CPSC 121, Winter 2017: Assignment 1

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Due Thursday 19 January 2017 at 16:00

1. An Island of Riddles (10 marks)
   There is an island on which everyone is a dragon or a troll. Dragons, being noble, always tell the truth. Trolls, being tricky, always lie. Alice, Ryan, and Steve, all inhabitants of the island, make the following statements.

   Alice: "I'm a dragon"
   Ryan: "Alice is a dragon"
   Steve: "Alice is a troll"

   (a) This part of the question is worth 1 of the 10 marks
   Is it possible that Alice is a dragon? In one sentence, justify your answer.

   (b) This part of the question is worth 1 of the 10 marks
   Is it possible that Alice is a troll? In one sentence, justify your answer.

   (c) This part of the question is worth 1 of the 10 marks
   Is it possible that Ryan is a dragon? In one sentence, justify your answer.

   (d) This part of the question is worth 1 of the 10 marks
   Is it possible that Ryan is a troll? In one sentence, justify your answer.

   (e) This part of the question is worth 1 of the 10 marks
   Is it possible that Steve is a dragon? In one sentence, justify your answer.

   (f) This part of the question is worth 1 of the 10 marks
   Is it possible that Steve is a troll? In one sentence, justify your answer.

   Answer:
   (a) It is possible that Alice is a dragon. If Alice is a dragon and telling the truth, then Ryan must be a dragon who is telling the truth about Alice, and Steve must be a troll who is lying about Alice.

   (b) It is possible that Alice is a troll. If Alice is a troll and lying, then Ryan must be a troll who is lying about Alice, and Steve must be a dragon who is telling the truth about Alice.

   (c,f): Yes, in the same situation as in (a).

   (d,e): Yes, in the same situation as in (b).

   Markers' Note: Treat these six questions as one six-point problem. Correct answers with no justification should get 2/6 points ("Yes" for all six). Correct

answers with poor justifications, e.g., just stating the row of the truth table ("Yes, if Alice and Ryan are dragons, and Steve is a troll") with no further explanation, should get 4/6 points. One point deducted for each incorrect ("No") answer. Cross-references are allowed (e.g., "The same situation as in (a)").

(g) This part of the question is worth 4 of the 10 marks
There are eight possible "assignments" of dragon or troll to Alice, Ryan, and Steve.

An "assignment" is a label of dragon or troll to each of Alice, Ryan, and Steve, *regardless of whether it is correct or incorrect.* For example, "Alice is a dragon, Ryan is a dragon, and Steve is a troll" is one of the eight possible assignments, regardless of whether that assignment is correct or not.

Are all of the eight possible assignments potentially correct, based on what Alice, Ryan, and Steve said? Justify your answer. Your answer will be judged on the brevity and preciseness of justifying why either "yes" or "no" is correct.

Answer:
No, they are not all possible. If Alice is a dragon and telling the truth about herself, that implies that Ryan is a dragon and telling the truth about her. If Ryan is a troll and lying about Alice, that would be a contradiction. So any assignment in which Alice is a dragon and Ryan is a troll is not possibly correct.

Markers' Note: A "No" answer with no justification should get 1/4.

A "No" answer with poor justification (e.g., just stating the row of the truth table with no further explanation, such as "All three being dragons cannot happen") gets 2/4.

Answers that waste time discussing multiple situations that cannot happen (e.g., why Ryan cannot be a troll when Alice is a dragon, *plus more explanation* about why Ryan cannot be a dragon when Alice is a troll) should receive 3/4. Note the provided answer covers off two situations, but *only uses a single explanation* to do it (Alice being a dragon, Ryan being a troll, and Steve being either assignment) — that is acceptable.

Any reasonable attempt to justify, in one explanation, one (or more) situations that cannot be possible, barring errors in the student's answer, should receive 4/4.

2. Everything is on the Table (15 marks)

(a) This part of the question is worth 4 of the 15 marks
Write the truth table for $((a \wedge b) \wedge (\sim (\sim (\sim b)) \vee b)) \vee (a \wedge c)$ showing the intermediate values for each of (1) $a \wedge b$; (2) $\sim (\sim (\sim b)) \vee b$; and, (3) $a \wedge c$.[1] You may choose to include other intermediate values if you wish, so long as the required three intermediate values and

[1] The textbook, Epp 4ed, uses **c** to indicate a contradiction — something that must be false. You can also use F or 0 to indicate a false proposition, which are generally what we will use in class. Here, we use c as a variable name. Note the slightly different typeface between **c** and c.

the final value are clearly indicated.

Answer:

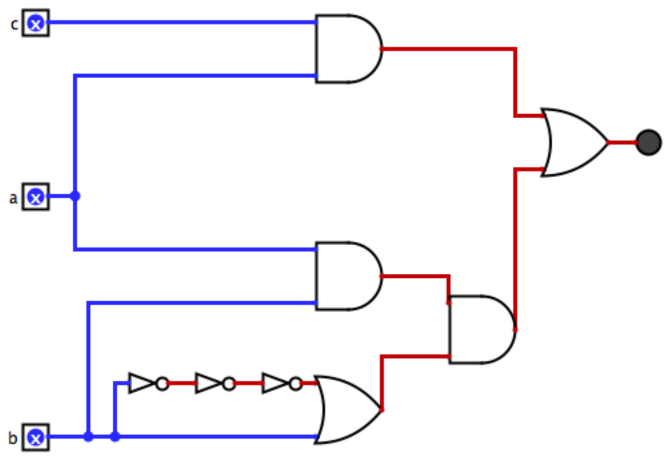
a	b	c	$a \wedge b$	$\sim (\sim (\sim b)) \vee b$	$a \wedge c$	$((a \wedge b) \wedge (\sim (\sim (\sim b)) \vee b)) \vee (a \wedge c)$
0	**0**	**0**	**0**	**1**	**0**	**0**
0	**0**	**1**	**0**	**1**	**0**	**0**
0	**1**	**0**	**0**	**1**	**0**	**0**
0	**1**	**1**	**0**	**1**	**0**	**0**
1	**0**	**0**	**0**	**1**	**0**	**0**
1	**0**	**1**	**0**	**1**	**1**	**1**
1	**1**	**0**	**1**	**1**	**0**	**1**
1	**1**	**1**	**1**	**1**	**1**	**1**

Markers' Note: One point per column.

(b) This part of the question is worth 4 of the 15 marks

Draw a circuit using *nothing other than* two-input AND and OR gates, and single-input NEGATION gates (i.e., without XOR, NAND, NOR, and XNOR gates) that implements the logical expression *as it is written*. (Note: this diagram will use more gates than strictly necessary, because there is a simpler equivalent logical expression, as you will see below.)

Answer:



Markers' Note: One point deducted for minor errors (e.g., miscounted the number of negation gates). Two points for each major error (e.g., incorrect order of gates).

(c) This part of the question is worth 6 of the 15 marks

Simplify the logical expression, using the logical equivalence laws (on page 35 of Epp 4ed), into its simplest logical equivalent (i.e., the equivalent expression that uses the fewest logical operations). Note: while the truth table from Question 1(a) might help you to verify your answer, you must use the equivalence laws to justify your conclusion.

Answer:

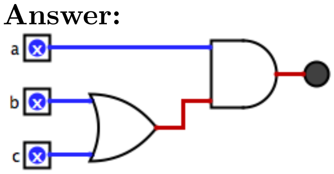
$$
\begin{aligned}
&((a \wedge b) \wedge (\sim (\sim (\sim b)) \vee b)) \vee (a \wedge c) && && (1)\\
&\equiv ((a \wedge b) \wedge (\sim b \vee b)) \vee (a \wedge c) && \textbf{by double negation} && (2)\\
&\equiv ((a \wedge b) \wedge 1) \vee (a \wedge c) && \textbf{by negation law} && (3)\\
&\equiv (a \wedge b) \vee (a \wedge c) && \textbf{by identity law} && (4)\\
&\equiv a \wedge (b \vee c) && \textbf{by distributive law} && (5)
\end{aligned}
$$

Markers' Note: One point for correctly applying each step, for a total of 3/6 points (students may skip the double-negation step without penalty, if you can see exactly what they did and where). The remaining 3/6 points are for correctly identifying the law used in each step. Deduct a point for unnecessary steps.

Line numbers are unnecessary, because you always operate on the previous line (numbers are just included in the answer above for reference).

(d) This part of the question is worth 1 of the 15 marks
Draw a circuit of the simplified equivalent expression, using *nothing other than* two-input AND and OR gates, and single-input NEGATION gates.

Answer:

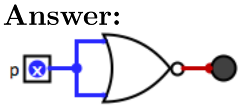


Markers' Note: All or nothing.

3. The Powerful NOR (10 marks)
In this question, you will prove that every truth table can be implemented using *only* two-input NOR gates.

(a) This part of the question is worth 2 of the 10 marks
Show that $\sim$ can be simulated using a NOR gate. That is, design a circuit whose only gate is a NOR gate, that takes as input a signal p, and whose output is $\sim p$.

Answer:



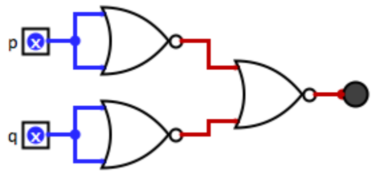
$$
\begin{aligned}
&p \textbf{ NOR } p && \\
&\equiv \sim (p \vee p) && \textbf{by def'n of NOR}\\
&\equiv \sim p && \textbf{by idempotence}
\end{aligned}
$$

Markers' Note: One point for a correct circuit, and one point for justification (equivalence laws or truth table). For the justification for this part of the question, plain-English descriptions are allowed, so long as the reasoning is sound and complete.

(b) This part of the question is worth 4 of the 10 marks

Show that $\wedge$ can be simulated using a NOR gate. That is, design a circuit whose only gates are all NOR gates, that takes as input two signals p and q, and whose output is $p \wedge q$.

Answer:



$(p$ **NOR** $p)$ **NOR** $(q$ **NOR** $q)$

$\equiv \sim p$ **NOR** $\sim q$ **by proof in Question 3(a)**

$\equiv \sim (\sim p \vee \sim q)$ **by def'n of NOR**

$\equiv \sim\sim p \wedge \sim\sim q$ **by de Morgan's**

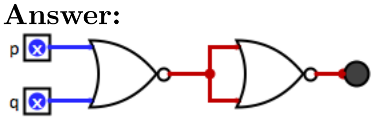
$\equiv p \wedge q$ **by Double Negation**

Markers' Note: Two points for a correct circuit, and two points for justification (equivalence laws or truth table). Minor errors in the circuit will lose 1 of the 2 points. Errors in the justification, or reliance or non-formal English statements, score at most 1 of the two 2 points.

(c) This part of the question is worth 4 of the 10 marks

Show that $\vee$ can be simulated using a NOR gate. That is, design a circuit whose only gates are all NOR gates, that takes as input two signals p and q, and whose output is $p \vee q$.

Answer:



$(p$ **NOR** $q)$ **NOR** $(p$ **NOR** $q)$

$\equiv \sim (p$ **NOR** $q)$ **by proof in Question 3(a)**

$\equiv \sim\sim (p \vee q)$ **by def'n of NOR**

$\equiv p \vee q$ **by Double Negation**

Markers' Note: Two points for a correct circuit, and two points for justification (equivalence laws or truth table). Minor errors in the circuit will lose 1 of the two 2 points. Errors in the justification, or reliance or non-formal English statements, score at most 1 of the two 2.

Since for every truth table over k atomic propositions, we can write a propositional formula that matches the truth table using $\wedge$, $\vee$, and $\sim$, your answers to this question show that you can implement any specified logic function with a circuit that uses only 2-input NOR gates.

Did you know: you can do the same with two-input NAND gates?

4. Not Udderly Alone (6 marks)

Ryan has a barn with four stalls, denoted s_1, s_2, s_3, and s_4. The stalls are arranged side-by-side in a single row.

Unfortunately, being forgetful, Ryan has forgotten exactly how many of each type of animal he is raising in the four-stall barn. But, he remembers that he has exactly four animals, with each stall serving as home to exactly one animal. Additionally, each animal is either a cow or a pig.

Ryan has a problem: namely, when cows are resting in their stalls, they never want to be left without at least one cow in a stall beside them (so they can share cheese recipes easily with at least one other cow).

Ryan has hired Alice and Steve to (over-)engineer a solution to his problem. First, Alice and Steve rig a sensor to each barn stall: the sensor for stall s_i produces output x_i, where $x_i =$ true if the animal in s_i is a cow, or false if the animal in s_i is not a cow (i.e., if it is a pig).

Design a circuit that takes the four sensor outputs as inputs (x_1, x_2, x_3, and x_4), and outputs true if and only if no cow is standing in a stall without a cow beside it. Should this circuit ever output false, Ryan will know to rearrange the animals to keep the cows happy.

Answer:
There is a problem (the circuit should output false) in precisely the four times there is a cow with no cow beside it:

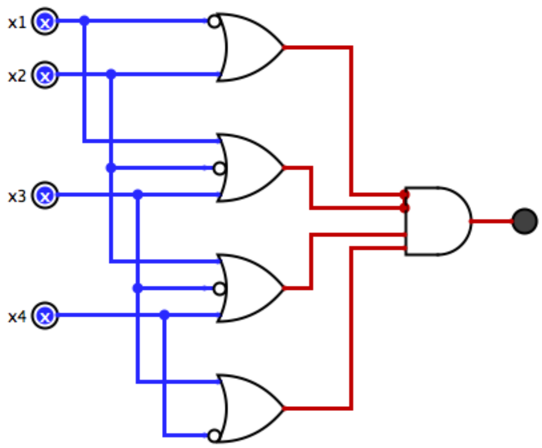
- $x_1 \wedge \sim x_2$ **(the cow in s_1 is alone);**
- $\sim x_1 \wedge x_2 \wedge \sim x_3$ **(the cow in s_2 is alone);**
- $\sim x_2 \wedge x_3 \wedge \sim x_4$**; (the cow in s_3 is alone); or,**
- $\sim x_3 \wedge x_4$ **(the cow in s_4 is alone).**

The circuit should output false if there is a cow-tastrophe, so the circuit can be a negation of the disjunctive-normal form expression of the above four cases. That is, the circuit can be expressed as:

$$\sim [(x_1 \wedge \sim x_2) \vee (\sim x_1 \wedge x_2 \wedge \sim x_3) \vee (\sim x_2 \wedge x_3 \wedge \sim x_4) \vee (\sim x_3 \wedge x_4)]$$

We could rewrite the formula to find alternative forms, but it does not reduce the number of gates used (there are still five). However, some people may find it easier to read:

$$\begin{aligned}
& \sim [(x_1 \wedge \sim x_2) \vee (\sim x_1 \wedge x_2 \wedge \sim x_3) \vee (\sim x_2 \wedge x_3 \wedge \sim x_4) \vee (\sim x_3 \wedge x_4)] \\
\equiv\ & \sim (x_1 \wedge \sim x_2) \wedge \sim (\sim x_1 \wedge x_2 \wedge \sim x_3) \wedge \sim (\sim x_2 \wedge x_3 \wedge \sim x_4) \wedge \sim (\sim x_3 \wedge x_4) \\
& \textbf{by de Morgan's} \\
\equiv\ & (\sim x_1 \vee \sim\sim x_2) \wedge (\sim\sim x_1 \vee \sim x_2 \vee \sim\sim x_3) \wedge (\sim\sim x_2 \vee \sim x_3 \vee \sim\sim x_4) \wedge (\sim\sim x_3 \vee \sim x_4) \\
& \textbf{by de Morgan's} \\
\equiv\ & (\sim x_1 \vee x_2) \wedge (x_1 \vee \sim x_2 \vee x_3) \wedge (x_2 \vee \sim x_3 \vee x_4) \wedge (x_3 \vee \sim x_4) \\
& \textbf{by Double Negation}
\end{aligned}$$



Markers' Note: Full marks can be earned for any correct solution that uses the minimal number of gates (five). If students use the "NOR" solution (written before I rearranged the formula), but use two gates to implement the NOR (NOT + OR), also give full marks.

Inefficient circuits (e.g., implementing the disjunctive normal for the 7 times that the circuit should output true, directly) should receive at most 5/6.

Deduct one mark for minor errors in the circuit drawing, including if the output is inverted. In particular, students presenting a 7-truth DNF implementation, while getting at most a score of 5/6, will almost certainly also make errors in drawing it out as a circuit.

While there are no marks required for "exploring the problem" (that is, a correct circuit alone can get full marks), doing such exploring such as writing out the logic formulas for when there is a problem with the cows can earn partial marks. Up to 3/6 marks can be earned for writing a logical formula or plain-English description that expresses how to solve the problem (3/6 for the negation of a DNF, as above; 2/6 for a vanilla DNF with 7 cases in which the circuit should output true), even if no circuit or an incomplete or flawed circuit is present.

5. Because I Say So (2 marks)
Write a truth table that takes two inputs, p and q, and outputs a tautology. You can name your tautological function with any letter or symbol you please, except p or q (to avoid confusion).

Answer:

p	q	☺
0	**0**	**1**
0	**1**	**1**
1	**0**	**1**
1	**1**	**1**

Markers' Note: Many students will likely attempt to write out a tautological function such as $p \vee \sim p$ as their function. Such work is unnecessary, but do not remove marks if it is actually a tautology. If the student's written function is not a tautology, the answer gets zero.

Marking should be all-or-nothing.

6. The Same or Different? (2 marks)
   Is $\sim (a \wedge b)$ equivalent to $\sim a \wedge \sim b$? Provide a real-life (concrete) example, giving any propositional definitions to a and b that you please, to demonstrate that they are the same or that they are different. Note that it is *not sufficient* merely to define meanings for a and b; you must also justify why the two compound propositional statements are equivalent, or describe a situation in which they evaluate differently.

   Answer:
   Let a mean that Alice likes apples, and let b mean that Ryan likes bananas. Consider the case where Alice likes apples, but Ryan does not like bananas. Then $\sim (a \wedge b)$ will evaluate to true, because "It is not the case that Alice likes apples and Ryan likes bananas" is a true statement. However, $\sim a \wedge \sim b$ will evaluate to false, because "Alice does not like apples, and Ryan does not like bananas" is not a true statement, as Alice likes apples.

   Markers' Note: An answer that simply gives definitions to a and b, then states without justification that the two compound statements are different, receives 0/2.

   An answer that identifies a truth-table row in which the two answers are different, but provides no other justification (e.g., "Alice likes apples and Ryan doesn't like bananas"), receives 1/2.

   To get the full two points, students must actually use their chosen definitions in some sort of explanation about the two compound propositions (be generous with how detailed explanations need to be, but it needs to go beyond simple identification of a truth-table row).