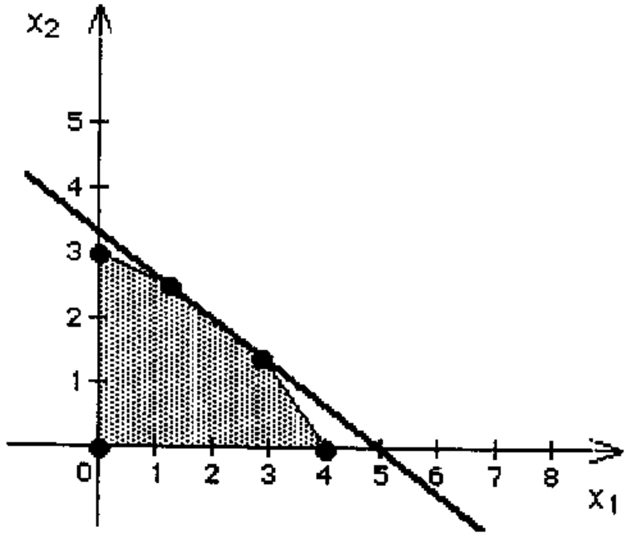
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$7	Solution Activity 1	1.25	0.000	20	0	8
\$C\$7	Solution Activity 2	2.5	0.000	30	20	0

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
SD\$3	A Totals	16.25	0.000	20	1E+30	3.75
SD\$4	B Totals	30	3.333	30	2.647058823	3
SD\$5	C Totals	15	0	15	1.666666667	2.142857143

b) The sensitivity report indicates that the problem has other optimal solutions because the allowable increase [illegible] and the allowable decrease of [illegible]

d) Optimal Solution: $(x_1, x_2) = (2.857, 1.429)$, $(1.25, 2.5)$ and all points on the connecting line. Profit = \$100 million.



4.7-10 a)

Resource	Resource Usage Per Unit of Each Activity: Activity 1	Activity 2	Totals		Resource Available
A	15	5	300	≤	300
B	10	6	240	≤	240
C	8	12	300	≤	450
Unit Profit	500	300	**\$ 12,000**		
Solution	**15**	**15**			

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
SB$7	Solution Activity 1	15	0.00	500	400	0
SC$7	Solution Activity 2	15	0.0	300	0	133.3333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$D\$3	A Totals	300	0.00	300	60	83.33333333
\$D\$4	B Totals	240	50	240	42.85714286	40
\$D\$5	C Totals	300	0	450	1E+30	150

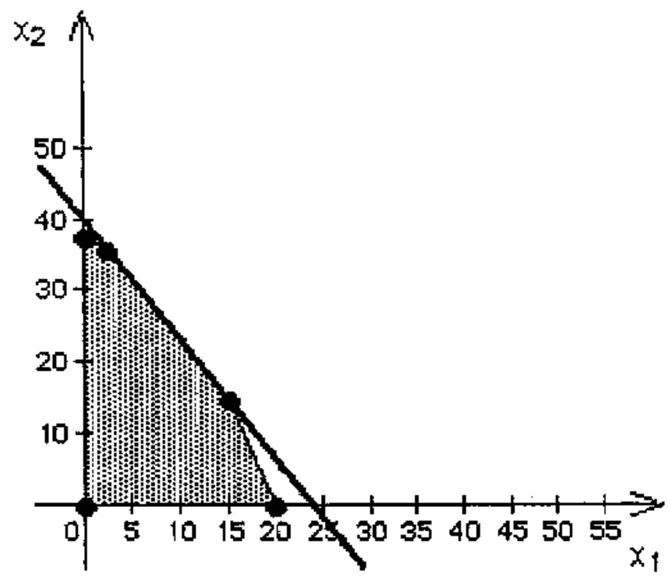
4.7-10.

b) The sensitivity report indicates that the problem has other optimal solutions because the allowable decrease of Activity 1 and the allowable increase of Activity 2 are 0.

	Resource Usage Per Unit of Each Activity				Resource
Resource	Activity 1	Activity 2	Totals		Available
A	15	5	217	≤	300
B	10	6	240	≤	240
C	8	12	450	≤	450
Unit Profit	500	301	**$ 12,036**		
Solution	**2.5**	**35.833**			

c) The other optimal solutions will be located on the line segment connecting the two optimal solutions found in parts a) and b).

d) Optimal Solution: $(x_1, x_2) = (15,15)$, $(2.5, 35.833)$ and all points on the connecting line. Profit = $12,000.



Cases

4-1 a) The fixed design and fashion costs are sunk costs and therefore should not be considered when setting the production now in July. Since the velvet shirts have a positive contribution to covering the sunk costs, they should be produced or at least considered for production according to the linear programming model. Had Ted raised these concerns before any fixed costs were made, then he would have been correct to advise against designing and producing the shirts. With a contribution of $22 and a demand of 6000 units, maximum expected profit will be only $132,000. This amount will not be enough to cover the $500,000 in fixed costs directly attributable to this product.

b) The following insight greatly simplifies the analysis of the problem. The production processes of the various clothing items are not all linked together. We can separate the clothing items according to the materials that are used in their production and instead of one large linear programming problem we can formulate 4 smaller problems.

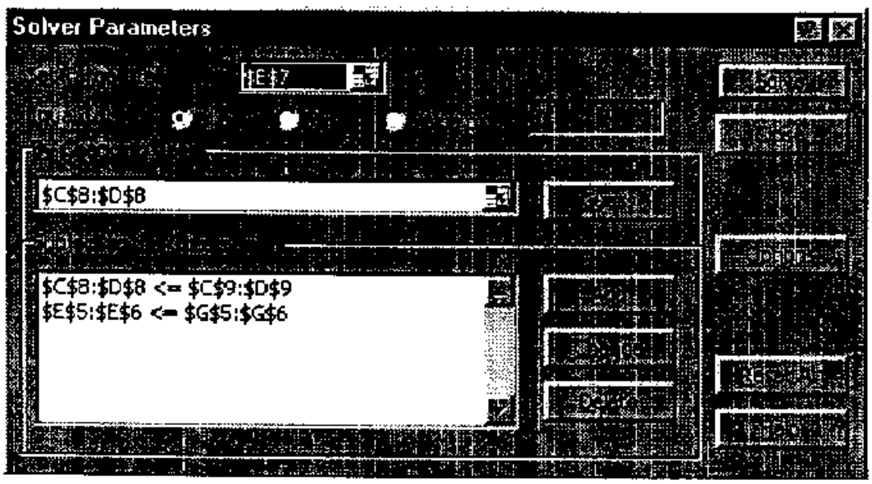
We use the term net contribution of a sales item to describe the difference between its total revenues and variable costs. The net contribution does not reflect any part of the fixed costs.

The cashmere sweater is the only item consisting of cashmere. The net contribution of one cashmere sweater equals $450 - $150 – 1.5*$60 = $210. TrendLines can sell at most 4000 sweaters and has 9000 yards of cashmere as raw material. It is optimal to produce 4000 sweaters using 6000 yards of cashmere yielding a net contribution of 4000*$210 = $840,000.

The silk blouse and camisole are the only items using silk and no other materials are used for these items. We can determine the optimal production amounts of these two items through a simple linear program. The first constraint models the resource limitation in the production process that Katherine has ordered 18,000 yards of silk. The second constraint models the production condition that whenever a silk blouse is produced automatically also a silk camisole is produced. Finally we must include the stated upper bounds on the number of silk items we can sell.

	A	B	C	D	E	F	G
1							
2							
3			Activity				
4		Constraint	silk blouse	silk camisole	Totals		Constraint RHS
5		silk	1.5	0.5	18000	<=	18000
6		production	1	-1	-8000	<=	0
7		unit profit	60.5	53.5	**1226000**		
8		Solution	**7000**	**15000**			
9		Maximum	12000	15000			

	E
3	
4	Totals
5	=SUMPRODUCT(C5:D5,C8:D8)
6	=SUMPRODUCT(C6:D6,C8:D8)
7	**=SUMPRODUCT(C7:D7,C8:D8)**
8	

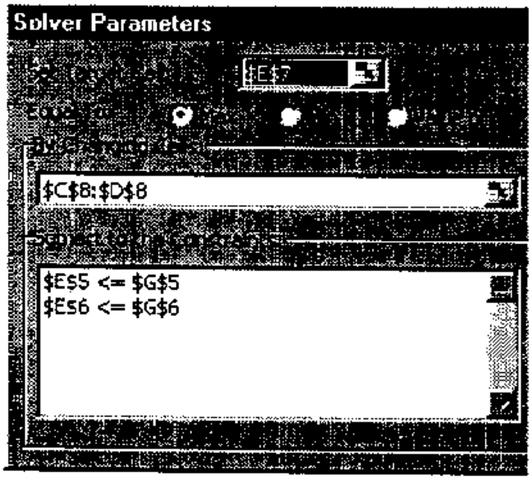


Throughout this case we use the following solver options.

TrendLines should produce 7000 silk blouses and 15000 silk camisoles yielding a net contribution of $1,226,000.

We can determine the optimal production plan for the items made from cotton in a similar fashion. There are no demand limitations for the cotton items.

	A	B	C	D	E	F	G
1							
2							
3			Activity				
4		Constraint	cotton sweater	cotton m-s	Totals		Constraint RHS
5		wool	1.5	0.5	30000	<=	30000
6		production	1	-1	-60000	<=	0
7		unit profit	66.25	33.75	**2025000**		
8		Solution	**0**	**60000**			

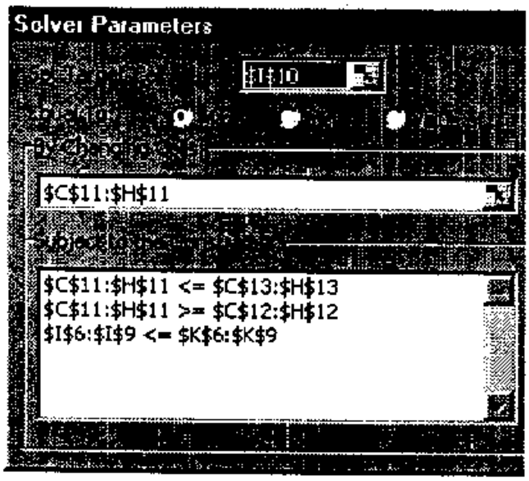


TrendLines should produce 60000 cotton mini-skirts but no cotton sweaters yielding a net contribution of $2,025,000.

It remains to develop a linear programming problem for determining the optimal production quantities of the tailored wool slacks, the tailored skirt, the wool blazer, the velvet pants and shirts, and the button-down blouse. We include four constraints for the resource limitations on wool, velvet, rayon, and acetate. Upper and lower bounds are given for many items. When there is no lower bound, we insert 0, when there is no upper bound, we determine a safe upper bound as a consequence of the resource limitations.

	A	B	C	D	E	F	G	H	I	J	K
1											
2											
3			Resource Usage Per Unit of Each Activity								
4			Activity								Resource
5		Resource	tail.wool s acks	tail.skirt	wool blazer	velvet pants	velvet shirt	b.-d. blouse	Totals		Available
6		woo	3	0	2.5	0	0	0	25100	<=	4500C
7		acetate	2	1.5	1.5	2	0	0	28000	<=	2800C
8		rayon	0	2	0	0	0	1.5	30000	<=	3000C
9		velvet	0	0	0	3	1.5	0	9000	<=	2000C
10		unit profit	110	143.25	155.25	136	22	26.625	**2771933.333**		
11		Solution	**4200**	**8066.666667**	**5000**	**0**	**6000**	**9244.444444**			
12		Minimum	4200	2800	3000	0	0	0			
13		Maximum	7000	20000	5000	5500	6000	20000			

	I
4	
5	Totals
6	=SUMPRODUCT(C6:H6,C11:H11)
7	=SUMPRODUCT(C7:H7,C11:H11)
8	=SUMPRODUCT(C8:H8,C11:H11)
9	=SUMPRODUCT(C9:H9,C11:H11)
10	**=SUMPRODUCT(C10:H10,C11:H11)**
11	



TrendLines should produce 4200 wool slacks, 8066.67 skirts, 5000 wool blazers, no velvet pants, 6000 velvet shirts, and 9244.44 button-down blouses. The net contribution of these items equals $2,771,933.33. (Of course, TrendLines cannot produce two-thirds of a skirt, so the actual solution should be integer. You will learn about integer programming in chapter 8.)

The net contribution of all clothing items equals S840,000 + $1,226,00 + $ 2,025,000 + $2,771,933.33 = $6,862,933.33. So far we have not considered the sunk costs for the three fashion shows and the designers which total $8,960,000. The total profit equals $6,862,933.33 - $8,960,000 = -$2,097,066.67. So, TrendLines actually loses almost $2.1 million.

c) If velvet cannot be sent back to the textile wholesaler, then the whole quantity will be considered as a sunk cost and therefore added to the fixed costs. The objective function coefficients of items using velvet will no longer include the material cost. The objective function coefficients of the velvet pants and shirts are now \$175 and \$40, respectively.

	A	B	C	D	E	F	G	H	I	J	K
1											
2											
3			Resource Usage Per Unit of Each Activity								
4			Activity								Resource
5		Resource	tail.wool slacks	tail.skirt	wool blazer	velvet pants	velvet shirt	b.-d. blouse	Totals		Available
6		wool	3	0	2.5	0	0	0	25100	<=	45000
7		acetate	2	1.5	1.5	2	0	0	28000	<=	28000
8		rayon	0	2	0	0	0	1.5	30000	<=	30000
9		velvet	0	0	0	3	1.5	0	20000	<=	20000
10		Unit profit	110	143.25	155.25	172	40	26.625	2983822.22		
11		Solution	**4200**	**3177.777778**	**5000**	**3666.666667**	**6000**	**15762.96296**			
12		Minimum	4200	2800	3000	0	0	0			
13		Maximum	7000	20000	5000	5500	6000	20000			

The production plan changes considerably. TrendLines should produce 4200 wool slacks, 3177.77 skirts, 5000 wool blazers, 3666.67 velvet pants, 6000 velvet shirts, and 15762.92 button-down blouses. The production decisions for all other items are unaffected by the change. The net contribution of all clothing items equals \$840,000 + \$1,226,00 + \$ 2,025,000 + \$2,983,822.22 = \$7,074,822.22. The sunk costs now include the material cost for velvet and total \$9,200,000. The loss equals \$9,200,000 - \$7,074,822.22 = \$2,125,177.78.

d) When TrendLines cannot return the velvet to the wholesaler, the costs for velvet cannot be recovered. These cost are no longer variable cost but now are sunk cost. As a consequence the increased net contribution of the velvet items makes them more attractive to produce. This way the revenues from selling these items can contribute to the recovery of at least some of the fixed costs. Instead of zero TrendLines produces now 3666.67 velvet pants. These pants also require some acetate and thus their production affects the production plan for all other items. Since it is not optimal to make full use of the ordered velvet in part (b) it comes as no surprise that the loss in part (c) is even bigger than in part (b).

e) The unit contribution of a wool blazer changes to \$75.25.

	A	B	C	D	E	F	G	H	I	J	K
2											
3			Resource Usage Per Unit of Each Activity								
4			Activity								Resource
5		Resource	tail.wool slacks	tail.skirt	wool blazer	velvet pants	velvet shirt	b.-d. blouse	Totals		Available
6		wool	3	0	2.5	0	0	0	20100	<=	45000
7		acetate	2	1.5	1.5	2	0	0	28000	<=	28000
8		rayon	0	2	0	0	0	1.5	30000	<=	30000
9		velvet	0	0	0	3	1.5	0	9000	<=	20000
10		Unit profit	110	143.25	75.25	136	22	26.625	2436933.333		
11		Solution	**4200**	**10066.66667**	**3000**	**0**	**6000**	**6577.777778**			
12		Minimum	4200	2800	3000	0	0	0			
13		Maximum	7000	20000	5000	5500	6000	20000			

TrendLines should produce 4200 wool slacks, 10066.67 skirts, the minimum of 3000 wool blazers, no velvet pants, 6000 velvet shirts, and 6577.78 button-down blouses. The production decisions for all other items are unaffected by the change. The net contribution of all clothing items equals \$840,000 + \$1,226,00 + \$ 2,025,000 + \$2,436,933.33 = \$6,527,933.33. The loss equals \$8,960,000 - \$6,527,933.33= \$2,432,066.67.

f) The right-hand-side of the acetate constraint changes.

	A	B	C	D	E	F	G	H	I	J	K
1											
2											
3			Resource Usage Per Unit of Each Activity								
4			Activity								
5		Resource	tail.wool slacks	tail.skirt	wool blazer	velvet pants	velvet shirt	b.-d. blouse	Totals		Resource Available
6		wool	3	0	2.5	0	0	0	25100	<=	45000
7		acetate	2	1.5	1.5	2	0	0	38000	<=	38000
8		rayon	0	2	0	0	0	1.5	30000	<=	30000
9		velvet	0	0	0	3	1.5	0	9000	<=	20000
10		unit profit	110	143.25	155.25	136	22	26.625	3490266.667		
11		Solution	4200	14733.33333	5000	0	6000	355.5555556			
12		Minimum	4200	2800	3000	0	0	0			
13		Maximum	7000	15000	5000	5500	6000	20000			

TrendLines should produce 4200 wool slacks, 14733.33 skirts, the minimum of 5000 wool blazers, no velvet pants, 6000 velvet shirts, and 355.55 button-down blouses. The production decisions for all other items are unaffected by the change. The net contribution of all clothing items equals \$840,000 + \$1,226,00 + \$ 2,025,000 + \$3,490,266.67 = \$7,581,266.67. The loss equals \$8,960,000 - \$7,581,266.67 = \$1,378,733.33.

g) The net contribution of one cashmere sweater sold in the November sale equals 0.6*$450 - $150 – 1.5*$60 = $30. After producing 4000 sweaters to be sold in September and October TrendLines has 3000 yards of cashmere as raw material left. It is optimal to produce 2000 more sweaters using the remaining 3000 yards of cashmere yielding an additional contribution of 2000*$30 = $60,000.

For the three linear programming problems determining the production plans for all other clothing items we need to include new decision variables representing the number of clothing items that are sold during the November sale. Clearly TrendLines does not want to produce items with a negative net contribution. Therefore, we need to consider only those clothing items that have a positive net contribution after taking the sales price into account.

	A	B	C	D	E	F	G	H
1								
2								
3				Activity				
4		Constraint	silk blouse	silk camisole	silk camisole(sale)	Totals		Constraint RHS
5		silk	1.5	0.5	0.5	18000	<=	18000
6		production	1	-1	-1	-8000	<=	0
7		unit profit	60.5	53.5	5.5	**1226000**		
8		Solution	**7000**	**15000**	**0**			
9		Maximum	12000	15000	36000			

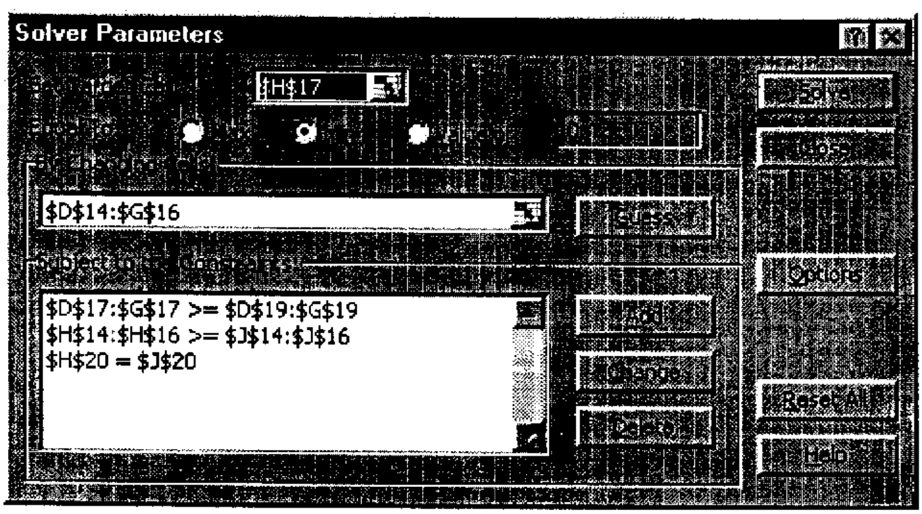
	A	B	C	D	E	F	G	H	I
1									
2									
3				Activity					
4		Constraint	cotton sweater	sweater(sale)	cotton m-s	m-s (sale)	Totals		Constraint RHS
5		wool	1.5	1.5	0.5	0.5	30000	<=	30000
6		production	1	1	-1	-1	-60000	<=	0
7		unit profit	66.25	14.25	33.75	3.75	**2025000**		
8		Solution	**0**	**0**	**60000**	**0**			

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2													
3						Resource Usage Per Unit of Each Activity							
4						Activity							
5		Resource	tail.wool slacks	tail.skirt	skirt (sale)	wool blazer	blazer (sale)	velvet pants	velvet shirt	b.-d. blouse	Totals		Resource Available
6		wool	3	0	0	2.5	2.5	0	0	0	25100	<=	45000
7		acetate	2	1.5	1.5	1.5	1.5	2	0	0	28000	<=	28000
8		rayon	0	2	2	0	0	0	0	1.5	30000	<=	30000
9		velvet	0	0	0	0	0	3	1.5	0	9000	<=	20000
10		unit profit	110	143.25	35.25	155.25	27.25	136	22	26.625	**2771933.333**		
11		Solution	**4200**	**8066.666667**	**0**	**5000**	**0**	**0**	**6000**	**9244.444444**			
12		Minimum	4200	2800	0	3000	0	0	0	0			
13		Maximum	7000	15000	15000	5000	20000	5500	6000	20000			

It only pays to produce 2000 more Cashmere sweaters. The production plan for all other items is the same as in part (b). The sale of the Cashmere sweaters reduces the loss by $60,000 to $2,037,066.67.

4-2 a) We define 12 decision variables, one for each age group surveyed in each region. Rob's restrictions are easily modeled as constraints. For example, his condition that at least 20 percent of the surveyed customers have to be from the first age group requires that the sum of the variables for the age group "18 to 25" across all three regions is at least 400. All his other requirements are modeled similarly. Finally, the sum of all variables has to equal 2000, because that is the number of customers Rob wants to have interviewed.

	A	B	C	D	E	F	G	H	I	J	K
1											
2					Cost per Person						
3					Age Group						
4				18 to 25	26 to 40	41 to 50	51 and over				
5			Silicon Valley	$4.75	$6.50	$6.50	$5.00				
6		Region	Big Cities	$5.25	$5.75	$6.25	$6.25				
7			Small Towns	$6.50	$7.50	$7.50	$7.25				
8											
9											
10											
11					Number of People Surveyed						
12					Age Group						
13				18 to 25	26 to 40	41 to 50	51 and over	Totals	Survey restrictions		
14			Silicon Valley	**600**	**0**	**0**	**300**	900	>=	300	
15		Region	Big Cities	**0**	**550**	**150**	**0**	700	>=	700	
16			Small Towns	**250**	**0**	**150**	**0**	400	>=	400	
17		Totals		850	550	300	300	**$11,200**	=	Total Cost	
18				>=	>=	>=	>=	$12,880.00	=	Bid	
19	Survey restrictions			400	550	300	300		Total Surveys		
20								2000	=	2000	
21											
22			Formula in cell H14:		=SUM(D14:G14)						
23			Formula in cell H15:		=SUM(D15:G15)						
24			Formula in cell H16:		=SUM(D16:G16)						
25			Formula in cell D17:		=SUM(D14:D16)						
26			Formula in cell E17:		=SUM(E14:E16)						
27			Formula in cell F17:		=SUM(F14:F16)						
28			Formula in cell G17:		=SUM(G14:G16)						
29			Formula in cell H20:		=SUM(D14:G16)						
30			Formula in cell H17:		=SUMPRODUCT(D5:G7,D14:G16)						
31			Formula in cell H18:		=1.15*H17						

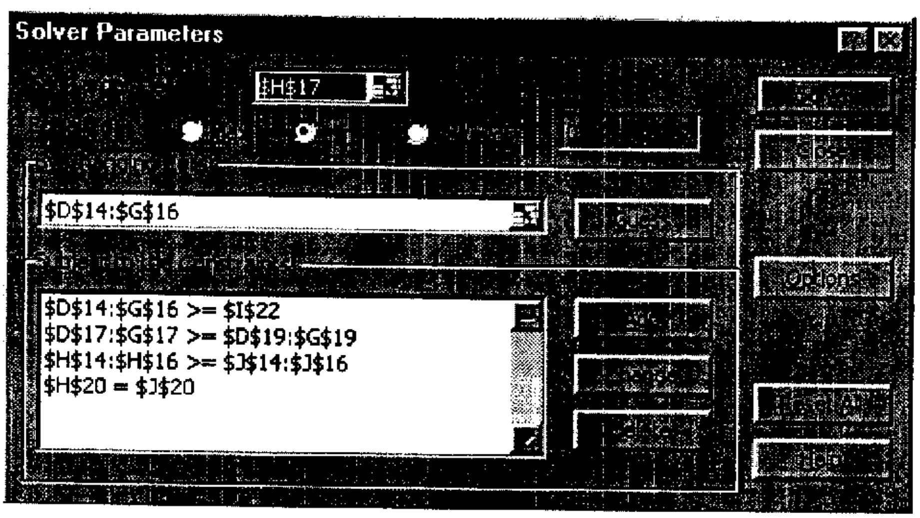


The cost of conducting the survey meeting all constraints imposed by AmeriBank incurs cost of $11,200. The mix of customers is displayed in the spreadsheet above.

b) Sophisticated Surveys will submit a bid of 1.15*$11200 = $12,880.

c) We need to include the new lower-bound constraint on all variables.

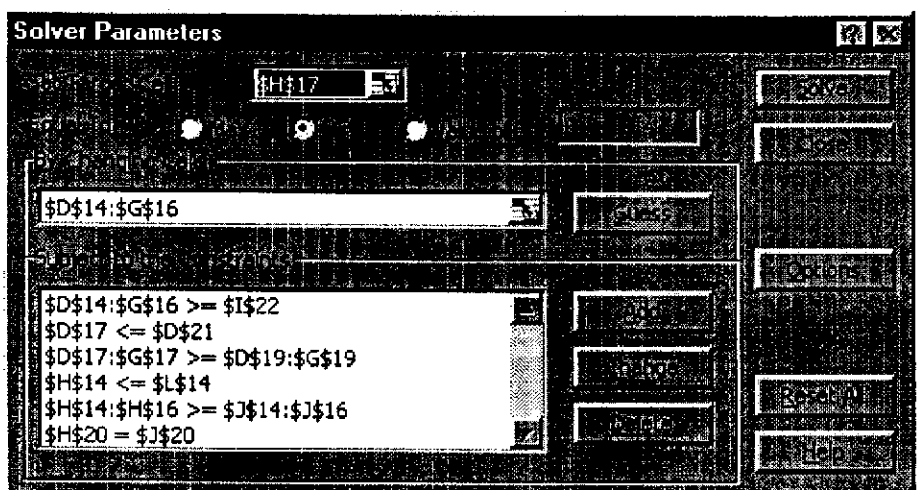
	A	B	C	D	E	F	G	H	I	J	K
1											
2				Cost per Person							
3				Age Group							
4				18 to 25	26 to 40	41 to 50	51 and over				
5			Silicon Valley	$4.75	$6.50	$6.50	$5.00				
6		Region	Big Cities	$5.25	$5.75	$6.25	$6.25				
7			Small Towns	$6.50	$7.50	$7.50	$7.25				
8											
9											
10											
11				Number of People Surveyed							
12				Age Group							
13				18 to 25	26 to 40	41 to 50	51 and over	Totals	Survey restrictions		
14			Silicon Valley	**600**	**50**	**50**	**200**	900	>=	300	
15		Region	Big Cities	**150**	**450**	**50**	**50**	700	>=	700	
16			Small Towns	**100**	**50**	**200**	**50**	400	>=	400	
17		Totals		850	550	300	300	**11387.5**	=	Total Cost	
18				>=	>=	>=	>=	$13,095.62	=	Bid	
19	Survey restrictions			400	550	300	300		Total Surveys		
20								2000	=	2000	
21								Minimum value for each variable			
22			Formula in cell H14:		"=SUM(D14:G14)"				50		
23			Formula in cell H15:		"=SUM(D15:G15)"						
24			Formula in cell H16:		"=SUM(D16:G16)"						
25			Formula in cell D17:		"=SUM(D14:D16)"						
26			Formula in cell E17:		"=SUM(E14:E16)"						
27			Formula in cell F17:		"=SUM(F14:F16)"						
28			Formula in cell G17:		"=SUM(G14:G16)"						
29			Formula in cell H20:		"=SUM(D14:G16)"						
30			Formula in cell H17:		"=SUMPRODUCT(D5:G7,D14:G16)"						
31			Formula in cell H18:		"=1.15*H17"						



The new requirement increases the bid to $13,095.62.

d) We include upper bounds on the total number of people surveyed in Silicon Valley and from the age group of 18 to 25 year-olds.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2				Cost per Person								
3				Age Group								
4				18 to 25	26 to 40	41 to 50	51 and over					
5			Silicon Valley	$4.75	$6.50	$6.50	$5.00					
6		Region	Big Cities	$5.25	$5.75	$6.25	$6.25					
7			Small Towns	$6.50	$7.50	$7.50	$7.25					
8												
9												
10												
11				Number of People Surveyed								
12				Age Group								
13				18 to 25	26 to 40	41 to 50	51 and over	Totals	Survey restrictions			
14			Silicon Valley	**100**	**50**	**50**	**450**	650	>=	300	<=	650
15		Region	Big Cities	**400**	**450**	**50**	**50**	950	>=	700		
16			Small Towns	**100**	**50**	**200**	**50**	400	>=	400		
17		Totals		600	550	300	550	**$11,575**	=	Total Cost		
18				>=	>=	>=	>=	$13,311.25	=	Bid		
19	Survey restrictions			400	550	300	300	Total Surveys				
20				<=				2000	=	2000		
21				600				Minimum value for each variable				
22									50			
23												
24			Formula in cell H14:		"=SUM(D14:G14)"							
25			Formula in cell H15:		"=SUM(D15:G15)"							
26			Formula in cell H16:		"=SUM(D16:G16)"							
27			Formula in cell D17:		"=SUM(D14:D16)"							
28			Formula in cell E17:		"=SUM(E14:E16)"							
29			Formula in cell F17:		"=SUM(F14:F16)"							
30			Formula in cell G17:		"=SUM(G14:G16)"							
31			Formula in cell H20:		"=SUM(D14:G16)"							
32			Formula in cell H17:		"=SUMPRODUCT(D5:G7,D14:G16)"							
33			Formula in cell H18:		"=1.15*H17"							



The new requirements increase the bid to $13,311.25.

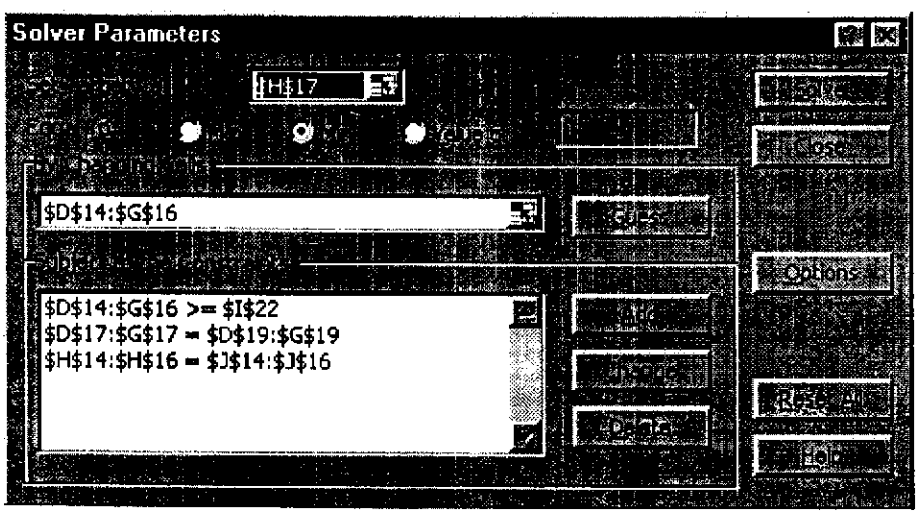
e) The three cost factors for the age group "18 to 25" are changed.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2				Cost per Person								
3				Age Group								
4				18 to 25	26 to 40	41 to 50	51 and over					
5			Silicon Valley	$6.50	$6.50	$6.50	$5.00					
6		Region	Big Cities	$6.75	$5.75	$6.25	$6.25					
7			Small Towns	$7.00	$7.50	$7.50	$7.25					
8												
9												
10												
11				Number of People Surveyed								
12				Age Group								
13				18 to 25	26 to 40	41 to 50	51 and over	Totals		Survey restrictions		
14			Silicon Valley	**50**	**50**	**50**	**500**	650	>=	300	<=	650
15		Region	Big Cities	**100**	**600**	**200**	**50**	950	>=	700		
16			Small Towns	**250**	**50**	**50**	**50**	400	>=	400		
17		Totals		400	700	300	600	**$ 12,025**	=	Total Cost		
18				>=	>=	>=	>=	$13,828.75	=	Bid		
19	Survey restrictions			400	550	300	300		Total Surveys			
20				<=				2000	=	2000		
21				600				Minimum value for each variable				
22									50			
23												
24			Formula in cell H14:		"=SUM(D14:G14)"							
25			Formula in cell H15:		"=SUM(D15:G15)"							
26			Formula in cell H16:		"=SUM(D16:G16)"							
27			Formula in cell D17:		"=SUM(D14:D16)"							
28			Formula in cell E17:		"=SUM(E14:E16)"							
29			Formula in cell F17:		"=SUM(F14:F16)"							
30			Formula in cell G17:		"=SUM(G14:G16)"							
31			Formula in cell H20:		"=SUM(D14:G16)"							
32			Formula in cell H17:		"=SUMPRODUCT(D5:G7,D14:G16)"							
33			Formula in cell H18:		"=1.15*H17"							

With the new cost factors the bid increases to $13,828.75.

f) We eliminate all lower and upper bounds on the age groups and regions and replace them with Rob's strict requirements. These requirements also ensure that exactly 2000 people are surveyed so that we can drop that constraint too.

	A	B	C	D	E	F	G	H	I	J	K
1											
2					Cost per Person						
3					Age Group						
4				18 to 25	26 to 40	41 to 50	51 and over				
5			Silicon Valley	$6.50	$6.50	$6.50	$5.00				
6		Region	Big Cities	$6.75	$5.75	$6.25	$6.25				
7			Small Towns	$7.00	$7.50	$7.50	$7.25				
8											
9											
10											
11					Number of People Surveyed						
12					Age Group						
13				18 to 25	26 to 40	41 to 50	51 and over	Totals		Survey restrictions	
14			Silicon Valley	**50**	**50**	**50**	**250**	400	=	400	
15		Region	Big Cities	**50**	**600**	**300**	**50**	1000	=	1000	
16			Small Towns	**400**	**50**	**50**	**100**	600	=	600	
17		Totals		500	700	400	400	**$ 12,475**	=	Total Cost	
18				=	=	=	=	$14,346.25	=	Bid	
19	Survey restrictions			500	700	400	400				
20											
21								Minimum value for each variable			
22			Formula in cell H14:		"=SUM(D14:G14)"				50		
23			Formula in cell H15:		"=SUM(D15:G15)"						
24			Formula in cell H16:		"=SUM(D16:G16)"						
25			Formula in cell D17:		"=SUM(D14:D16)"						
26			Formula in cell E17:		"=SUM(E14:E16)"						
27			Formula in cell F17:		"=SUM(F14:F16)"						
28			Formula in cell G17:		"=SUM(G14:G16)"						
29			Formula in cell H20:		"=SUM(D14:G16)"						
30			Formula in cell H17:		"=SUMPRODUCT(D5:G7,D14:G16)"						
31			Formula in cell H18:		"=1.15*H17"						



Rob's strict requirements increase the cost of the survey by $450. The new bid of Sophisticated Surveys is $14,346.25.

4-3

a & b)

Data:

Area	Number of Students	Percentage in 6th Grade	Percentage in 7th Grade	Percentage in 8th Grade	Bussing Cost ($/Student) School 1	School 2	School 3
1	450	0.32	0.38	0.3	300	0	700
2	600	0.37	0.28	0.35	-	400	500
3	550	0.3	0.32	0.38	600	300	200
4	350	0.28	0.4	0.32	200	500	-
5	500	0.39	0.34	0.27	0	-	400
6	450	0.34	0.28	0.38	500	300	0
				Capacity:	900	1100	1000

Solution:

Number of Students Assigned

	School 1	School 2	School 3	Total		
Area 1	0	450	0	450	=	450
Area 2	0	422.222222	177.777778	600	=	600
Area 3	0	227.777778	322.222222	550	=	550
Area 4	350	0	0	350	=	350
Area 5	366.666667	0	133.333333	500	=	500
Area 6	83.3333333	0	366.666667	450	=	450
Total	800	1100	1000			
	≤	≤	≤			
Capacity	900	1100	1000			

Total Bussing Cost = $ 555,555.56

Grade Constraints:

	School 1	School 2	School 3
6th Graders	269.333333	368.555556	339.111111
7th Graders	258	362.111111	300.888889
8th Graders	242.666667	369.333333	360
30% of Total	240	330	300
36% of Total	258	396	360

c) The recommendation to the school board is to assign students to schools as shown in the above solution section of the spreadsheet. Quantities that are not integers must be rounded since partial students cannot be sent.

d) The following solution decreases total bussing costs by over $135,000 but violates the grade constraints that were imposed. Solutions will vary and those than satisfy the grade constraints will be likely to increase the total bussing costs.

Data:

Area	Number of Students	Percentage in 6th Grade	Percentage in 7th Grade	Percentage in 8th Grade	Bussing Cost ($/Student) School 1	School 2	School 3
1	450	0.32	0.38	0.3	300	0	700
2	600	0.37	0.28	0.35	-	400	500
3	550	0.3	0.32	0.38	600	300	200
4	350	0.28	0.4	0.32	200	500	-
5	500	0.39	0.34	0.27	0	-	400
6	450	0.34	0.28	0.38	500	300	0
				Capacity:	900	1100	1000

Solution:

	Number of Students Assigned School 1	School 2	School 3	Total		
Area 1	0	450	0	450	=	450
Area 2	0	600	0	600	=	600
Area 3	0	0	550	550	=	550
Area 4	350	0	0	350	=	350
Area 5	500	0	0	500	=	500
Area 6	0	0	450	450	=	450
Total	850	1050	1000			
	≤	≤	≤			
Capacity	900	1100	1000			

Total Bussing Cost = $ 420,000.00

Grade Constraints:

	School 1	School 2	School 3
6th Graders	293	366	318
7th Graders	310	339	302
8th Graders	247	345	380
30% of Total	255	315	300
36% of Total	306	378	360

e) The number of students assigned from each area to each school changes to the solution shown below and the total bussing cost is reduced by almost $162,000.

Data:

Area	Number of Students	Percentage in 6th Grade	Percentage in 7th Grade	Percentage in 8th Grade	Bussing Cost ($/Student) School 1	School 2	School 3
1	450	0.32	0.38	0.3	300	0	700
2	600	0.37	0.28	0.35	-	400	500
3	550	0.3	0.32	0.38	600	300	0
4	350	0.28	0.4	0.32	0	500	-
5	500	0.39	0.34	0.27	0	-	400
6	450	0.34	0.28	0.38	500	300	0
				Capacity:	900	1100	1000

Solution:

	Number of Students Assigned School 1	School 2	School 3	Total		
Area 1	0	450	0	450	=	450
Area 2	0	600	0	600	=	600
Area 3	0	0	550	550	=	550
Area 4	350	0	0	350	=	350
Area 5	318.181818	0	181.818182	500	=	500
Area 6	131.818182	50	268.181818	450	=	450
Total	800	1100	1000			
	≤	≤	≤			
Capacity	900	1100	1000			

Total Bussing Cost = $ 393,636.36

Grade Constraints:

	School 1	School 2	School 3
6th Graders	266.909091	383	327.090909
7th Graders	285.090909	353	312.909091
8th Graders	248	364	360
30% of Total	240	330	300
36% of Total	288	396	360

f) The number of students assigned from each area to each school changes to the solution shown below and the total bussing cost is reduced by over S215,000.

Data:

Area	Number of Students	Percentage in 6th Grade	Percentage in 7th Grade	Percentage in 8th Grade	Bussing Cost ($/Student) School 1	School 2	School 3
1	450	0.32	0.38	0.3	0	0	700
2	600	0.37	0.28	0.35	-	400	500
3	550	0 3	0.32	0.38	600	0	0
4	350	0.28	0 4	0.32	0	500	-
5	500	0.39	0.34	0.27	0	-	400
6	450	0.34	0.28	0.38	500	0	0
				Capacity:	900	1100	1000

Solution:

	Number of Students Assigned School 1	School 2	School 3	Total		
Area 1	38.7096771	411.290323	0	450	=	450
Area 2	0	236.559139	363.440861	600	=	600
Area 3	0	77.95699	472.04301	550	=	550
Area 4	350	0	0	350	=	350
Area 5	435.483871	0	64.5161288	500	=	500
Area 6	75.8064517	374.193548	0	450	=	450
Tota	900	1100	900			
	≤	≤	≤			
Capacity	900	1100	1000			

Total Bussing Cost = $ 340,053.76

Grade Constraints:

	School 1	School 2	School 3
6th Graders	306	369.752688	301.247312
7th Graders	324	352.247312	274.752688
8th Graders	270	378	324
30% of Total	270	330	270
36% of Total	324	396	324

g)

Option	Cost	# students walking 1 to 1.5 miles	# students walking more than 1.5 miles
current	$555,555.56	0	0
1	$393,636.36	900	0
2	$340,053.76	900	491

h) Answers will vary.

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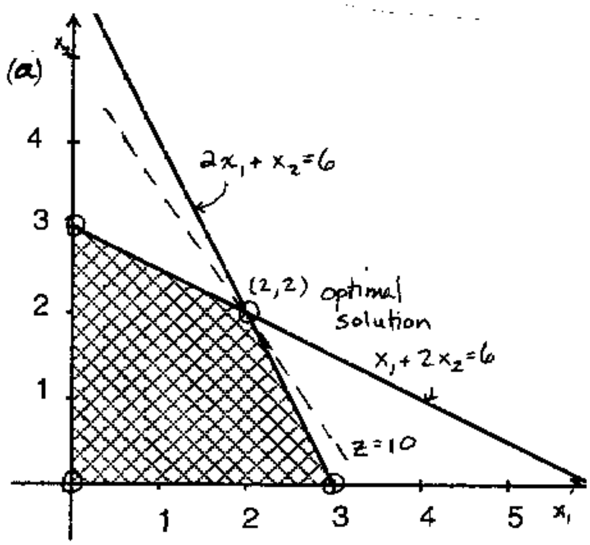
5.1-1.

(a)



$(x_1, x_2) = (2,2)$ is optimal with $z = 10$

(c) Maximize $Z = 3x_1 + 2x_2$

subject to: $2x_1 + x_2 + x_3 = 6$

$x_1 + 2x_2 + x_4 = 6$

$x_i \geq 0$ for $i = 1, 2, 3, 4$.

b) – d)

Defining Equations	CP	Feasible?	Basic Solution	Indicating Var's	Equations
$x_1 = 0$ $x_2 = 0$	(0,0)	Yes	(0,0,6,6)	x_1 x_2	$x_3 = 6$ $x_4 = 6$
$x_1 = 0$ $x_1 + 2x_2 = 6$	(0,3)	Yes	(0,3,3,0)	x_1 x_4	$x_2 + x_3 = 6$ $2x_2 = 6$
$x_1 = 0$ $2x_1 + x_2 = 6$	(0,6)	No	(0,6,0,-6)	x_1 x_3	$x_2 = 6$ $2x_2 + x_4 = 6$
$x_2 = 0$ $x_1 + 2x_2 = 6$	(6,0)	No	(6,0,-6,0)	x_2 x_4	$2x_1 + x_3 = 6$ $x_1 = 6$
$x_2 = 0$ $2x_1 + x_2 = 6$	(3,0)	Yes	(3,0,0,3)	x_2 x_3	$2x_1 = 6$ $x_1 + x_4 = 6$
$2x_1 + x_2 = 6$ $x_1 + 2x_2 = 6$	(2,2)	Yes	(2,2,0,0)	x_3 x_4	$2x_1 + x_2 = 6$ $x_1 + 2x_2 = 6$

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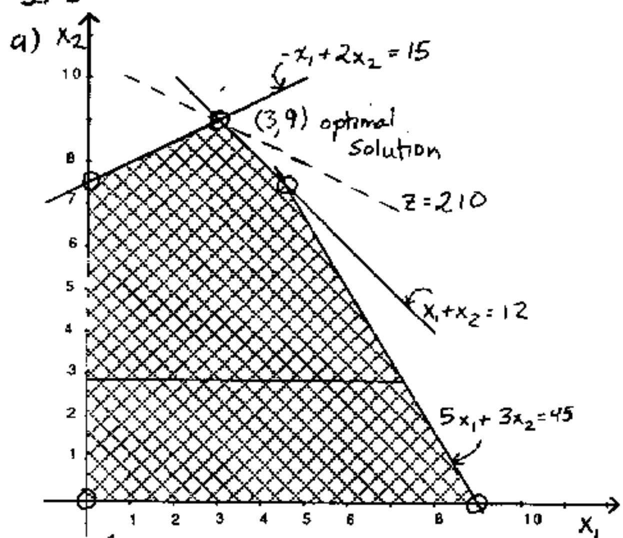
5.1-1.e)

Step #	CPF Sol'n	Deleted Defining Eqn.	Added Defining Eqn	Deleted Ind. Var.	Added Ind. Var
1	(0, 0)	$x_1 = 0$	$2x_1 + x_2 = 6$	x_1	x_3
2	(3, 0)	$x_2 = 0$	$x_1 + 2x_2 = 6$	x_2	x_4
3	(2, 2) Optimal				

5.1-2

a)



$(x_1, x_2) = (3, 9)$ is optimal with $z = 210$

(c) Maximize $Z = 10x_1 + 20x_2$

subject to

$$-x_1 + 2x_2 + x_3 = 15$$
$$x_1 + x_2 + x_4 = 12$$
$$5x_1 + 3x_2 + x_5 = 45$$

$x_i \geq 0$ for $i = 1, 2, 3, 4, 5$.

b)-d)

Defining Eqns	CP	Feasible?	Basic Sol'n	Ind. Variables	Equations
$x_1 = 0$ $x_2 = 0$	(0,0)	Yes	(0, 0, 15, 12, 45)	x_1 x_2	$x_3 = 15$ $x_4 = 12$ $x_5 = 45$
$x_1 = 0$ $-x_1 + 2x_2 = 15$	(0, 7.5)	Yes	(0, 7.5, 0, 4.5, 22.5)	x_1 x_3	$2x_2 = 15$ $x_2 + x_4 = 12$ $3x_2 + x_5 = 45$
$x_1 = 0$ $x_1 + x_2 = 15$	(0, 12)	No	(0, 12, -9, 0, 9)	x_1 x_4	$2x_2 + x_3 = 15$ $x_2 = 12$ $3x_2 + x_5 = 45$
$x_1 = 0$ $5x_1 + 3x_2 = 45$	(0, 15)	No	(0, 15, -15, -3, 0)	x_1 x_5	$2x_2 + x_3 = 15$ $x_2 + x_4 = 12$ $3x_2 = 45$
$x_2 = 0$ $-x_1 + 2x_2 = 15$	(-15, 0)	No	(-15, 0, 0, 3, 120)	x_2 x_3	$-x_1 = 15$ $x_1 + x_4 = 12$ $5x_1 + x_5 = 45$
$x_2 = 0$ $x_1 + x_2 = 12$	(12, 0)	No	(12, 0, 27, 0, -15)	x_2 x_4	$-x_1 + x_3 = 15$ $x_1 = 12$ $5x_1 + x_5 = 45$
$x_2 = 0$ $5x_1 + 3x_2 = 45$	(9, 0)	Yes	(9, 0, 24, 3, 0)	x_2 x_5	$-x_1 + x_3 = 15$ $x_1 + x_4 = 12$ $5x_1 = 45$
$-x_1 + 2x_2 = 15$ $x_1 + x_2 = 12$	(3, 9)	Yes	(3, 9, 0, 0, 3)	x_3 x_4	$-x_1 + 2x_2 = 15$ $x_1 + x_2 = 12$ $\quad 5x_1 + 3x_2 + x_5 = 45$
$-x_1 + 2x_2 = 15$ $5x_1 + 3x_2 = 45$	$(\frac{45}{13}, \frac{120}{13})$	No	$(\frac{45}{13}, \frac{120}{13}, 0, -\frac{9}{13}, 0)$	x_3 x_5	$-x_1 + 2x_2 = 15$ $x_1 + x_2 + x_4 = 12$ $\quad 5x_1 + 3x_2 = 45$
$x_1 + x_2 = 12$ $5x_1 + 3x_2 = 45$	(4.5, 7.5)	Yes	(4.5, 7.5, 3.5, 0, 0)	x_4 x_5	$-x_1 + 2x_2 + x_3 = 15$ $x_1 + x_2 = 12$ $\quad 5x_1 + 3x_2 = 45$

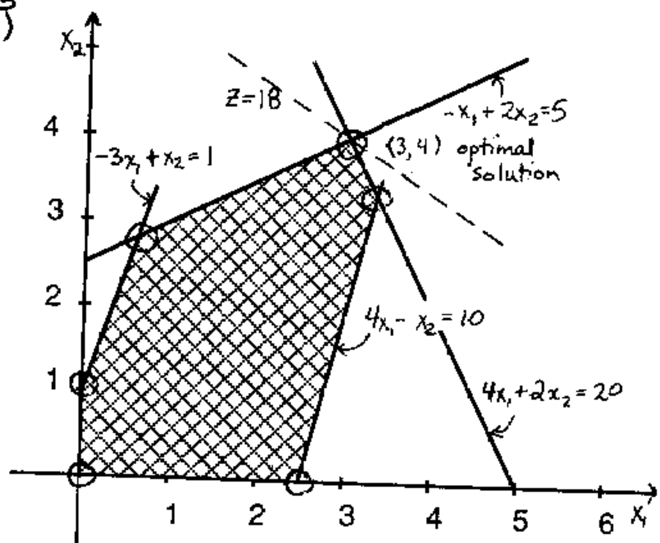
5.1-2

e)

Step #	CPF Sol'n	Deleted Defining Eqn.	Added Defining Eqn.	Deleted Ind. Var.	Added Ind. Var
1	(0, 0)	$x_2 = 0$	$-x_1 + 2x_2 = 15$	x_2	x_3
2	(0, 7.5)	$x_1 = 0$	$x_1 + x_2 = 12$	x_1	x_4
3	(3, 9)	Optimal			

5.1-3

a)



$(x_1, x_2) = (3,4)$ is optimal with $z = 18$

b)

CPF Sol'ns	Defining Eqns	BF Solutions	Nonbasic Var's	Z
(0,0)	$x_1 = 0$ $x_2 = 0$	(0, 0, 1, 20, 10, 5)	x_1 & x_2	0
(0,1)	$x_1 = 0$ $-3x_1 + x_2 = 1$	(0, 1, 0, 18, 11, 3)	x_1 & x_3	3
(0.6, 2.8)	$-3x_1 + x_2 = 1$ $-x_1 + 2x_2 = 5$	(0.6, 2.8, 0, 12, 10.4, 0)	x_3 & x_6	9.6
(3, 4)*	$-x_1 + 2x_2 = 5$ $4x_1 + 2x_2 = 20$	(3, 4, 6, 0, 2, 0)	x_4 & x_6	18 *
(3.33, 3.33)	$4x_1 + 2x_2 = 20$ $4x_1 - x_2 = 10$	(3.33, 3.33, 7.67, 0, 0, 1.67)	x_4 & x_5	16.67
(2.5, 0)	$4x_1 - x_2 = 10$ $x_2 = 0$	(2.5, 0, 8.5, 10, 0, 7.5)	x_2 & x_5	5

* Optimal

5.1-3. c)

CP Infeas Sol'ns	Defining Eqns	B Infeas Sol'ns	Non-basic Var's
$(-\frac{1}{3}, 0)$	$-3x_1+x_2=1$ $x_2=0$	$(-\frac{1}{3}, 0, 0, 21\frac{1}{3}, 11\frac{1}{3}, 4\frac{2}{3})$	x_3 x_2
$(-5, 0)$	$-x_1+2x_2=5$ $x_2=0$	$(-5, 0, -14, 40, 30, 0)$	x_6 x_2
$(0, 10)$	$4x_1+2x_2=20$ $x_1=0$	$(0, 10, -9, 0, 20, -15)$	x_4 x_1
$(0, \frac{5}{2})$	$-x_1+2x_2=5$ $x_1=0$	$(0, \frac{5}{2}, -\frac{3}{2}, 15, 12\frac{1}{2}, 0)$	x_6 x_1
$(\frac{9}{5}, \frac{32}{5})$	$4x_1+2x_2=20$ $-3x_1+x_2=1$	$(\frac{9}{5}, \frac{32}{5}, 0, 0, \frac{46}{5}, -6)$	x_4 x_3
$(11, 34)$	$-3x_1+x_2=1$ $4x_1-x_2=10$	$(11, 34, 0, -92, 0, -52)$	x_3 x_5
$(\frac{25}{7}, \frac{30}{7})$	$4x_1-x_2=10$ $-x_1+2x_2=5$	$(\frac{25}{7}, \frac{30}{7}, \frac{52}{7}, \frac{-20}{7}, 0, 0)$	x_5 x_6
$(5, 0)$	$4x_1+2x_2=20$ $x_2=0$	$(5, 0, 16, 0, -10, 10)$	x_4 x_2
$(0, -10)$	$4x_1-x_2=10$ $x_1=0$	$(0, -10, 11, 40, 0, 25)$	x_5 x_1

All sets yield a solution.

5.1-4. a) $(x_1, x_2, x_3) = (10, 0, 0)$

b) $x_2 = 0$

$x_3 = 0$

$x_1 - x_2 + 2x_3 = 10$

5.1-5

(a)

Corner Point Feasible Solution	Defining Equations
$(0,0,0)$	$x_1=0$, $x_2=0$, $x_3=0$
$(4,0,0)$	$x_1=4$, $x_2=0$, $x_3=0$
$(4,2,0)$	$x_1=4$, $x_1+x_2=6$, $x_3=0$
$(2,4,0)$	$x_2=4$, $x_1+x_2=6$, $x_3=0$
$(0,4,0)$	$x_1=0$, $x_2=4$, $x_3=0$
$(0,4,2)$	$x_1=0$, $x_2=4$, $-x_1+2x_3=4$
$(2,4,3)$	$x_1+x_2=6$, $x_2=4$, $-x_1+2x_3=4$
$(4,2,4)$	$x_1+x_2=6$, $x_1=4$, $-x_1+2x_3=4$
$(4,0,4)$	$x_2=0$, $x_1=4$, $-x_1+2x_3=4$
$(0,0,2)$	$x_2=0$, $x_1=0$, $-x_1+2x_3=4$

(b)

$$x_1+x_2=6$$
$$x_2=0$$
$$-x_1+2x_3=4$$

(c)

$$x_1=4$$
$$x_1=0$$
$$x_2=0$$

The system is inconsistent.

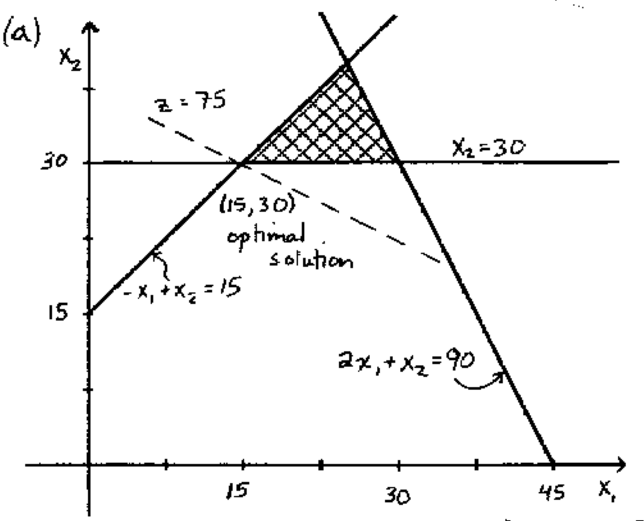
5.1-6

(a) & (b)

Defining Equations	Corner Point	Feasible?	Basic Solution $(y_1, y_2, y_3, y_4, y_5)$	Nonbasic Variables
$y_1=0$, $y_2=0$, $y_3=0$	$(0,0,0)$	No	$(0,0,0,-3,-5)$	y_1, y_2, y_3
$y_1=0$, $y_2=0$, $y_1+3y_3=3$	$(0,0,1)$	No	$(0,0,1,0,-3)$	y_1, y_2, y_4
$y_1=0$, $y_2=0$, $2y_2+2y_3=5$	$(0,0,2.5)$	Yes	$(0,0,2.5,4.5,0)$	y_1, y_2, y_5
$y_1=0$, $y_1+3y_3=3$, $y_3=0$	No Solution			y_1, y_3, y_4
$y_1=0$, $2y_2+2y_3=5$, $y_3=0$	$(0,2.5,0)$	No	$(0,2.5,0,-3,0)$	y_1, y_3, y_5
$y_2=0$, $y_1+3y_3=3$, $y_3=0$	$(3,0,0)$	No	$(3,0,0,0,-5)$	y_2, y_3, y_4
$y_2=0$, $2y_2+2y_3=5$, $y_3=0$	No Solution			y_2, y_3, y_5
$y_2=0$, $y_1+3y_3=3$, $2y_2+2y_3=5$	$(-4.5,0,2.5)$	No	$(-4.5,0,2.5,0,0)$	y_2, y_4, y_5
$y_3=0$, $y_1+3y_3=3$, $2y_2+2y_3=5$	$(3,2.5,0)$	Yes	$(3,2.5,0,0,0)$	y_3, y_4, y_5
$y_1=0$, $y_1+3y_3=3$, $2y_2+2y_3=5$	$(0,1.5,1)$	Yes	$(0,1.5,1,0,0)$	y_1, y_4, y_5

S.1-7

(a)



$(x_1, x_2) = (15, 30)$ is optimal with $z = 75$

(b)

Corner Point	Defining Equations	Basic Feasible Solution $(x_1, x_2, x_3, x_4, x_5)$	Nonbasic Variables
(15, 30)	$x_2 = 30$, $-x_1 + x_2 = 15$	(15, 30, 0, 30, 0)	x_3 and x_5
(30, 30)	$x_2 = 30$, $2x_1 + x_2 = 90$	(30, 30, 15, 0, 0)	x_4 and x_5
(25, 40)	$-x_1 + x_2 = 15$, $2x_1 + x_2 = 90$	(25, 40, 0, 0, 10)	x_3 and x_4

S1-8

(a) and (b)

Defining Equations	Corner Point	Feasible?	Basic Solution $(x_1, x_2, x_3, x_4, x_5)$	Nonbasic Variables
$x_1 = 0$, $x_2 = 0$	(0, 0)	No	(0, 0, -10, 6, -6)	x_1 and x_2
$x_1 = 0$, $2x_1 + x_2 = 10$	(0, 10)	No	(0, 10, 0, -14, 4)	x_1 and x_3
$x_1 = 0$, $-3x_1 + 2x_2 = 6$	(0, 3)	No	(0, 3, -7, 0, -3)	x_1 and x_4
$x_1 = 0$, $x_1 + x_2 = 6$	(0, 6)	No	(0, 6, -4, -6, 0)	x_1 and x_5
$x_2 = 0$, $2x_1 + x_2 = 10$	(5, 0)	No	(5, 0, 0, 21, -11)	x_2 and x_3
$x_2 = 0$, $-3x_1 + 2x_2 = 6$	(-2, 0)	No	(-2, 0, -14, 0, -8)	x_2 and x_4
$x_2 = 0$, $x_1 + x_2 = 6$	(6, 0)	Yes	(6, 0, 2, 24, 0)	x_2 and x_5
$2x_1 + x_2 = 10$, $-3x_1 + 2x_2 = 6$	(2, 6)	Yes	(2, 6, 0, 0, 8)	x_3 and x_4
$2x_1 + x_2 = 10$, $x_1 + x_2 = 6$	(4, 2)	Yes	(4, 2, 0, 14, 0)	x_3 and x_5
$-3x_1 + 2x_2 = 6$, $x_1 + x_2 = 6$	(1.2, 4.8)	No	(1.2, 4.8, -2.8, 0, 0)	x_4 and x_5

5.1-9 a) and b)

Defining Equations	Corner Point	Feasible?	Basic Solution $(x_1, x_2, x_3, x_4, x_5, x_6)$	Nonbasic Variables
$x_1=0$, $x_2=0$	(0,0)	Yes	(0, 0, 10, 60, 18, 44)	x_1 and x_2
$x_1=0$, $x_2=10$	(0, 10)	Yes	(0, 10, 0, 10, 8, 34)	x_1 and x_3
$x_1=0$, $2x_1+5x_2=60$	(0, 12)	No	(0, 12, -2, 0, 6, 32)	x_1 and x_4
$x_1=0$, $x_1+x_2=18$	(0, 18)	No	(0, 18, -8, -30, 0, 26)	x_1 and x_5
$x_1=0$, $3x_1+x_2=44$	(0, 44)	No	(0, 44, -34, -160, -26, 0)	x_1 and x_6
$x_2=0$, $x_2=10$	No Solution			x_2 and x_3
$x_2=0$, $2x_1+5x_2=60$	(30, 0)	No	(30, 0, 10, 0, -12, -46)	x_2 and x_4
$x_2=0$, $x_1+x_2=18$	(18, 0)	No	(18, 0, 10, 24, 0, -10)	x_2 and x_5
$x_2=0$, $3x_1+x_2=44$	(14.67, 0)	Yes	(14.67, 0, 10, 30.67, 3.33, 0)	x_2 and x_6
$x_2=10$, $2x_1+5x_2=60$	(5, 10)	Yes	(5, 10, 0, 0, 3, 19)	x_3 and x_4
$x_2=10$, $x_1+x_2=18$	(8, 10)	No	(8, 10, 0, -6, 0, 10)	x_3 and x_5
$x_3=10$, $3x_1+x_2=44$	(11.33, 10)	No	(11.33, 10, 0, -12.67, -3.33, 0)	x_3 and x_6
$2x_1+5x_2=60$, $x_1+x_2=18$	(10, 8)	Yes	(10, 8, 2, 0, 0, 6)	x_4 and x_5
$2x_1+5x_2=60$, $3x_1+x_2=44$	(12.31, 7.08)	No	(12.31, 7.08, 2.92, 0, -1.38, 0)	x_4 and x_6
$x_1+x_2=18$, $3x_1+x_2=44$	(13, 5)	Yes	(13, 5, 5, 9, 0, 0)	x_5 and x_6

5.1-10
a) If the feasible region is unbounded then there may be no optimal solution.
b) An optimal solution may contain all points on a line segment between two corner points.
c) If an adjacent corner point has an equal objective function value then all the points on the connecting line segment will also be optimal.

5.1-11
a) False, (p. 5-10) Property 1: (a) If there is exactly one optimal solution, then it must be a CPF. (b) If there are multiple optimal solutions, then at least 2 must be adjacent CPF Sol'ns. We can take a convex combination of 2 optimal CPF Sol'n's to get a non-CPF optimal solution, for example.

b) False, the number of CPF solutions is at most $\frac{(m+n)!}{m!\,n!} = \binom{m+n}{n}$ (p. 5-12).

c) False, Property 3 (p. 5-13). If this statement were true, it would mean no matter which CPF we started on, it or one of its adjacent CPF solutions would be optimal! (which, of course, is not the case)

5.1-12. (a) True. By property 1(a), there must be multiple solutions (since this optimal solution is not a CPF). This then means there are an infinite number of optimal solutions since any convex combination of optimal solutions is also optimal.

(b) True. The points on the line segment connecting x^* and x^{**} can be represented by $x = \alpha Z^* + (1 - \alpha)x^{**}$, α ranging from 0 to 1. Let $Z^* = Z^{**}$ represent the objective function values for x^* and x^{**}. Then the objective function value at x would be (by linearity) $Z = \alpha Z^* + (1 - \alpha)Z^{**} = \alpha Z^* + (1 - \alpha) Z^* = Z^*$. So the objective function value is the same at all these points. Furthermore, since the feasible region is convex, all these points must be feasible.

(c) False. The simultaneous solution of any set of n constraint boundary equations may be infeasible, or it may not even exist (e.g., $x_1 = 2$, $x_1 = 4$).

5.1-13. (a). True. If there are *no* optimal solutions, then the problem must have no feasible solutions *or* the objective value can be increased indefinitely (Chap. 3). The former is not the case (assumed in the problem) and the latter cannot be true since the feasible region is *bounded.* Thus, there must be at least one optimal solution.

(b) False. If a solution is optimal, it need not be a BF solution. A convex combination of BF solutions can give an optimal but not Basic (i.e. CP) solution. However, it is true that if optimal solutions exist, then at least one of them must be a BF solution. This follows straight from Property 1 (since BF solutions $\Rightarrow$ CPF solutions).

(c) True. Since BF solutions correspond to CPF solutions, this follows directly from Property 2.

5.1-14
$$\left.\begin{array}{l} x_1 = 0 \\ 2x_1 + x_2 + 3x_3 = 60 \\ 3x_1 + 3x_2 + 5x_3 = 120 \end{array}\right\}$$ This system has solution $(x_1, x_2, x_3) = (0, 15, 15)$

5.1-15
Since $X_2 > 0$ and $X_3 > 0$, $X_2 = 0$ and $X_3 = 0$ cannot be part of the three boundary equations. Therefore, the boundary equations are: $X_1 = 0$

$$2X_1 + X_2 + X_3 = 20$$
$$3X_1 + X_2 + 2X_3 = 30$$

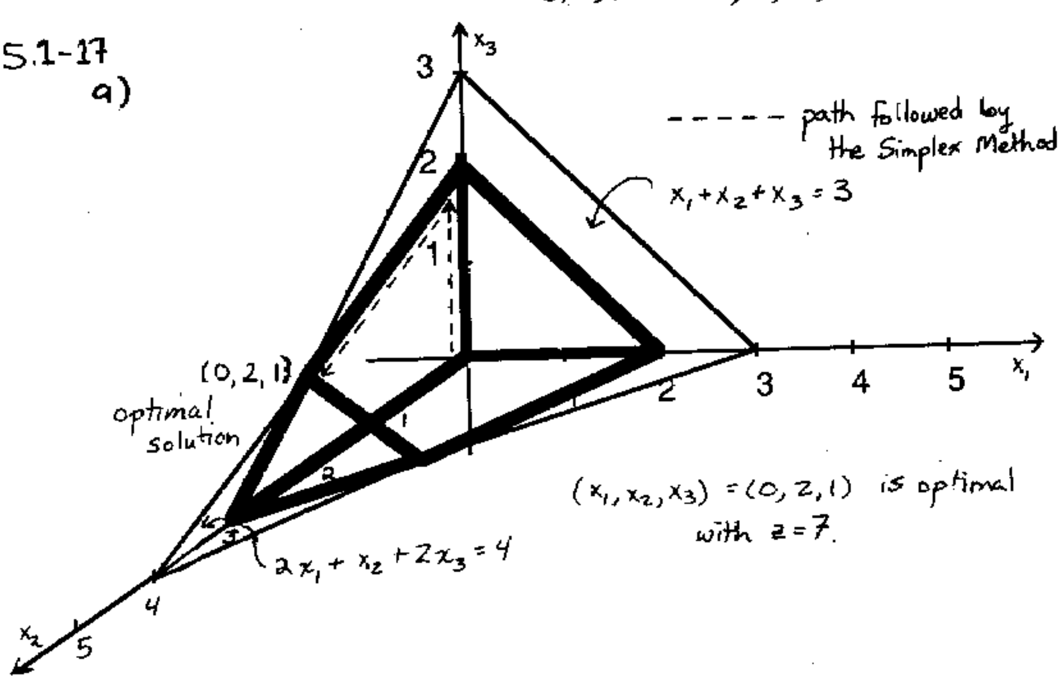
The solution to this set of equation (and, therefore, the optimal) is: $(X_1, X_2, X_3) = (0, 10, 10)$

5.1-16
Since $x_1 > 0$ and $x_3 > 0$, $x_1 = 0$ and $x_3 = 0$ can't be part of the three boundary equations. If $x_2 = 0$, then $x_1 + x_3 \le 24$ and $3x_1 + 3x_3 \le 60$ or, equivalently, $x_1 + x_3 \le 24$ and $x_1 + x_3 \le 20$. This implies the second constraint must have some slack and can't be a boundary equation.
Therefore, the boundary equations are: $x_2 = 0$

$$x_1 + 3x_2 + 2x_3 = 30$$
$$3x_1 + 5x_2 + 3x_3 = 60.$$

The optimal solution is $(x_1, x_2, x_3) = (10, 0, 10)$

5.1-17
a)



5.1-17

(b) The path is chosen because moving along the edges chosen provides the greatest increase in the objective value for a unit move in the chosen direction for any possible edge at each vertex/decision point.

(c)

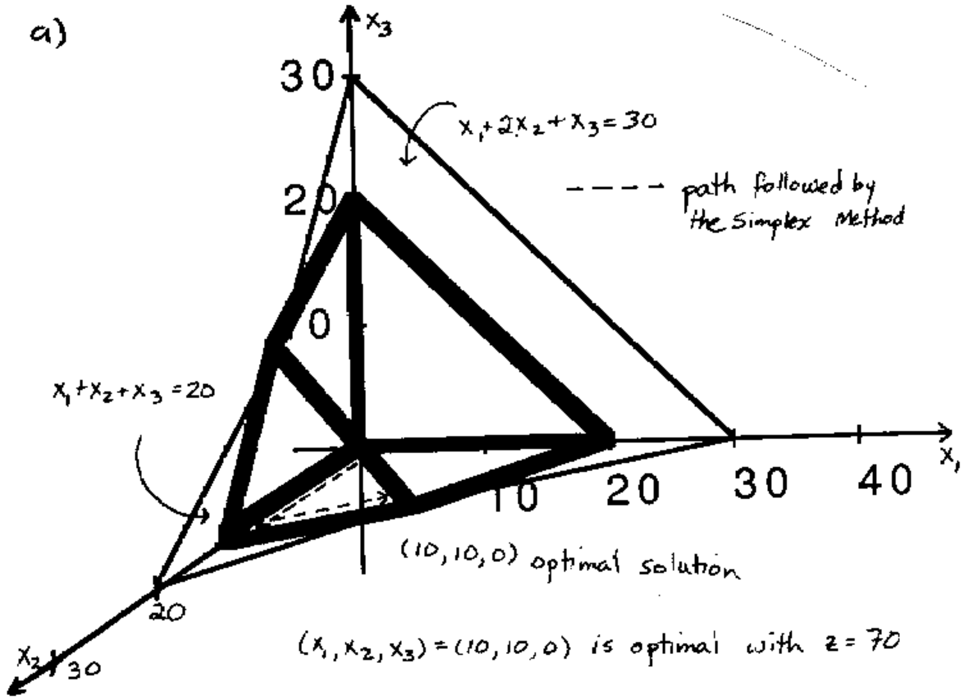
Edge	Constraint Boundary Equations	End Points	Additional Constraint
1	$x_2=0$, $x_1=0$	$(0,0,0)$ $(0,0,2)$	$x_3=0$ $2x_1+x_2+2x_3=4$
2	$2x_1+x_2+2x_3=4$, $x_1=0$	$(0,0,2)$ $(0,2,1)$	$x_2=0$ $x_1+x_2+x_3=3$

(d) and (e)

Corner Point	Defining Equations	Basic Feasible Solution (x_1,x_2,x_3,x_4,x_5)	Nonbasic Variables
$(0,0,0)$	$x_1=0$, $x_2=0$, $x_3=0$	$(0,0,0,4,3)$	x_1, x_2, x_3
$(0,0,2)$	$x_1=0$, $x_2=0$, $2x_1+x_2+2x_3=4$	$(0,0,2,0,1)$	x_1, x_2, x_4
$(0,2,1)$	$x_1=0$, $2x_1+x_2+2x_3=4$, $x_1+x_2+x_3=3$	$(0,0,2,0,1)$	x_1, x_4, x_5

The nonbasic variables, having value zero, are equivalent to indicating variables. They indicate that their associated inequality constraints are actually equalities. These associated equalities are the defining equations.

5.1-18 a)



$(x_1, x_2, x_3) = (10, 10, 0)$ is optimal with $z = 70$

(b) The path is chosen because moving along these edges provides the greatest increase in the objective value for a unit move in the chosen direction for any possible edge at each vertex/decision point.

(c)

Edge	Constraint Boundary Equations	End Points	Additional Constraint
1	$x_1=0$, $x_3=0$	(0,0,0)	$x_2=0$
		(0,15,0)	$x_1+2x_2+x_3=30$
2	$x_3=0$, $x_1+2x_2+x_3=30$	(0,15,0)	$x_1=0$
		(10,10,0)	$x_1+x_2+x_3=20$

(d) and (e)

Corner Point	Defining Equations	Basic Feasible Solution $(x_1, x_2, x_3, x_4, x_5)$	Nonbasic Variables
(0,0,0)	$x_1=0$, $x_2=0$, $x_3=0$	(0,0,0,20,30)	x_1, x_2, x_3
(0,15,0)	$x_1=0$, $x_3=0$, $x_1+2x_2+x_3=30$	(0, 15, 0, 5, 0)	x_1, x_3, x_5
(10,10,0)	$x_3=0$, $x_1+2x_2+x_3=30$, $x_1+x_2+x_3=20$	(10, 10, 0, 0, 0)	x_3, x_4, x_5

The nonbasic variables, having value zero, are equivalent to indicating variables. They indicate that their associated inequality constraints are actually equalities. These associated equalities are the defining equations.

5.1-19

(a) For the objective function Maximize $z = x_3$, both corner points $(4,2,4)$ and $(4,0,4)$ provide the maximum value $z=4$

(b) For the objective function Maximize $z = -x_1 + 2x_3$, all the corner points, $(0,0,2)$, $(4,0,4)$, $(4,2,4)$, $(2,4,3)$ and $(0,4,2)$ provide the maximum value $z=4$

5.1-20

(a) Geometrically each constraint is a plane and the points feasible for a given (inequality) constraint form a half-space. The line segment defined by any two feasible points must lie entirely on the feasible side of the plane and, therefore, all points on the line segment are feasible implying that the set of solutions for any one constraint is a convex set.

(b) Because the points in the feasible region of the linear program satisfy all constraints simultaneously, it must be the case that for any two feasible points the points on the line segment joining them must also satisfy each constraint (from part (a)). Therefore, the set of solutions that satisfy all constraints simultaneously is a convex set.

5.1-21

For the objective function Maximize $z = 3x_1 + 4x_2 + 3x_3$, starting at the point $(0,0,0)$ the first edge chosen would be that connecting to $(0,4,0)$ because moving along that edge increases z faster than any other edge.
From $(0,4,0)$ the edge that increases z fastest connects to either $(0,4,2)$ or $(2,4,0)$. From either of these the edge with fastest increase connects to $(2,4,3)$. From $(2,4,3)$ the only edge that will provide an increase in z connects to the optimal solution $(4,2,4)$

5.1-22 (a)

Original Constraint	Boundary Equation	Indicating Variable.
$x_1 \geq 0$	$x_1 = 0$	x_1
$x_2 \geq 0$	$x_2 = 0$	x_2
$x_3 \geq 0$	$x_3 = 0$	x_3
$x_1 + x_4 = 4$	$x_1 = 4$	x_4
$x_2 + x_5 = 4$	$x_2 = 4$	x_5
$x_1 + x_2 + x_6 = 6$	$x_1 + x_2 = 6$	x_6
$-x_1 + 2x_3 + x_7 = 4$	$-x_1 + 2x_3 = 4$	x_7

(b)

Corner Point Feasible Solution	Defining Equations	Basic Feasible Solutions $(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$	Nonbasic Variables
$(2,4,3)$	$x_1 + x_2 = 6$, $x_2 = 4$, $-x_1 + 2x_3 = 4$	$(2,4,3,2,0,0,0)$	x_5, x_6, x_7
$(4,2,4)$	$x_1 + x_2 = 6$, $-x_1 + 2x_3 = 4$, $x_1 = 4$	$(4,2,4,0,2,0,0)$	x_4, x_6, x_7
$(0,4,2)$	$x_1 = 0$, $x_2 = 4$, $-x_1 + 2x_3 = 4$	$(0,4,2,4,0,2,0)$	x_1, x_5, x_7
$(2,4,0)$	$x_3 = 0$, $x_1 + x_2 = 6$, $x_2 = 4$	$(2,4,0,2,0,0,6)$	x_3, x_5, x_6

(c) Because the sets of defining equations of $(4,2,4)$, $(0,4,2)$ and $(2,4,0)$ differ from the set of defining equations of $(2,4,3)$ by only one equation, they are adjacent to $(2,4,3)$. Since the sets of defining equations of $(4,2,4)$, $(0,4,2)$ and $(2,4,0)$ differ by more than one equation they are not adjacent.

If we substitute "nonbasic variables" for "defining equations" and "variable" for "equation," we see the same statement is true.

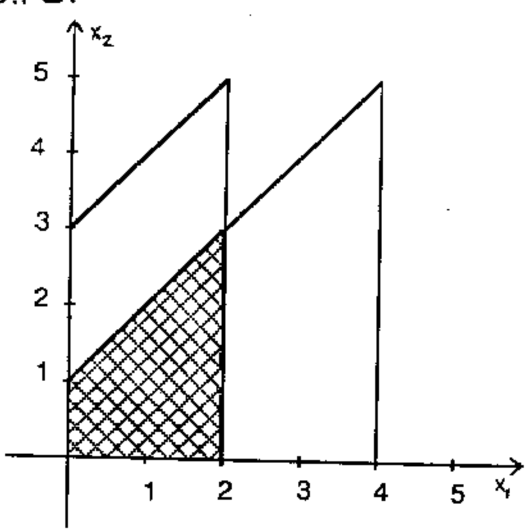
5.1-23

(a) x_5 enters

(b) x_4 leaves

(c) $(42,4,0,2,0,0)$

5.1-24



(a) The optimal solution is:

$$\begin{pmatrix} x_3 \\ x_1 \\ x_5 \end{pmatrix} = B^{-1}b = \frac{1}{27}\begin{pmatrix} 11 & -3 & 1 \\ -6 & 9 & -3 \\ 2 & -3 & 10 \end{pmatrix}\begin{pmatrix} 180 \\ 270 \\ 180 \end{pmatrix} = \begin{pmatrix} 50 \\ 30 \\ 50 \end{pmatrix}$$

$$z = c^T x = (8,4,6,3,9)\begin{pmatrix} 30 \\ 0 \\ 50 \\ 0 \\ 50 \end{pmatrix} = 990$$

(b) The shadow prices are:

$$c_B B^{-1} = \frac{1}{27}(6,8,9)\begin{pmatrix} 11 & -3 & 1 \\ -6 & 9 & -3 \\ 2 & -3 & 10 \end{pmatrix} = \begin{pmatrix} 1.33 \\ 1 \\ 2.67 \end{pmatrix}$$

5.2-1 (a) The optimal solution is:

$$\begin{pmatrix} x_3 \\ x_1 \\ x_5 \end{pmatrix} = B^{-1} b = \frac{1}{27} \begin{pmatrix} 11 & -3 & 1 \\ -6 & 9 & -3 \\ 2 & -3 & 10 \end{pmatrix} \begin{pmatrix} 180 \\ 270 \\ 180 \end{pmatrix} = \begin{pmatrix} 50 \\ 30 \\ 50 \end{pmatrix}$$

$$Z = c^T x = (8, 4, 6, 3, 9) \begin{pmatrix} 30 \\ 0 \\ 50 \\ 0 \\ 50 \end{pmatrix} = 990$$

(b) The shadow prices are:

$$c_B B^{-1} = \frac{1}{27} (6, 8, 9) \begin{pmatrix} 11 & -3 & 1 \\ -6 & 9 & -3 \\ 2 & -3 & 10 \end{pmatrix} = \begin{pmatrix} 1.33 \\ 1 \\ 2.67 \end{pmatrix}$$

5.2-2

$c = (5, 8, 7, 4, 6, 0, 0)$, $A = \begin{pmatrix} 2 & 3 & 3 & 2 & 2 & 1 & 0 \\ 3 & 5 & 4 & 2 & 4 & 0 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$

Iteration 0 : $B = B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $x_B = \begin{pmatrix} x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 20 \\ 30 \end{pmatrix} = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$

$c_B = (0, 0)$ $-c = (-5, -8, -7, -4, -6, 0, 0)$ so x_2 enters

Revised x_2 coefficients : $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ so x_7 leaves the basis

Iteration 1: $\eta = \begin{pmatrix} -3/5 \\ 1/5 \end{pmatrix}$ $B^{-1}_{NEW} = \begin{pmatrix} 1 & -3/5 \\ 0 & 1/5 \end{pmatrix}$ $x_B = \begin{pmatrix} x_6 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{3}{5} \\ 0 & \frac{1}{5} \end{pmatrix}\begin{pmatrix} 20 \\ 30 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

$c_B = (0, 8)$ Revised Row 0 : $(0, \frac{8}{5})\begin{pmatrix} 2 & 3 & 3 & 2 & 2 & 1 & 0 \\ 3 & 5 & 4 & 2 & 4 & 0 & 1 \end{pmatrix} - (5, 8, 7, 4, 6, 0, 0)$

$= (-\frac{1}{5}, 0, -\frac{3}{5}, -\frac{4}{5}, \frac{2}{5}, 0, \frac{8}{5})$

so x_4 enters the basis

Revised x_4 coefficients : $\begin{pmatrix} 1 & -3/5 \\ 0 & 1/5 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4/5 \\ 2/5 \end{pmatrix}$ so x_6 leaves the basis

Iteration 2 : $\eta = \begin{pmatrix} 5/4 \\ -1/2 \end{pmatrix}$ $B^{-1}_{NEW} = \begin{pmatrix} 5/4 & -3/4 \\ -1/2 & 1/2 \end{pmatrix}$ $x_B = \begin{pmatrix} x_4 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5/4 & -3/4 \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 20 \\ 30 \end{pmatrix} = \begin{pmatrix} 5/2 \\ 5 \end{pmatrix}$

$c_B = (4, 8)$ Revised Row 0 : $(1, 1)\begin{pmatrix} 2 & 3 & 3 & 2 & 2 & 1 & 0 \\ 3 & 5 & 4 & 2 & 4 & 0 & 1 \end{pmatrix} - (5, 8, 7, 4, 6, 0, 0)$

$= (0, 0, 0, 0, 0, 1, 1)$

Current Solution is optimal

$(x_1, x_2, x_3, x_4, x_5) = (0, 5, 0, \frac{5}{2}, 0)$ $Z = 8 \cdot 5 + 4 \cdot \frac{5}{2} = 50$

5.2-3

$c = (4, 3, 6, 0, 0)$, $A = \begin{pmatrix} 3 & 1 & 3 & 1 & 0 \\ 2 & 2 & 3 & 0 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 30 \\ 40 \end{pmatrix}$

Iteration 0 : $B = B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $x_B = \begin{pmatrix} x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 30 \\ 40 \end{pmatrix} = \begin{pmatrix} 30 \\ 40 \end{pmatrix}$

$c_B = (0, 0)$ $-c = (-4, -3, -6, 0, 0)$ so x_3 enters the basis

Revised x_3 coefficients: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ so x_4 leaves the basis

Iteration 1: $\eta = \begin{pmatrix} 1/3 \\ -1 \end{pmatrix}$ $B^{-1}_{NEW} = \begin{pmatrix} 1/3 & 0 \\ -1 & 1 \end{pmatrix}$ $x_B = \begin{pmatrix} x_3 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1/3 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 30 \\ 40 \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$

5.2-3 (cont')

$c_B = (6,0)$ Revised Row 0 : $(2,0)\begin{pmatrix} 3 & 1 & 3 & 1 & 0 \\ 2 & 2 & 3 & 0 & 1 \end{pmatrix} - (4,3,6,0,0)$

$= (2,-1,0,2,0)$ so x_2 enters the basis.

Revised x_2 coefficients : $\begin{pmatrix} 1/3 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1 \end{pmatrix}$ so x_5 leaves the basis.

Iteration 2 : $\eta = \begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix}$ $B_{NEW}^{-1} = \begin{pmatrix} 2/3 & -1/3 \\ -1 & 1 \end{pmatrix}$ $x_B = \begin{pmatrix} x_3 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2/3 & -1/3 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 30 \\ 40 \end{pmatrix} = \begin{pmatrix} 20/3 \\ 10 \end{pmatrix}$

$c_B = (6,3)$ Revised Row 0 : $(1,1)\begin{pmatrix} 3 & 1 & 3 & 1 & 0 \\ 2 & 2 & 3 & 0 & 1 \end{pmatrix} - (4,3,6,0,0)$

$= (1,0,0,1,1)$

Current Solution is optimal.

$(x_1, x_2, x_3) = (0, 10, \frac{20}{3})$ $z = 10 \cdot 3 + \frac{20}{3} \cdot 6 = 70$

5.2-4

For Corner Point $(0,0)$: $x_B = \begin{pmatrix} x_3 \\ x_4 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = B^{-1}$

$x_B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$ Row 0 = $(-3,-2,0,0)$

For Corner Point $(3,0)$: $x_B = \begin{pmatrix} x_1 \\ x_4 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$ $B^{-1} = \begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{pmatrix}$

$x_B = \begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ $c_B = (3,0)$

Row 0 $= \left(\frac{3}{2}, 0\right)\begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix} - (3,2,0,0) = \left(0, -\frac{1}{2}, \frac{3}{2}, 0\right)$

For Corner Point $(2,2)$: $x_B = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ $B^{-1} = \begin{pmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{pmatrix}$

$x_B = \begin{pmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{pmatrix}\begin{pmatrix} 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ $c_B = (3,2)$

Row 0 : $(3,2)\begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix} - (3,2,0,0) = (0,0,\frac{1}{3},\frac{1}{3})$

$(x_1, x_2) = (2,2)$ is optimal with $z = 3 \cdot 2 + 2 \cdot 2 = 10$

5.2-5

$c = (1, 2, 0, 0) \quad A = \begin{pmatrix} 1 & 3 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

Iteration 0: $B = B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad x_B = \begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 8 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

$c_B = (0, 0) \quad -c = (-1, -2, 0, 0)$ so x_2 enters the basis

Revised x_2 coefficients : $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ so x_3 leaves the basis

Iteration 1: $\eta = \begin{pmatrix} 1/3 \\ -1/3 \end{pmatrix} \quad B^{-1}_{NEW} = \begin{pmatrix} 1/3 & 0 \\ -1/3 & 1 \end{pmatrix} \quad x_B = \begin{pmatrix} x_2 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1/3 & 0 \\ -1/3 & 1 \end{pmatrix}\begin{pmatrix} 8 \\ 4 \end{pmatrix} = \begin{pmatrix} 8/3 \\ 4/3 \end{pmatrix}$

$c_B = (2, 0)$ Revised Row 0: $(2/3, 0)\begin{pmatrix} 1 & 3 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} - (1, 2, 0, 0)$

$= (-1/3, 0, 2/3, 0)$ x_1 enters the basis

Revised x_1 coefficients : $\begin{pmatrix} 1/3 & 0 \\ -1/3 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix}$ x_4 leaves the basis

Iteration 2: $\eta = \begin{pmatrix} -\frac{1}{2} \\ 3/2 \end{pmatrix} \quad B^{-1}_{NEW} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 3/2 \end{pmatrix} \quad x_B = \begin{pmatrix} x_2 \\ x_1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}\begin{pmatrix} 8 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$c_B = (2, 1)$ Revised Row 0 : $(\frac{1}{2}, \frac{1}{2})\begin{pmatrix} 1 & 3 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} - (1, 2, 0, 0)$

$= (0, 0, \frac{1}{2}, \frac{1}{2})$

Current Solution is optimal

$(x_1, x_2) = (2, 2) \quad z = 2 \cdot 1 + 2 \cdot 2 = 6$

5.2-6

$c = (2, -2, 3, 0, 0, 0) \quad A = \begin{pmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 4 \\ 2 \\ 12 \end{pmatrix}$

Iteration 0: $B = B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad x_B = \begin{pmatrix} x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \\ 12 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 12 \end{pmatrix}$

$c_B = (0, 0, 0) \quad -c = (-2, 2, -3, 0, 0, 0)$ x_3 enters the basis

Revised x_3 coefficients : $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ x_5 leaves the basis.

Iteration 1: $\eta = \begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix} \quad B^{-1}_{NEW} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix} \quad x_B = \begin{pmatrix} x_4 \\ x_3 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \\ 12 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix}$

$c_B = (0, 3, 0)$ Revised Row 0: $(0, 3, 0)\begin{pmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{pmatrix} - (2, -2, 3, 0, 0, 0)$

$= (4, -1, 0, 0, 3, 0)$ x_2 enters the basis

5.2-6 (cont')

Revised x_2 coefficients: $\begin{pmatrix}1&-1&0\\0&1&0\\0&-3&1\end{pmatrix}\begin{pmatrix}1\\-1\\1\end{pmatrix}=\begin{pmatrix}2\\-1\\4\end{pmatrix}$ x_4 leaves the basis.

Iteration 2: $\eta=\begin{pmatrix}1/2\\1/2\\-2\end{pmatrix}$ $B_{NEW}^{-1}=\begin{pmatrix}1/2&-1/2&0\\1/2&1/2&0\\-2&-1&1\end{pmatrix}$ $x_B=\begin{pmatrix}x_2\\x_3\\x_6\end{pmatrix}=\begin{pmatrix}\frac12&-\frac12&0\\\frac12&\frac12&0\\-2&-1&1\end{pmatrix}\begin{pmatrix}4\\2\\12\end{pmatrix}=\begin{pmatrix}1\\3\\2\end{pmatrix}$

$c_B=(-2,3,0)$ Revised Row 0: $(\frac12,\frac52,0)\begin{pmatrix}-1&1&1&1&0&0\\2&-1&1&0&1&0\\1&1&3&0&0&1\end{pmatrix}-(2,-2,3,0,0,0)$

$=(\frac52,0,0,\frac12,\frac52,0)$

Current Solution is optimal

$(x_1,x_2,x_3)=(0,1,3)$ $Z=-2\cdot1+3\cdot3=7$

5.2-7

a) $c=(10,20,0,0,0)$ $A=\begin{pmatrix}-1&2&1&0&0\\1&1&0&1&0\\5&3&0&0&1\end{pmatrix}$ $b=\begin{pmatrix}15\\12\\45\end{pmatrix}$

Iteration 0: $B=B^{-1}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$ $x_B=\begin{pmatrix}x_3\\x_4\\x_5\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}15\\12\\45\end{pmatrix}=\begin{pmatrix}15\\12\\45\end{pmatrix}$

$c_B=(0,0,0)$ $-c=(-10,-20,0,0,0)$ x_2 enters the basis

Revised x_2 coefficients: $\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}2\\1\\3\end{pmatrix}=\begin{pmatrix}2\\1\\3\end{pmatrix}$ x_3 leaves the basis

Iteration 1: $\eta=\begin{pmatrix}1/2\\-1/2\\-3/2\end{pmatrix}$ $B_{NEW}^{-1}=\begin{pmatrix}\frac12&0&0\\-\frac12&1&0\\-\frac32&0&1\end{pmatrix}$ $x_B=\begin{pmatrix}x_2\\x_4\\x_5\end{pmatrix}=\begin{pmatrix}\frac12&0&0\\-\frac12&1&0\\-\frac32&0&1\end{pmatrix}\begin{pmatrix}15\\12\\45\end{pmatrix}=\begin{pmatrix}15/2\\9/2\\45/2\end{pmatrix}$

$c_B=(20,0,0)$ Revised Row 0: $(10,0,0)\begin{pmatrix}-1&2&1&0&0\\1&1&0&1&0\\5&3&0&0&1\end{pmatrix}-(10,20,0,0,0)$

$=(-20,0,10,0,0)$ x_1 enters the basis

Revised x_1 coefficients: $\begin{pmatrix}1/2&0&0\\-1/2&1&0\\-3/2&0&1\end{pmatrix}\begin{pmatrix}-1\\1\\5\end{pmatrix}=\begin{pmatrix}-1/2\\3/2\\13/2\end{pmatrix}$ x_4 leaves the basis.

Iteration 2: $\eta=\begin{pmatrix}1/3\\2/3\\-13/3\end{pmatrix}$ $B_{NEW}^{-1}=\begin{pmatrix}1/3&1/3&0\\-1/3&2/3&0\\2/3&-13/3&1\end{pmatrix}$ $x_B=\begin{pmatrix}x_2\\x_1\\x_5\end{pmatrix}=\begin{pmatrix}\frac13&\frac13&0\\-\frac13&\frac23&0\\\frac23&-\frac{13}{3}&1\end{pmatrix}\begin{pmatrix}15\\12\\45\end{pmatrix}=\begin{pmatrix}9\\3\\3\end{pmatrix}$

$c_B=(20,10,0)$ Revised Row 0: $(10/3,40/3,0)\begin{pmatrix}-1&2&1&0&0\\1&1&0&1&0\\5&3&0&0&1\end{pmatrix}-(10,20,0,0,0)$

$=(0,0,10/3,40/3,0)$

Current Solution is optimal

$(x_1,x_2)=(3,9)$ $Z=10\cdot3+20\cdot9=210$

5.2-7(b) $c = (5, 4, -1, 3, 0, 0)$ $A = \begin{pmatrix} 3 & 2 & -3 & 1 & 1 & 0 \\ 3 & 3 & 1 & 3 & 0 & 1 \end{pmatrix}$ $b = \begin{pmatrix} 24 \\ 36 \end{pmatrix}$

Iteration 0: $B = B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $x_B = \begin{pmatrix} x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 24 \\ 36 \end{pmatrix} = \begin{pmatrix} 24 \\ 36 \end{pmatrix}$

$c_B = (0, 0)$ $-c = (-5, -4, 1, -3, 0, 0)$ x_1 enters the basis.

Revised x_1 coefficients: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ x_5 leaves the basis.

Iteration 1: $\eta = \begin{pmatrix} 1/3 \\ -1 \end{pmatrix}$ $B^{-1}_{New} = \begin{pmatrix} 1/3 & 0 \\ -1 & 1 \end{pmatrix}$ $x_B = \begin{pmatrix} x_1 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1/3 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 24 \\ 36 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix}$

$c_B = (5, 0)$ Revised Row 0: $(5/3, 0)\begin{pmatrix} 3 & 2 & -3 & 1 & 1 & 0 \\ 3 & 3 & 1 & 3 & 0 & 1 \end{pmatrix} - (5, 4, -1, 3, 0, 0)$

$= (0, -2/3, -4, -4/3, 5/3, 0)$ x_3 enters

Revised x_3 coefficients: $\begin{pmatrix} 1/3 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ x_6 leaves the basis.

Iteration 2: $\eta = \begin{pmatrix} 1/4 \\ 1/4 \end{pmatrix}$ $B^{-1}_{NEW} = \begin{pmatrix} \frac{1}{12} & \frac{1}{4} \\ -1/4 & \frac{1}{4} \end{pmatrix}$ $x_B = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1/12 & 1/4 \\ -1/4 & 1/4 \end{pmatrix}\begin{pmatrix} 24 \\ 36 \end{pmatrix} = \begin{pmatrix} 11 \\ 3 \end{pmatrix}$

$c_B = (5, -1)$ Revised Row 0: $(2/3, 1)\begin{pmatrix} 3 & 2 & -3 & 1 & 1 & 0 \\ 3 & 3 & 1 & 3 & 0 & 1 \end{pmatrix} - (5, 4, -1, 3, 0, 0)$

$= (0, 1/3, 0, 2/3, 2/3, 1)$

Current Solution is optimal

$(x_1, x_2, x_3, x_4) = (11, 0, 3, 0)$ $z = 11 \cdot 5 + 3(-1) = 52$

5.3-1

(a) $B^{-1} = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$. The final constraint columns for (x_1, x_2, x_3) will be

$$B^{-1}\begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 1 & 0 \\ 2 & 0 & 0 \\ 4 & 0 & 1 \end{pmatrix}$$

$C_B = (-1, 0, 2)$. The final objective coefficients for (x_1, x_2, x_3) will be

$$(-1, 0, 2)\begin{pmatrix} 5 & 1 & 0 \\ 2 & 0 & 0 \\ 4 & 0 & 1 \end{pmatrix} - (1, -1, 2) = (2, 0, 0)$$

The right-hand side is $B^{-1}b = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 14 \\ 5 \\ 11 \end{pmatrix}$. $Z = (-1, 0, 2)\begin{pmatrix} 14 \\ 5 \\ 11 \end{pmatrix} = 8$

Final Tableau becomes:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	2	0	0	1	1	0	8
X2	1	0	5	1	0	1	3	0	14
X6	2	0	2	0	0	0	1	1	5
X3	3	0	4	0	1	1	2	0	11

(b) The Defining Equations are:

$$2x_1 - 2x_2 + 3x_3 = 5$$
$$x_1 + x_2 - x_3 = 3$$
$$x_1 = 0$$

5.3-2 (a) The final constraint columns for (x_1, x_2, x_3, x_4) will be:

$$B^{-1}\begin{pmatrix} 4 & 2 & 1 & 1 \\ 3 & 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 4 & 2 & 1 & 1 \\ 3 & 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 2 & 0 & 3 & 1 \end{pmatrix}$$

The final objective coefficients for (x_1, x_2, x_3, x_4) will be:

$$-c + c_B B^{-1} A = -(4,3,1,2) + (3,2)\begin{pmatrix} 1 & 1 & -1 & 0 \\ 2 & 0 & 3 & 1 \end{pmatrix} = (3,0,2,0)$$

The right-hand side is $B^{-1}b = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$. $Z = (3,2)\begin{pmatrix} 1 \\ 3 \end{pmatrix} = 9$

The final tableau becomes:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	3	0	2	0	1	1	9
X2	1	0	1	1	-1	0	1	-1	1
X4	2	0	2	0	3	1	-1	2	3

(b) The Defining Equations are:

$$4x_1 + 2x_2 + x_3 + x_4 = 5$$
$$3x_1 + x_2 + 2x_3 + x_4 = 4$$
$$x_1 = 0$$
$$x_3 = 0$$

5.3-3 The final constraint columns for (x_1, x_2, x_3) will be:

$$B^{-1}\begin{pmatrix} 2 & 2 & \frac{1}{2} \\ -4 & -2 & -\frac{3}{2} \\ 1 & 2 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 2 & 2 & \frac{1}{2} \\ -4 & -2 & -\frac{3}{2} \\ 1 & 2 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0 & 4 & 0 \\ 0 & 4 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

The final objective coefficients for (x_1, x_2, x_3) will be:

$$-(6,1,2) + (0,2,6)\begin{pmatrix} 0 & 4 & 0 \\ 0 & 4 & 1 \\ 1 & 0 & 0 \end{pmatrix} = (0,7,0)$$

The right-hand side will be $B^{-1}b = \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \\ 1 \end{pmatrix}$

$$Z = (0,2,6)\begin{pmatrix} 7 \\ 0 \\ 1 \end{pmatrix} = 6$$

The final tableau becomes:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	7	0	2	0	2	6
X5	1	0	0	4	0	1	1	2	7
X3	2	0	0	4	1	-2	0	4	0
X1	3	0	1	0	0	1	0	-1	1

5.3-4 (a) The final constraint coefficients for (x_1, x_2, x_3) will be:

$$B^{-1}\begin{pmatrix} 1 & 1 & 3 \\ 2 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -2 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 1 & 3 \\ 2 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 0 & 1 \\ -\frac{3}{2} & 1 & 0 \end{pmatrix}.$$

The final objective coefficients will be:

$$-(1,-1,2) + (0,2,-1)\begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 0 & 1 \\ -\frac{3}{2} & 1 & 0 \end{pmatrix} = \left(\tfrac{3}{2}, 0, 0\right)$$

The right-hand side will be: $B^{-1}b = \begin{pmatrix} 1 & -1 & -2 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 15 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$

$$Z = (0, 2, -1)\begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} = 5$$

The final tableau becomes:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	1.5	0	0	0	1.5	0.5	5
X4	1	0	1	0	0	1	-1	-2	5
X3	2	0	0.5	0	1	0	0.5	0.5	3
X2	3	0	-1.5	1	0	0	-0.5	0.5	1

(b) The Defining Equations are:

$$2x_1 - x_2 + x_3 = 2$$
$$-x_1 + x_2 + x_3 = 4$$
$$x_1 = 0$$

5.3-5 (a) The constraint coefficients for (x_1, x_2, x_3) will be:

$$B^{-1}\begin{pmatrix} 8 & 2 & 3 \\ 4 & 3 & 0 \\ 2 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{16} & -\frac{1}{8} & 0 & 0 \\ -\frac{1}{4} & \frac{1}{2} & 0 & 0 \\ -\frac{3}{8} & \frac{1}{4} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 8 & 2 & 3 \\ 4 & 3 & 0 \\ 2 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \frac{9}{16} \\ 0 & 1 & -\frac{3}{4} \\ 0 & 0 & -\frac{1}{8} \\ 0 & 0 & 1 \end{pmatrix}$$

The objective coefficients for (x_1, x_2, x_3) will be:

$$-(20, 6, 8) + (20, 6, 0, 0)\begin{pmatrix} 1 & 0 & \frac{9}{16} \\ 0 & 1 & -\frac{3}{4} \\ 0 & 0 & -\frac{1}{8} \\ 0 & 0 & 1 \end{pmatrix} = \left(0, 0, -\tfrac{5}{4}\right)$$

The right-hand side is: $B^{-1}b = \begin{pmatrix} \frac{3}{16} & -\frac{1}{8} & 0 & 0 \\ -\frac{1}{4} & \frac{1}{2} & 0 & 0 \\ -\frac{3}{8} & \frac{1}{4} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 200 \\ 100 \\ 50 \\ 20 \end{pmatrix} = \begin{pmatrix} 25 \\ 0 \\ 0 \\ 20 \end{pmatrix}$

$$Z = c_B^T x_B = (20, 6, 0, 0)\begin{pmatrix} 25 \\ 0 \\ 0 \\ 20 \end{pmatrix} = 500$$

5.3-5. a) (cont')

The tableau is:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	Right side
Z	0	1	0	0	-1.25	2.25	0.5	0	0	500
X1	1	0	1	0	0.563	0.188	-0.13	0	0	25
X2	2	0	0	1	-0.75	-0.25	0.5	0	0	0
X6	3	0	0	0	-0.13	-0.38	0.25	1	0	0
X7	4	0	0	0	1	0	0	0	1	20

(b) The Revised Simplex Method would generate the reduced costs for Row 0 and then the revised column for x_3

(c) The Defining Equations are:

$$8x_1 + 2x_2 + 3x_3 = 200$$
$$4x_1 + 3x_2 = 100$$
$$x_3 = 0$$

$2x_1 + x_3 = 50$ is also a "tight" constraint

5.3-6. $B^{-1} = \begin{pmatrix} \frac{1}{12} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}$ The final right-hand side will be:

$$B^{-1}b = \begin{pmatrix} \frac{1}{12} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} 120 \\ 150 \end{pmatrix} = \begin{pmatrix} 55 \\ 15 \end{pmatrix}$$

$$Z^* = (6, 5, -1, 4)\begin{pmatrix} 55 \\ 0 \\ 15 \\ 0 \end{pmatrix} = 315$$

Also Z^* can be calculated by the reduced objective coefficients:

$$Z^* = \left(\tfrac{3}{4}, \tfrac{5}{4}\right)\begin{pmatrix} 120 \\ 150 \end{pmatrix} = 315$$

$b_1^* = 55$, $b_2^* = 15$, $Z^* = 315$

5.3-7. a) $[-c_1, -c_2, -c_3 \mid 0 \;\; 0 \mid 0] + [\tfrac{3}{5}, \tfrac{4}{5}]\begin{bmatrix} 1 & 2 & 1 & 1 & 0 & b \\ 2 & 1 & 3 & 0 & 1 & 2b \end{bmatrix} = [\tfrac{7}{10}, 0, 0, \tfrac{3}{5}, \tfrac{4}{5}, Z^*]$

Or:

$[-c_1, -c_2, -c_3 \mid 0 \;\; 0 \mid 0] + [\tfrac{11}{5}, 2, 3, \tfrac{3}{5}, \tfrac{4}{5}, \tfrac{11}{5}b] = [\tfrac{7}{10}, 0, 0, \tfrac{3}{5}, \tfrac{4}{5}, Z^*]$

So $c_1 = \frac{11}{5} - \frac{7}{10} = \frac{3}{2}$
$c_2 = 2$
$c_3 = 3$

b) $B^{-1} = \begin{pmatrix} 3/5 & -1/5 \\ -1/5 & 2/5 \end{pmatrix}$ $\quad B^{-1}b = b^* \Leftrightarrow \begin{pmatrix} 3/5 & -1/5 \\ -1/5 & 2/5 \end{pmatrix}\begin{pmatrix} b \\ 2b \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 1/5 \\ 3/5 \end{pmatrix} b = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \Rightarrow b = 5$$

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5.3-7. c) Using (a), $Z^* = c_B b^* = (c_2, c_3)\begin{pmatrix}1\\3\end{pmatrix} = (2,3)\begin{pmatrix}1\\3\end{pmatrix} = 11$.

Using (b), $Z^* = \tilde{c}_B b = \left(\frac{3}{5}, \frac{4}{5}\right)\begin{pmatrix}b\\2b\end{pmatrix} = \left(\frac{3}{5}, \frac{4}{5}\right)\begin{pmatrix}5\\10\end{pmatrix} = 11$.

5.3-8 First iteration: $\frac{5}{2}$ of row 2 is added to row 0

(i.e. premultiply $A_0 = \left[\begin{array}{cc:ccc:c} 1 & 0 & 1 & 0 & 0 & 4 \\ 0 & 2 & 0 & 1 & 0 & 12 \\ 3 & 2 & 0 & 0 & 1 & 18 \end{array}\right]$ by $[0, \frac{5}{2}, 0]$

and add to row 0)

Second iteration: row 3 is added to row 0

(i.e. premultiply $A_1 = \left[\begin{array}{cc:ccc:c} 1 & 0 & 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & 6 \\ 3 & 0 & 0 & -1 & 1 & 6 \end{array}\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix} A_0$

by $[0,0,1]$ and add to row 0)

Therefore, the final row 0 is:

Init row 0 $+ [0, \frac{5}{2}, 0] A_0 + [0,0,1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix} A_0$

Final row 0 $= [-3, -5 \,\vdots\, 0, 0, 0 \,\vdots\, 0] + [0, \frac{3}{2}, 1] A_0$ as desired

$$\left([0, \tfrac{5}{2}, 0] + [0,0,1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{bmatrix} = [0, \tfrac{3}{2}, 1] \right)$$

5.3-9. a) Use the columns corresponding to artificial variables in exactly the same way a slack column would have been used.

Note: The shadow price of this column may be positive or negative.

b) For the reversed inequalities, use the negative of the column corresponding to the slack variable (the artificial column may be discarded) in exactly the same formulae.

c) Same as (b)

d) No change, use slack and artificial variables as above.

5.3-10. For the tableau see the solution to problem 18 in chapter 4. The columns that will contain S^* are those corresponding to x_5 and x_6 since those columns contain the identity in the initial tableau.

5.3-11.

(a) The final constraint columns for $(x_1, x_2, x_3, x_4, x_6)$ are:

$$B^{-1}\begin{pmatrix} 1 & 4 & 2 & -1 & 0 \\ 3 & 2 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 3/10 & -1/10 \\ -2/10 & 2/5 \end{pmatrix}\begin{pmatrix} 1 & 4 & 2 & -1 & 0 \\ 3 & 2 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 3/5 & -3/10 & 1/10 \\ 1 & 0 & -2/5 & 2/10 & -2/5 \end{pmatrix}$$

The final objective coefficients for $(x_1, x_2, x_3, x_4, x_6)$ are:

$$(-4M+2, -6M+3, -2M+2, M, M) - (-6M+3, -4M+2)\begin{pmatrix} 0 & 1 & 3/5 & -3/10 & 1/10 \\ 1 & 0 & -2/5 & 2/10 & -2/5 \end{pmatrix}$$

$$= (0, 0, 1, \tfrac{1}{2}, \tfrac{1}{2})$$

The right-hand side is: $B^{-1}b = \begin{pmatrix} 3/10 & -1/10 \\ -2/10 & 2/5 \end{pmatrix}\begin{pmatrix} 8 \\ 6 \end{pmatrix} = \begin{pmatrix} 9/5 \\ 4/5 \end{pmatrix}$

$$Z = -14M + c_B^T x_B = -14M + (-6M+3, -4M+2)\begin{pmatrix} 9/5 \\ 4/5 \end{pmatrix} = 7$$

The final tableau is:

Bas Var	Eq No	Z	X1	X2	X3	X4	x_6	$\bar{X}5$	$\bar{X}7$	Right side
Z	0	-1	0	0	1	0.5	0.5	1M -0.5	1M -0.5	-7
X2	1	0	0	1	0.6	-0.3	0.1	0.3	-0.1	1.8
X1	2	0	1	0	-0.4	0.2	-0.4	-0.2	0.4	0.8

b) The constraints in the original tableau are $(A \mid I \mid I \mid b)$ with the second identity matrix corresponding to the artificial variables. When we pre-multiply by M, we get:

$$(A^* \mid S^* \mid L^* \mid b^*) = M\,(A \mid I \mid I \mid b) = (MA \mid M \mid M \mid Mb)$$

and we have $M = S^* = L^*$. For this problem $M = \begin{pmatrix} 3/10 & -1/10 \\ -1/5 & 2/5 \end{pmatrix}$

The original Row 0 $t = (c + e^T AM \mid Me^T \mid 0 \mid -Me^T b)$ and in the final tableau

$$t^* = t + vT = (z^* + c \mid -y^* \mid Me^T - y^* \mid Z^*)$$

$$= (c + e^T AM \mid Me^T \mid 0 \mid -Me^T b) + v^T(A \mid I \mid I \mid b)$$

5.3-11 b) (cont')

$$t^* = c + e^T A M + v^T A \mid M e^T + v \mid v \mid -M e^T b + v b)$$

We can see $v = -y^* + Me^T$

For this problem $v = \left(-\frac{1}{2} + M,\ -\frac{1}{2} + M\right)$

(c) If $t = (2, 3, 2, 0, M, 0, M, 0)$, then in matrix notation

$$t = (c \,/\, 0 \,/\, Me^T \,/\, Me^T b).$$

$$t^* = t + v^T T = (z^* + c \,/\, y^* \,/\, Me^T - y^* \,/\, Z^*)$$

$$= (c \,/\, 0 \,/\, Me^T \,/\, Me^T b) + v^T (A \mid I \,/\, I \,/\, b)$$

$$= c + v^T A \,/\, v \mid Me^T + v \,/\, Me^T b + v^T b).$$

We conclude $v = -y^*$. For this problem $v = \left(-\frac{1}{2}, -\frac{1}{2}\right)$

(d) The defining equations are $x = M b$ or $M^{-1} x = b$.

– or –

$$\begin{aligned} x_1 + 4x_2 + 2x_3 &= 8 \\ 3x_1 + 2x_2 &= 6 \\ x_3 &= 0. \end{aligned}$$

5.3-12

(a) The final constraint columns for (x_1, x_2, x_3) are:

$$B^{-1}\begin{pmatrix} 1 & 3 & 2 \\ 1 & 5 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 3 & 2 \\ 1 & 5 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \\ 0 & -2 & 2 \end{pmatrix}$$

The final objective coefficients for (x_1, x_2, x_3) are:

$$-(2, 4, 3) + (2, 0)\begin{pmatrix} 1 & 3 & 2 \\ 0 & -2 & 2 \end{pmatrix} = (0, 2, 1)$$

The right-hand side is $B^{-1} b = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 20 \\ 10 \end{pmatrix} = \begin{pmatrix} 20 \\ 10 \end{pmatrix}$. $z = c_B^T x_B = (2, 0)\begin{pmatrix} 20 \\ 10 \end{pmatrix} = 40$

The final tableau is:

Bas Var	Eq No	Z	X1	X2	X3	X_5	$\bar{X}_4$	X6	Right side
							1M	1M	
Z	0	1	0	2	1	0	2	0	40
X1	1	0	1	3	2	0	1	0	20
X5	2	0	0	-2	2	1	1	-1	10

(b) The Defining Equations are:

$$\begin{aligned} x_1 + 3x_2 + 2x_3 &= 20 \\ x_2 &= 0 \\ x_3 &= 0 \end{aligned}$$

5.3-13 a)

$$\begin{array}{l} -2x_1 + 2x_2 + x_3 + x_4 = 10 \\ 3x_1 + x_2 - x_3 + x_5 = 20 \\ \hline x_1 - x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 = -5 \\ 4x_2 + \frac{1}{2}x_3 + \frac{3}{2}x_4 + x_5 = 35 \\ \hline x_1 + 3x_2 + x_4 + x_5 = 30 \\ 8x_2 + x_3 + 3x_4 + 2x_5 = 70 \end{array}$$

$(x_1, x_2, x_3) = (30, 0, 70)$ is optimal

$z = 30 \cdot 3 + 70 \cdot 2 = 230$

(b) If we add (-3) of the 1st row from the result in (a) and (-2) of the 2nd row from the result in (a) to our original objective, we get:

$$\begin{array}{l} 3x_1 + 7x_2 + 2x_3 \\ -3x_1 - 9x_2 - 3x_4 - 3x_5 \\ -16x_2 - 2x_3 - 6x_4 - 4x_5 \\ \hline -18x_2 - 9x_4 - 7x_5 \end{array}$$

The shadow prices are 9 and 7

(c) The Defining Equations are:

$$\begin{array}{l} -2x_1 + 2x_2 + x_3 = 10 \\ 3x_1 + x_2 - x_3 = 20 \\ x_2 = 0 \end{array}$$

(d) $B = \begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix}$ $B^{-1} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$. $x_B = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 10 \\ 20 \end{pmatrix} = \begin{pmatrix} 30 \\ 70 \end{pmatrix}$

$y^* = (3, 2) \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix} = (9, 7)$

Revised Row 0 $= (9, 7) \begin{pmatrix} -2 & 2 & 1 & 1 & 0 \\ 3 & 1 & -1 & 0 & 1 \end{pmatrix} - (3, 7, 2, 0, 0) = (0, 18, 0, 9, 7)$

so, the current solution is optimal.

(e) The final tableau is:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	18	0	9	7	230
X1	1	0	1	3	0	1	1	30
X3	2	0	0	8	1	3	2	70