CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	203	All
Examination	**Date**	**Pages**
Final	December 2016	3
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Special Instructions:	Only approved calculators are allowed. **Show all your work for full marks.**	

MARKS

[10] **1.** (a) Suppose $f(x) = \sqrt[3]{x-1}$ and $g(x) = 1 + (\frac{x}{1+x^3})^3$. Find $f \circ g$ and $g \circ f$ and their domains.

(b) Find the inverse of the function $f(x) = \sqrt{2^x - 2}$. Determine the domain and range of f and f^{-1}.

[8] **2.** Evaluate the limits:

a) $\lim\limits_{x \to 5} \frac{\sqrt{2x-1} - 3}{x^3 - 125}$ b) $\lim\limits_{x \to \infty} \frac{(x^3+1)(2x-3)^2}{(x+1)^2(3x+2)^3}$.

Do not use L'Hôpital's rule.

[11] **3.** (a) Consider the function $f(x) = \frac{|x^2+4x-5|}{x^2-25}$. Calculate both one-sided limits at the point(s) where the function is undefined.

(b) Find the value of a and b so that the function

$$f(x) = \begin{cases} 5 + x^2 & \text{if} \quad x \le 0 \\ ax + b & \text{if} \quad 0 < x \le 1 \\ \dfrac{25}{x} & \text{if} \quad x > 1 \end{cases}$$

is continuous everywhere. Sketch the graph of this function.

[15] **4.** Find the derivatives of the following functions (you do not need to simplify the answers)

(a) $f(x) = \dfrac{\sqrt{x} + 3\sqrt[3]{x^2} + x^5}{2x\sqrt[3]{x}}$

(b) $f(x) = (x^3 + ex - \sin\pi)(\cos 2x)$

(c) $f(x) = \ln^3(x^2 + \tan(3x))$

(d) $f(x) = \dfrac{\arcsin^2 x}{\sqrt{1-x^2}}$

(e) $f(x) = (3x^2 + 5)^{\arctan x}$

[12] **5.** Given the function $f(x) = \sqrt{x^2 + 8}$

(a) Use appropriate differentiation rules to find the derivative $f'(x)$.

(b) Use the definition of derivative to verify the answer in part a.

(c) Find the differential of the function.

(d) Use the differential above, or (equivalently) use the linear approximation at $a = 1$ (with the appropriate choice of Δx) to find the approximate value of $\sqrt{8.49}$. Check the approximation with your calculator.

[16] 6. (a) The equation of a curve is $y^4 \tan x = xy^3 + y - 1$ and defines y implicitly as a function of x. Verify that the point $(0, 1)$ belongs to this curve and find an equation of the tangent line to the curve at this point.

(b) Let $f(x) = \dfrac{4 - 3x^7}{x^5}$. Find $f'''(x)$.

(c) Use L'Hôpital's rule to evaluate $\displaystyle\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$

[12] 7. (a) A particle is moving along the plane curve $2x^2 + 5y^2 = 22$. At the moment when $x = -1$ the x-coordinate is increasing at the rate of 5 cm/sec. If the y-coordinate is positive at this moment, is it increasing or decreasing? How fast?

(b) A rectangle $ABCD$ has sides parallel to the coordinate axes and point A is located at the origin. Point B is on the positive x-axis and point C is on the graph of the function $y = e^{-2x}$ and has positive x and y coordinates. Find the coordinates of the point C that maximizes the area of the rectangle.

[16] **8.** Given the function $f(x) = \dfrac{2x}{x^2 - 9}$,

(a) Find the domain and check for symmetry. Find all asymptotes (if there are any).

(b) Calculate $f'(x)$ and use it to determine interval(s) where the function is increasing, interval(s) where the function is decreasing, and local extrema (if there are any).

(c) Calculate $f''(x)$ and use it to determine interval(s) where the function is concave upward, interval(s) where the function is concave downward, and points of inflection (if there are any).

(d) Sketch the graph of the function.

[5] **Bonus Question**

Given the equation $x^5 + 5x = 5$,

(a) Use the Intermediate Value Theorem to show that there is a solution between 0 and 1.

(b) Use the Mean Value Theorem to show that there cannot be more than one solution between 0 and 1.