ANSWERS AND SOLUTIONS

1. [**3+3+3 = 9 Marks**] Circle only one of **(i)**, **(ii)**, **(iii)** or **(iv)** per question. No explanation required. Only the answer will be marked. **3** marks are given for a correct answer, **0** - otherwise.

(a) Which of the matrices below are in RREF (reduced row echelon form)?

$$A = \begin{bmatrix} 1 & 0 & 1 & 31 \\ 0 & 1 & -2 & 17 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(i) A and B; **(ii)** A and C; **(iii)** B and C; **(iv)** A, B and C.

Answer: (ii)

(b) A linear system with the augmented matrix $\begin{bmatrix} 7 & 1 & -4 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$ has

(i) a unique solution; **(ii)** infinitely many solutions; **(iii)** no solution;

Answer: (iii)

(c) A linear system with the augmented matrix $\begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & 2 & 2 & 1 \\ 0 & 0 & 3 & 6 \end{bmatrix}$ has

(i) a unique solution; **(ii)** infinitely many solutions; **(iii)** no solution;

Answer: (i)

2. [**6 Marks**] For what value(s) of k will the system below be consistent?

$$\begin{cases} 2x + y = 3 \\ -4x - 2y = k \end{cases}$$

Solution:

The augmented matrix of the system is

$$\begin{bmatrix} 2 & 1 & 3 \\ -4 & -2 & \text{k} \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 3 \\ 0 & 0 & k+6 \end{bmatrix}.$$

To be consistent, the system must not have any rows of the type $[0\ 0\ ...\ 0\ b]$ with $b \neq 0$. Thus, $k+6$ must be zero, which implies that $k = -6$.

3. [**10 Marks**]

Let $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 0 & 2 \end{bmatrix}$ and $D = [1\ -2]$.

(a) [3] Find, if possible, $B + 3C$ and $12A - 7B$.

(b) [4] Compute AB.

(c) [3] Compute DA.

Solution:

(a) The matrices B and C are of the same size, therefore $B + 3C$ is defined.

$$3C = 3\begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 6 & -3 \\ 0 & 6 \end{bmatrix},$$

$$B + 3C = \begin{bmatrix} 4 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 6 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 5 & -1 \\ 0 & 7 \end{bmatrix}.$$

Since A and B have a different size, then $12A - 7B$ is not defined.

(b) $\begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 1 \\ 1 & 7 \end{bmatrix}.$

(c) $DA = [1\ -2] \begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & 0 \end{bmatrix} = [0\ -7\ \ 1].$

4. [15 Marks]

Solve the following linear system by row-reducing the augmented matrix to either REF or RREF (your choice!).

$$\begin{cases} x_1 + x_2 - 3x_3 = 3 \\ -2x_1 - x_2 = -4 \\ 4x_1 + 2x_2 + 3x_3 = 7 \end{cases}$$

Solution:

I choose RREF. Row-reduce the augmented matrix:

$$\begin{bmatrix} 1 & 1 & -3 & 3 \\ -2 & -1 & 0 & -4 \\ 4 & 2 & 3 & 7 \end{bmatrix} \to R_2 + 2R_1 \begin{bmatrix} 1 & 1 & -3 & 3 \\ 0 & 1 & -6 & 2 \\ 4 & 2 & 3 & 7 \end{bmatrix} \to R_3 - 4R_1 \begin{bmatrix} 1 & 1 & -3 & 3 \\ 0 & 1 & -6 & 2 \\ 0 & -2 & 15 & -5 \end{bmatrix}$$

$$\to R_3{+}2R_2 \begin{bmatrix} 1 & 1 & -3 & 3 \\ 0 & 1 & -6 & 2 \\ 0 & 0 & 3 & -1 \end{bmatrix} \to \frac{1}{3}R_3 \begin{bmatrix} 1 & 1 & -3 & 3 \\ 0 & 1 & -6 & 2 \\ 0 & 0 & 1 & -\frac{1}{3} \end{bmatrix} \to R_1{-}R_2 \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -6 & 2 \\ 0 & 0 & 1 & -\frac{1}{3} \end{bmatrix}$$

$$\to R_1 - 3R_3 \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & -6 & 2 \\ 0 & 0 & 1 & -\frac{1}{3} \end{bmatrix} \to R_2 + 6R_3 \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} \end{bmatrix}.$$

Thus, the corresponding system of equations is

$$\begin{cases} x_1 = 2 \\ x_2 = 0 \\ x_3 = -\frac{1}{3} \end{cases}$$

and the unique solution $\vec{x} = (2, 0, -\frac{1}{3})$ is apparent.