

COMP233 Probability and Statistics for Computer Science

Tutorial for Week 2 covers sections 3.1-3.5 from the Ross textbook.

1. Chapter 3 Problems in textbook - Question 3

Let $S = \{1, 2, 3, 4, 5, 6, 7\}$, $E = \{1, 3, 5, 7\}$, $F = \{7, 4, 6\}$, $G = \{1, 4\}$.
Find

- (a) EF (c) EG^c (e) $E^c(F \cup G)$
(b) $E \cup FG$ (d) $EF^c \cup G$ (f) $EG \cup FG$

2. Suppose the sample space S consists of all five-letter words having distinct alphabetic characters. For this question, each alphabetic character is used only once in a word.

- (a) How many words are there in S ?
(b) How many “special” words are in S for which only the second and the fourth character are vowels, i.e., one of $\{a, e, i, o, u\}$ (since y can be a vowel or a consonant we’ll consider it to be a consonant), while the remaining characters are consonants?
(c) Assuming the outcomes in S to be equally likely, what is the probability of drawing such a special word?

3. Three balls are selected at random from a bag containing

2 red , 3 green , 4 blue balls.

The balls are not replaced back in the bag upon being selected.

- (a) What would be an appropriate sample space S ?
(b) What is the the number of outcomes in S ?
(c) What is the probability that all three balls are red?
(d) What is the probability that all three balls are green?
(e) What is the probability that all three balls are blue?
(f) What is the probability of one red, one green, and one blue ball?

4. Chapter 3 Problems in textbook - Question 5

A system is composed of four components, each of which is either working or failed. Consider an experiment that consists of observing the status of each component, and let the outcome of the experiment be given by the vector (x_1, x_2, x_3, x_4) where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed.

- (a) How many outcomes are in the sample space of this experiment?
 - (b) Suppose that the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working. Specify all the outcomes in the event that the system works.
 - (c) Let E be the event that components 1 and 3 are both failed. How many outcomes are contained in event E?
5. This question concerns forming committees of 4 persons from a group of 10 people.
- (a) How many ways are there to choose a committee of 4 persons from a group of 10 persons, if order is not important?
 - (b) If each of these outcomes from part (a) is equally likely then what is the probability that a particular person is on the committee?
 - (c) What is the probability that a particular person is not on the committee?
 - (d) How many ways are there to choose a committee of 4 persons from a group of 10 persons, if one is to be the chairperson?

6. Chapter 3 Problems in textbook - Question 17

Show that

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

7. Chapter 3 Problems in textbook - Question 18

A group of 5 boys and 10 girls is lined up in random order — that is, each of the $15!$ permutations is assumed to be equally likely.

- (a) What is the probability that the person in the 4^{th} position is a boy?
- (b) What about the person in the 12^{th} position?
- (c) What is the probability that a particular boy is in the 3^{rd} position?