Lecture 7: Equilibrium of Rigid Bodies

GNG 1105

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Application

Engineers designing this crane will need to determine the forces that act on this body under various conditions.

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Introduction*

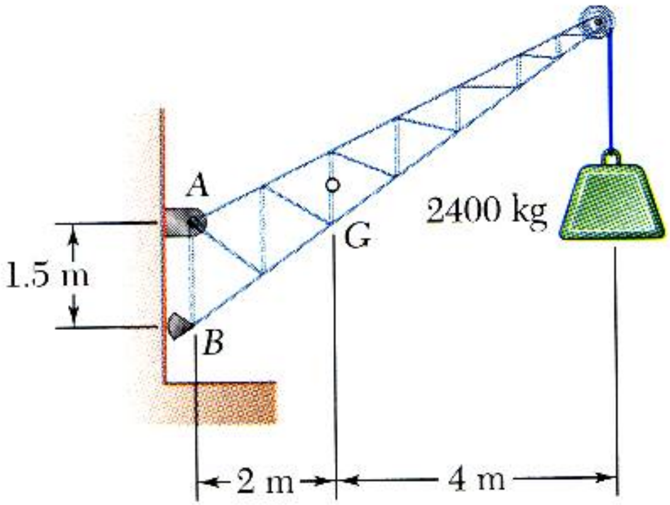
- The necessary and sufficient conditions for the **static equilibrium** of a body are that the forces sum to zero, and the moment about any point sum to zero:

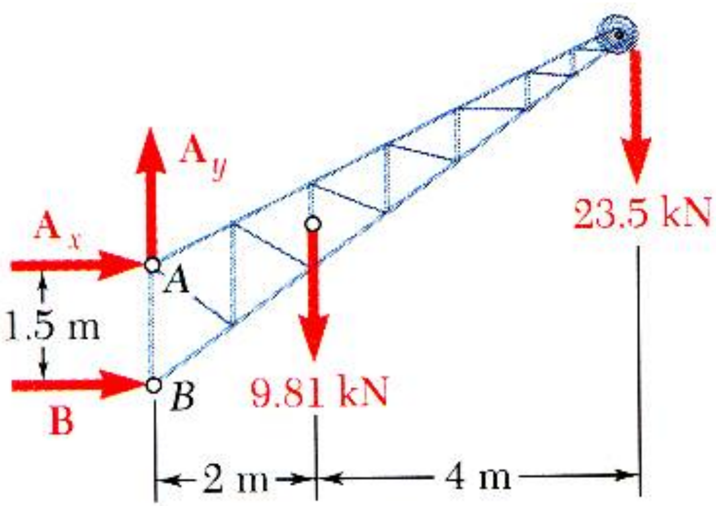
$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum\left(\vec{r} \times \vec{F}\right) = 0$$

which may also be written in their rectangular components:

- Equilibrium analysis can be applied to two-dimensional or three-dimensional bodies, but the first step in any analysis is the creation of the ***free body diagram***

Free-Body Diagram

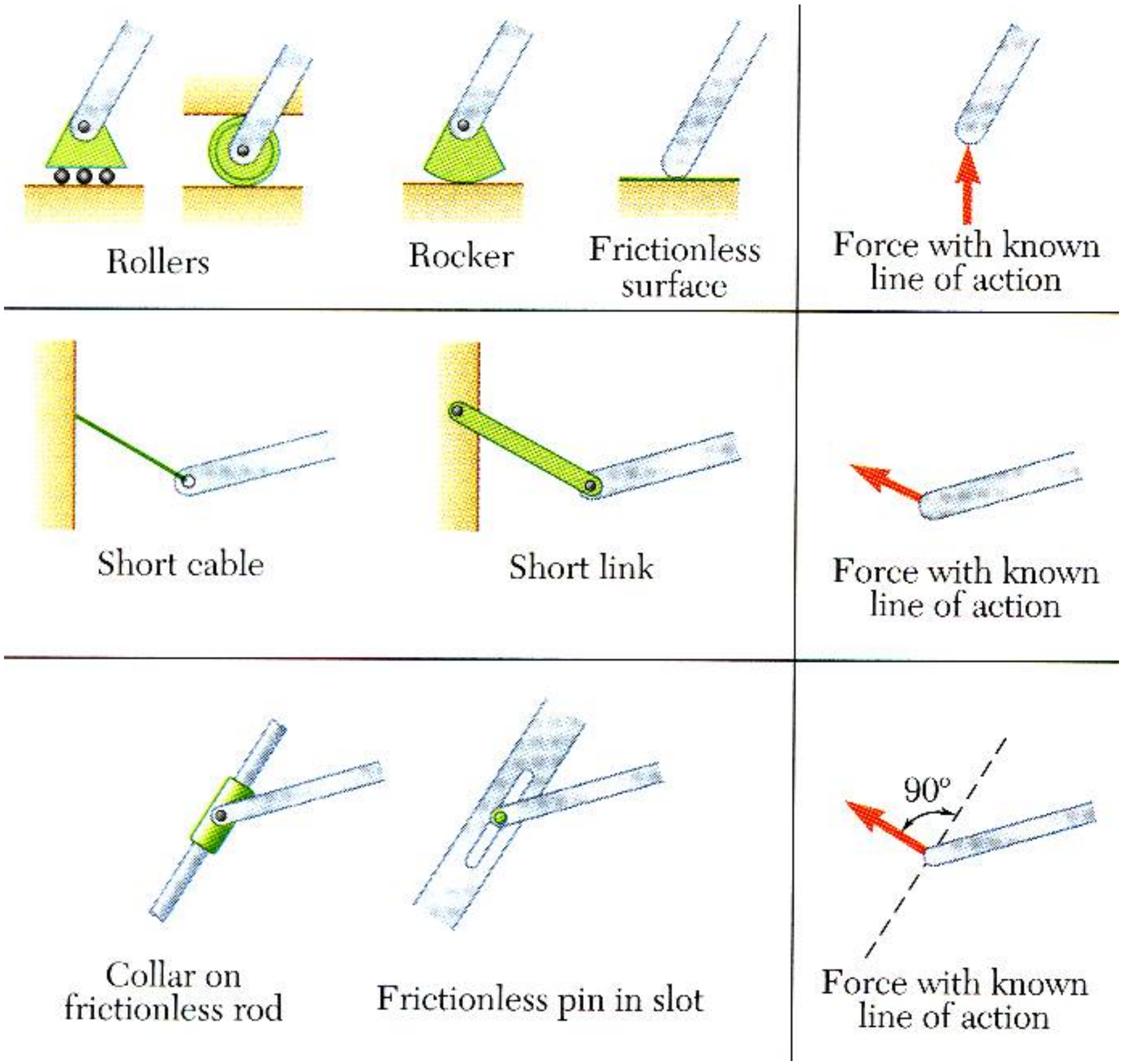




The first step in the static equilibrium analysis of a rigid body is identification of all forces acting on the body with a *free body diagram*.

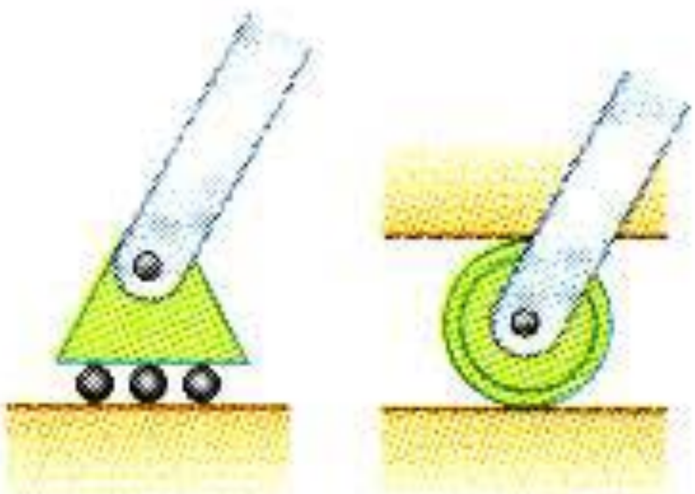
- Select the body to be analyzed and **detach it from the ground** and all other bodies and/or supports.
- Indicate **point of application, magnitude, and direction of external forces**, including the **rigid body weight**.
- Indicate point of application and assumed direction of unknown forces from **reactions** of the ground and/or other bodies, such as the supports.
- **Include the dimensions**, which will be needed to compute the moments of the forces.

Reactions at Supports and Connections for a Two-Dimensional Structure*

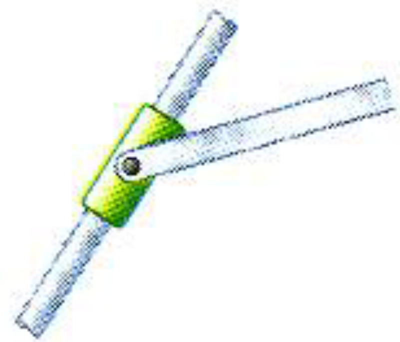


- Reactions equivalent to a force with known line of action.

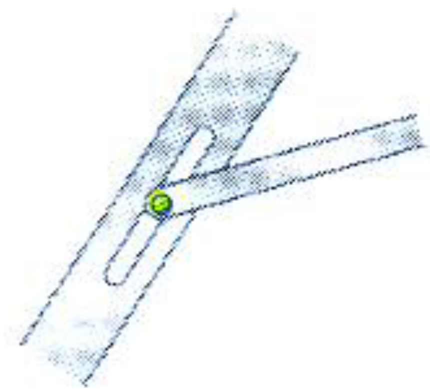
Reactions at Supports and Connections for a Two-Dimensional Structure*



Rollers



Collar on frictionless rod



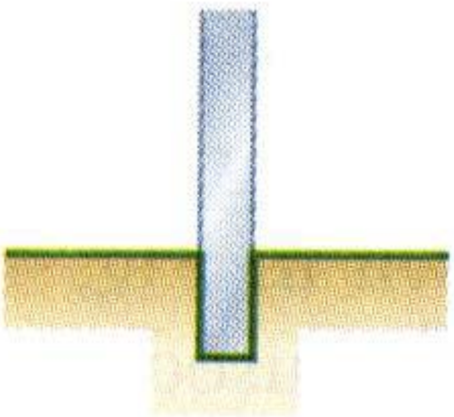
Frictionless pin in slot

Reactions at Supports and Connections for a Two-Dimensional Structure*

Frictionless pin or hinge

Rough surface

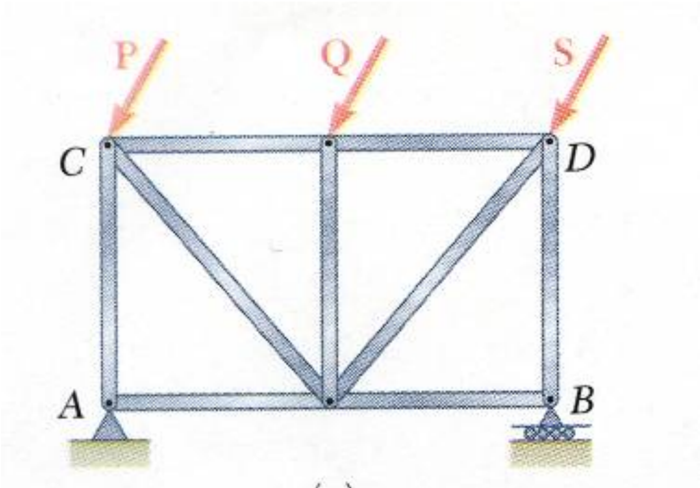
Reactions at Supports and Connections for a Two-Dimensional Structure



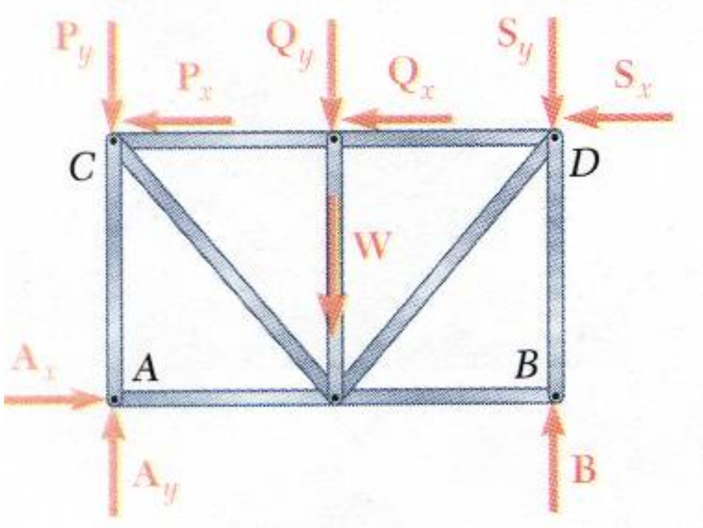
Fixed support

Sense of unknown reaction*

Equilibrium of a Rigid Body in Two Dimensions



(a)



(b)

- For known forces and moments that act on a two-dimensional structure, the following are true:

  $$F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$$

- Equations of equilibrium become

  $$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

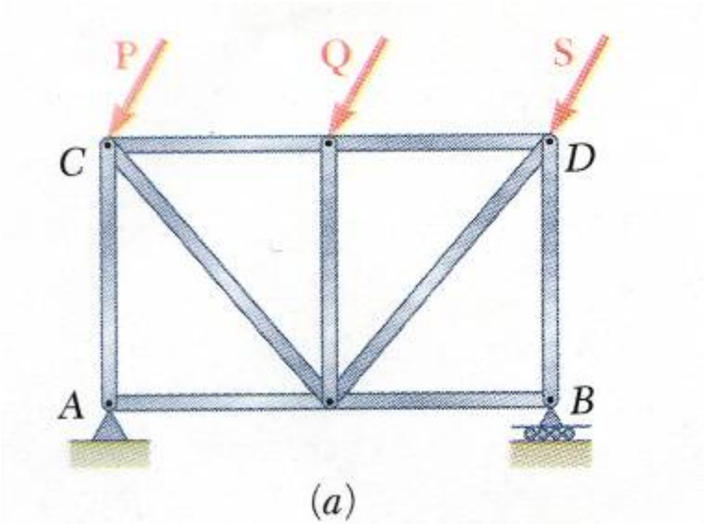
  where A can be any point in the plane of the body.

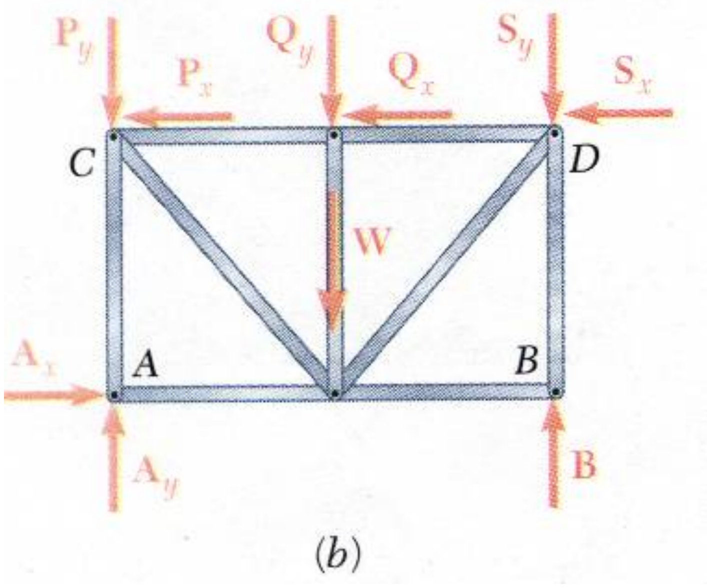
- The 3 equations can be solved for no more than 3 unknowns.

- The 3 equations cannot be augmented with additional equations, but they can be replaced

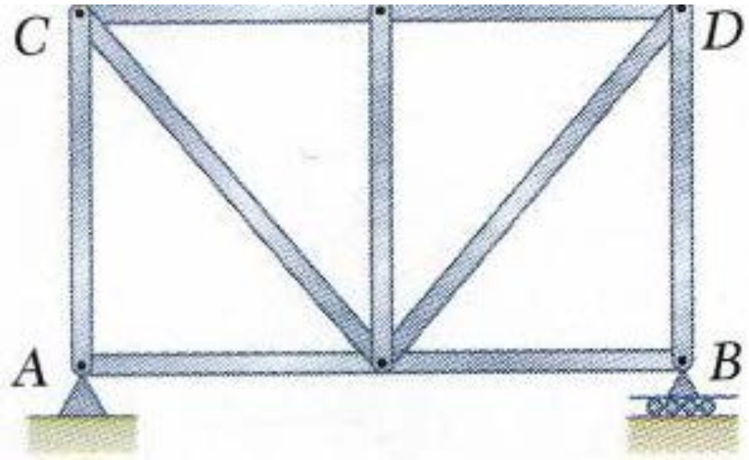
  $$\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$$

Equilibrium of a Rigid Body in Two Dimensions*

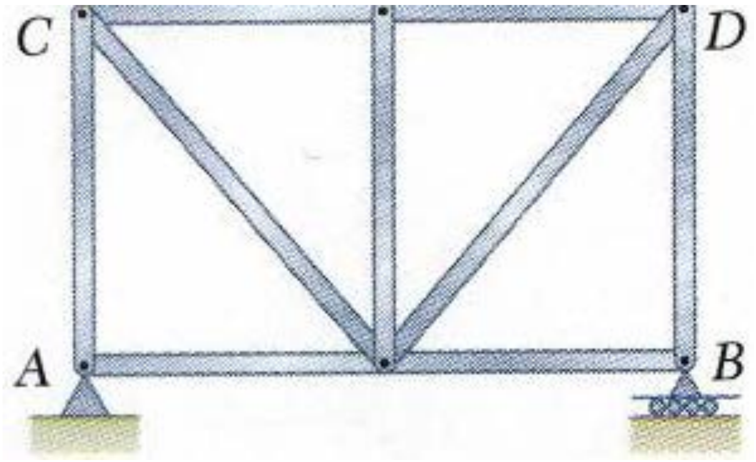




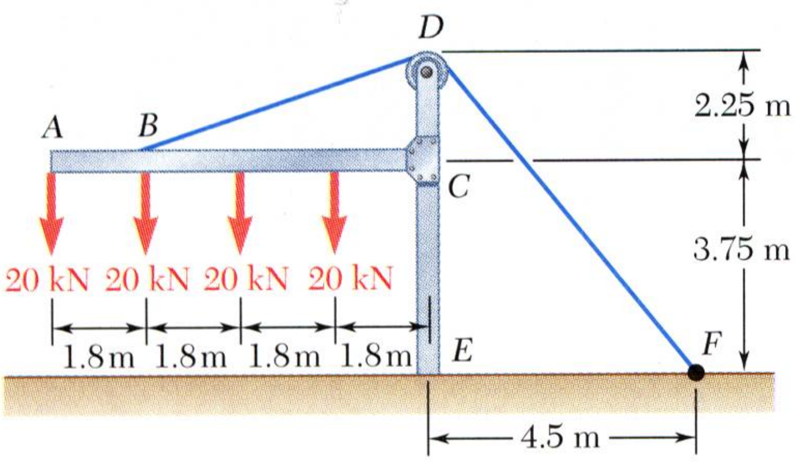
Simple Example*



Simple Example*



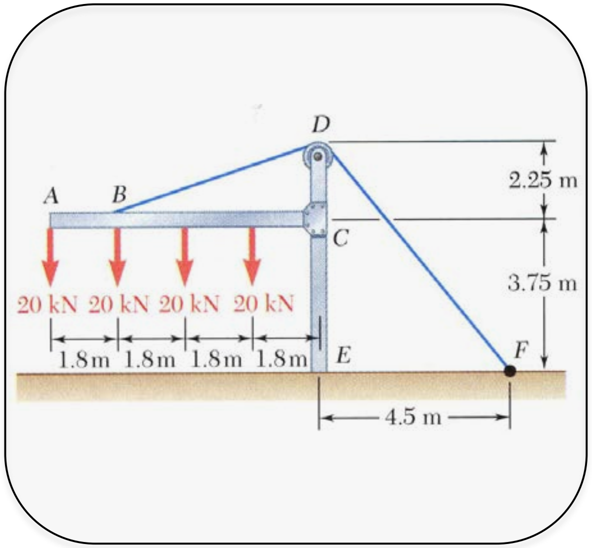
Practice - FBD



The frame shown supports part of the roof of a small building. Your goal is to draw the free body diagram (FBD) for the frame.

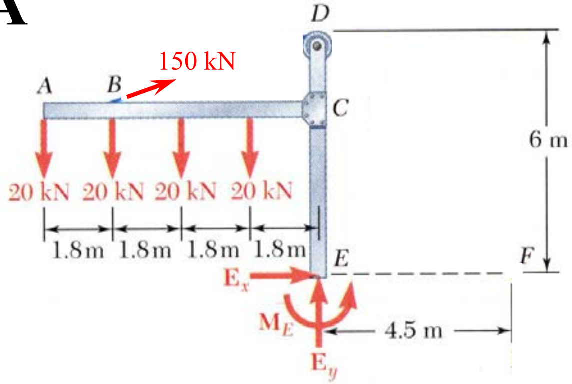
First, you should draw your own FBD.

Practice - FBD

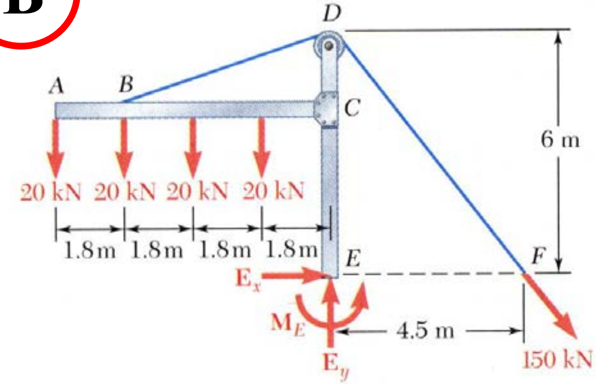


Choose the most correct FBD for the original problem.

A

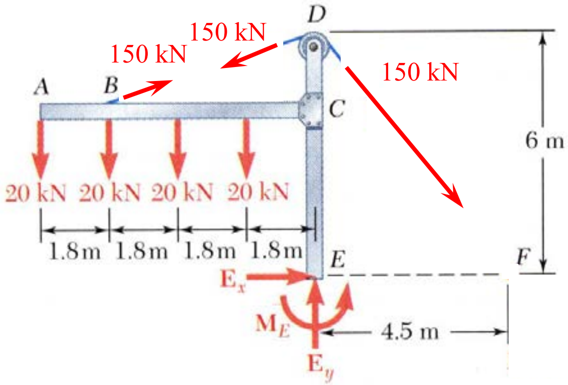


B

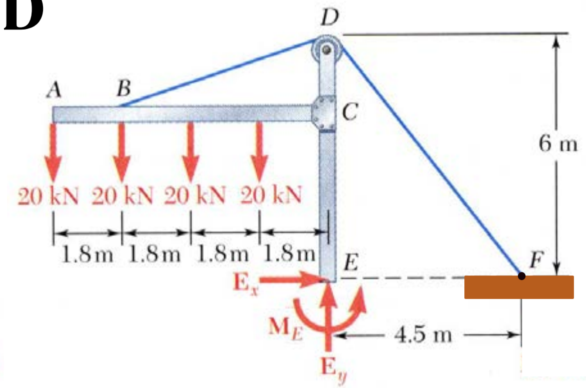


B is the most correct, though C is also correct. A & D are incorrect; why?

C



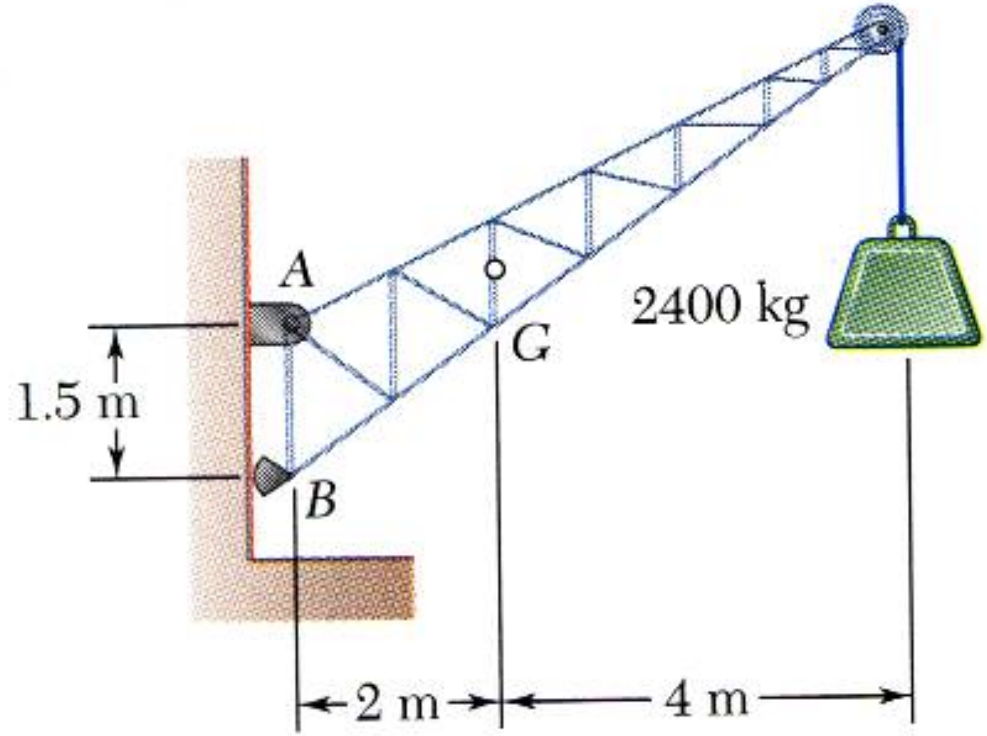
D



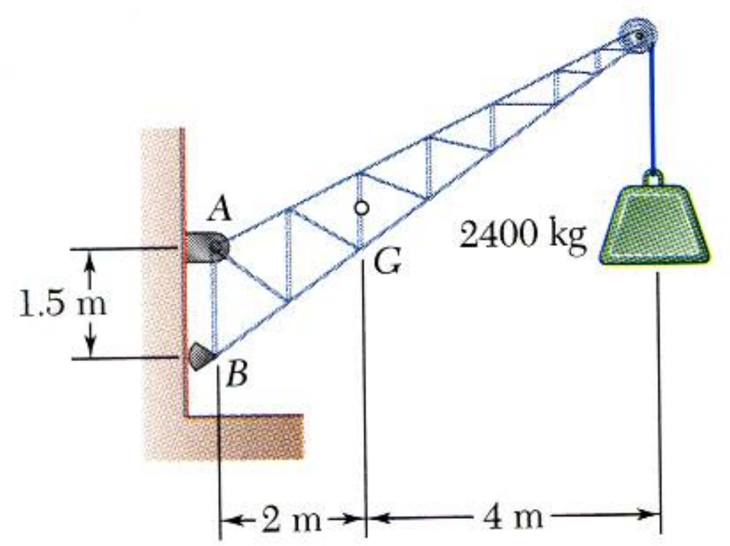
Sample Problem

A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*.

Determine the components of the reactions at *A* and *B*.



Sample Problem



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Determine the components of the reactions at *A* and *B*.

Solution steps:

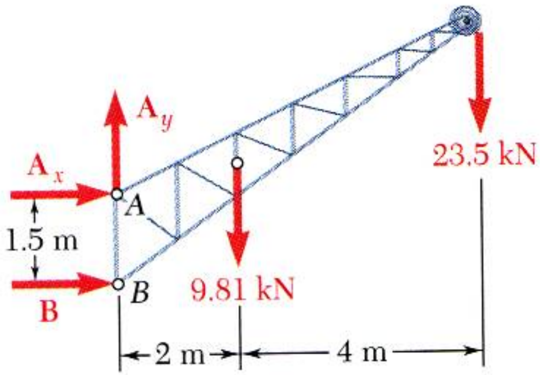
1) Create a FBD
2) $\sum M_A = 0 \rightarrow R_B$
3) Reaction at A by solving:

$$\sum F_x = 0$$
$$\sum F_y = 0$$

4) Check our work using

$$\sum M_B = 0$$

Sample Problem*

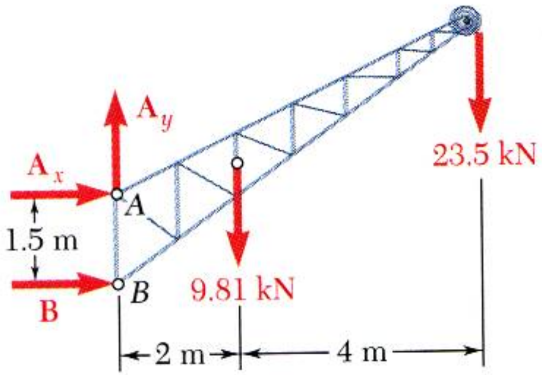


- Create the free-body diagram.

- Determine *B* by solving the equation for the sum of the moments of all forces about *A*.

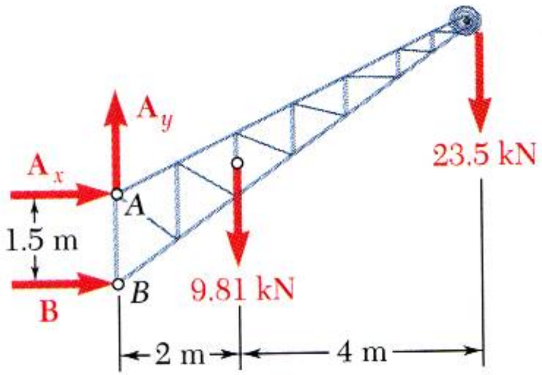
- Determine the reactions at *A* by solving the equations for the sum of all horizontal forces and all vertical forces.

Sample Problem*

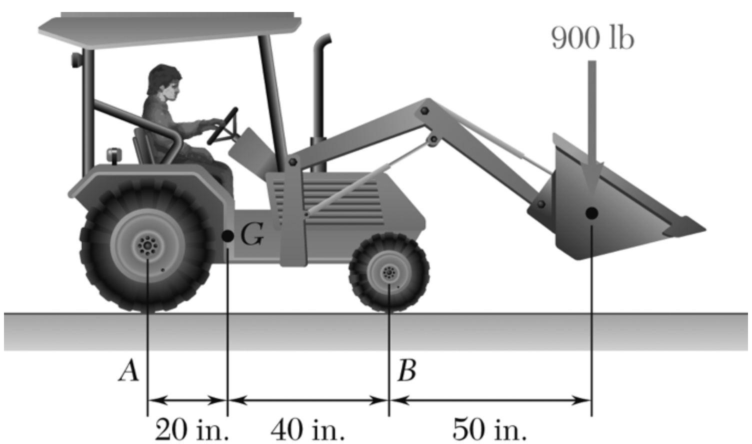


- Create the free-body diagram.

Sample Problem*



Practice

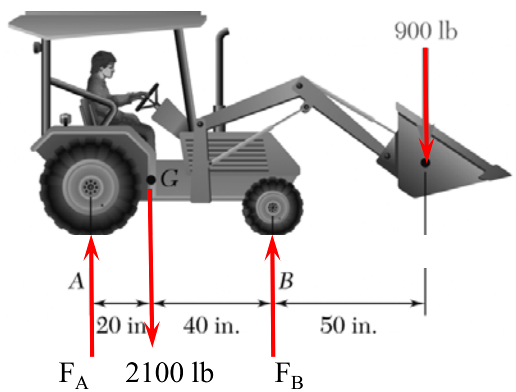


A 2100-lb tractor is used to lift 900 lb of gravel. Determine the reaction at each of the two rear wheels and two front wheels

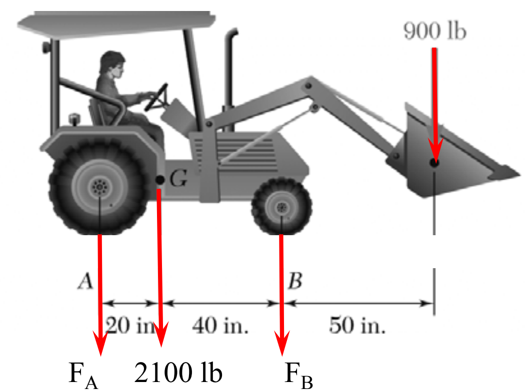
Practice

- Draw the free body diagram of the tractor (on your own first).
- From among the choices, choose the best FBD, and discuss the problem(s) with the other FBDs.

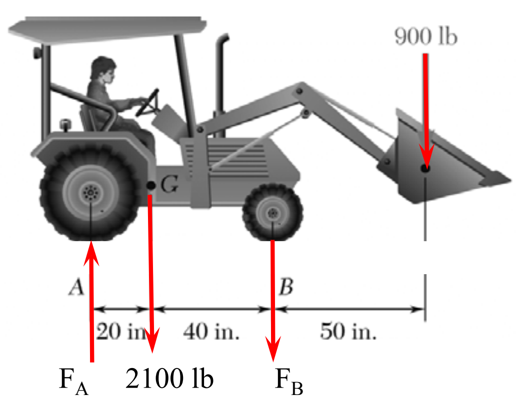
A.



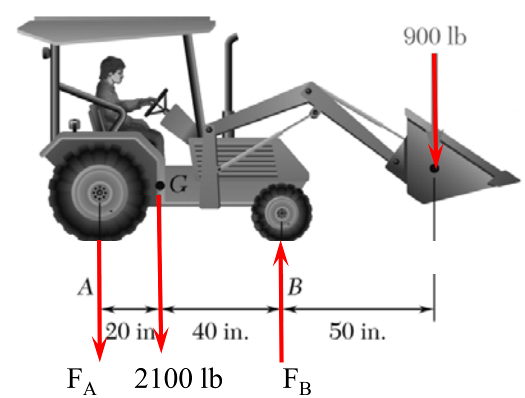
B.



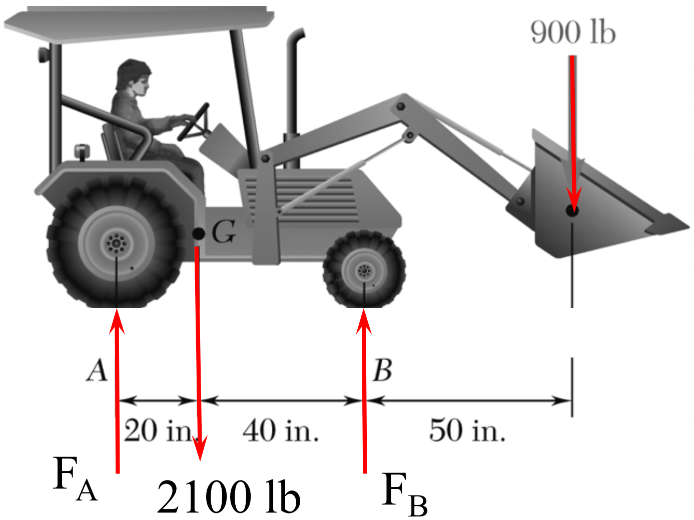
C.



D.



Practice



Now let's apply the equilibrium conditions to this FBD.

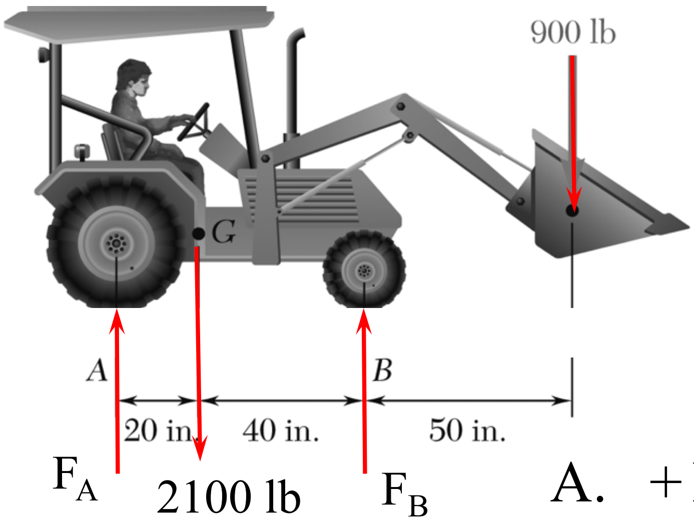
- Start with the moment equation:

$$\sum M_{pt} = 0$$

- What's the advantage to starting with this instead of the other conditions?
- About what point should we sum moments, and why?

Points A or B are equally good because each results in an equation with only one unknown.

Practice



Assume we chose to use point B.

Choose the correct equation for

$$\sum M_B = 0.$$

A. $+F_A(60\text{ in.}) - 2100\text{lb}\,(40\text{ in.}) - 900\text{ lb}\,(50\text{ in.}) = 0$

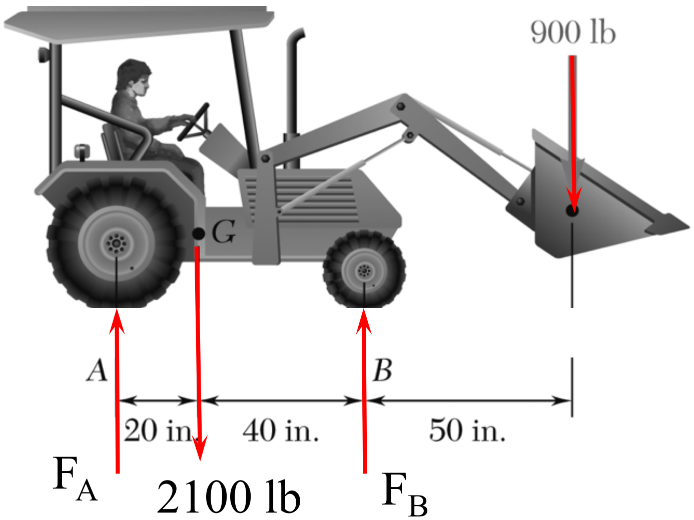
B. $+F_A(20\text{ in.}) - 2100\text{lb}\,(40\text{ in.}) - 900\text{ lb}\,(50\text{ in.}) = 0$

C. $-F_A(60\text{ in.}) - 2100\text{lb}\,(40\text{ in.}) + 900\text{ lb}\,(50\text{ in.}) = 0$

D. $-F_A(60\text{ in.}) + 2100\text{lb}\,(40\text{ in.}) - 900\text{ lb}\,(50\text{ in.}) = 0$

F_A=650 lb, so the reaction *at each wheel* is 325 lb

Practice



Now apply the final equilibrium condition, $\Sigma F_y = 0$.

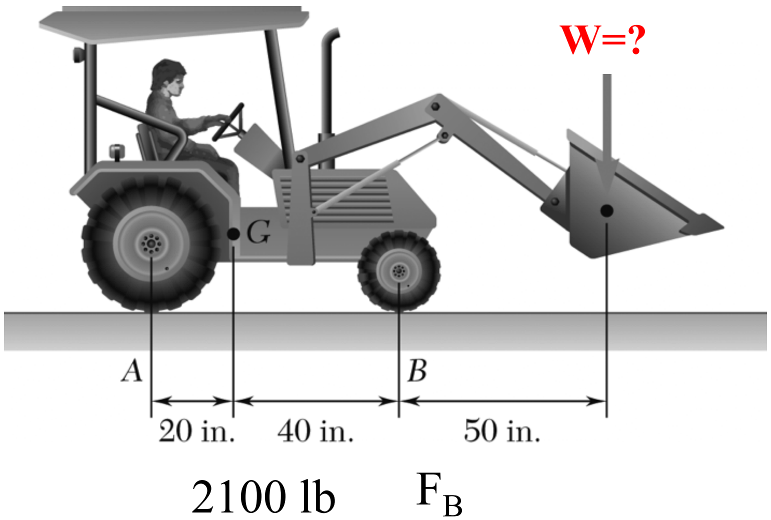
$$F_A - 2100 \text{ lb} + F_B - 900 \text{ lb} = 0$$

$$\text{or } \; +650 \text{ lb} - 2100 \text{ lb} + F_B - 900 \text{ lb} = 0$$

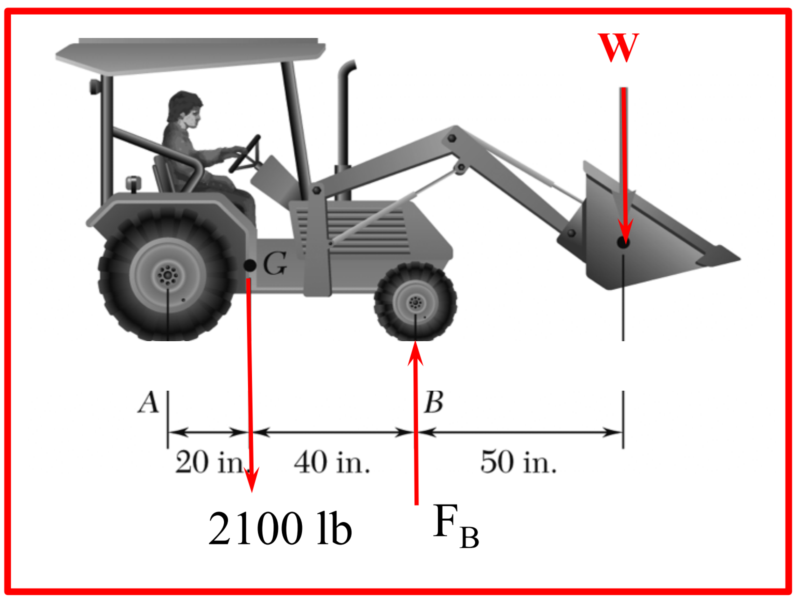
$$\Rightarrow \boxed{F_B = 2350 \text{ lb, or } 1175 \text{ lb at each front wheel}}$$

Why was the third equilibrium condition, $\Sigma F_x = 0$ not used?

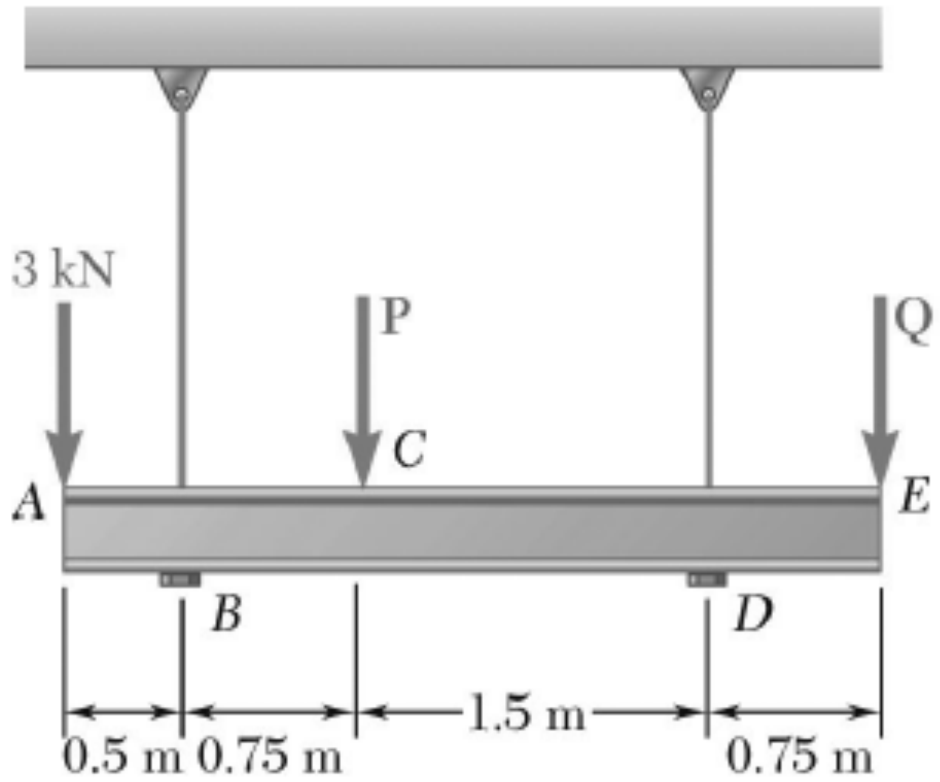
What if…?



- Now suppose we have a different problem: How much gravel can this tractor carry before it tips over?
- Hint: Think about what the free body diagram would be for this situation…



Problem*



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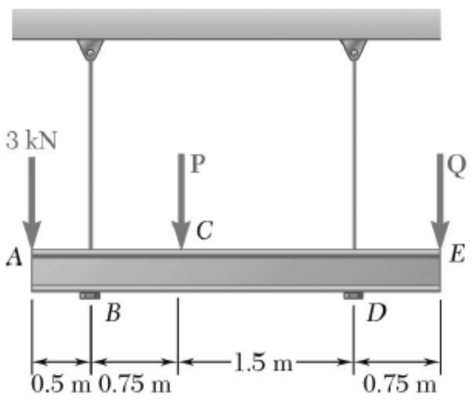
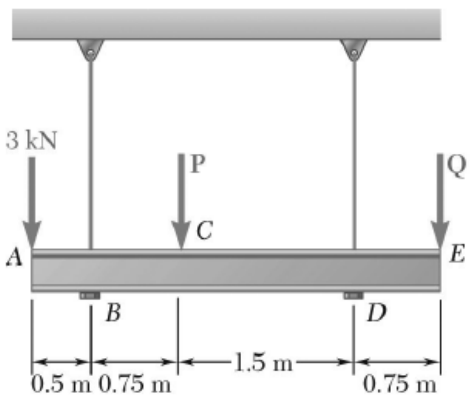
Problem



Problem



Problem