



uOttawa

PHY1321 AND PHY1331
Principles of Physics
LECTURE NOTES

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Fall 2017

Lecture Notes

to accompany "Physics for Scientists and Engineers"
by Serway and Jewett (9th edition)

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PHY1321/31 Fall 2016 Tentative (and Optimistic) Course Schedule:

PHY1321/1331A

PHY1321/1331B

PART I

THERMODYNAMICS

1	Sept 6	Intro Class	Sept 8
2	Sept 11	State Variables: Pressure and Temperature	Sept 12
3	Sept 13	Ideal Gas Law: PV diagram	Sept 15
4	Sept 18	First Law of Thermodynamics	Sept 19
5	Sept 20	Internal Energy, Work and Heat in Gas Transitions	Sept 22
6	Sept 25	Summary of Thermodynamics I	Sept 26
7	Sept 27	Kinetic Theory of Gases I	Sept 29
8	Oct 2	Kinetic Theory of Gases II	Oct 3
9	Oct 4	Heat Engines	Oct 6
10	Oct 11	Entropy /Summary of the Thermodynamics II	Oct 10

MIDTERM I

Oct 14

PART II

REVIEW OF LINEAR DYNAMICS

11	Oct 16	Kinematics I	Oct 13
12	Oct 18	Kinematics II	Oct 17
13	Oct 30	Dynamics I	Oct 20
14	Nov 1	Dynamics II	Oct 31
15	Nov 6	Energy, Work	Nov 3
16	Nov 8	Linear Momentum/Centre of Mass	Nov 7
17	Nov 13	Centre Of Mass	Nov 10

MIDTERM II

Nov 11

18	Nov 15	Rotational Motion I	Nov 14
19	Nov 20	Rotational Motion II	Nov 17
20	Nov 22	Rotational Motion III	Nov 21
21	Nov 27	Fluids I	Nov 24
22	Nov 29	Fluids II	Nov 28
23	Dec 4	Strategic Reserve	Dec 1
24	Dec 6	Strategic Reserve	Dec 5

Yes ,it is a lot! I have taught this course for last 13 years and only few times I have been able to discuss all of the topics listed above. Still, we will do our best.

HOW TO USE THE NOTES?

THE PHY 1321/1331 LECTURE NOTES HAVE BEEN CREATED, COMPILED AND REFINED OVER THE COURSE OF LAST TEN YEARS WHILE I HAVE BEEN TEACHING THIS COURSE, WHAT I AM SAYING HERE IS THAT THESE ARE VERY GOOD NOTES!

Why notes?

1. It is rather difficult to copy ppt slides during the lecture in a large auditorium equipped with small foldable "elbow-rests" pretending to be tables.
2. Also we will frequently solve problems from the textbook (end of chapter problems) so instead of carrying the heavy textbooks on campus, students should bring the notes and use them in class (annotate them, add their own notes/comments etc.) the same idea is behind putting the DGD problems at the end of your notes

THESE NOTES ARE MEANT TO BE READ BEFORE THE CLASS!

ALSO, EACH TOPIC LISTS CORRESPONDING CHAPTERS FROM THE TEXTBOOK.

THERE ARE TWO SUBCHAPTERS FOR SELF - STUDY! (roughly 1% of the discussed material: THEY WILL NOT BE DISCUSSED IN CLASS BUT THEY ARE TESTABLE)

IT IS ABSOLUTELY VITAL THAT STUDENTS READ THE NOTES BEFORE THE CLASS. (THAT IS ABSOLUTE MINIMUM OF PREPARATION FOR THE CLASS).

SERIOUS STUDENTS WOULD:

- i) **READ THE NOTES,**
- ii) **READ THE RELEVANT CHAPTERS FROM THE TEXTBOOK,**
- iii) **THINK ABOUT THE SUGGESTED PROBLEMS.**
- iv) **COMPLETE ONLINE PRE-LECTURE QUIZ (typically due on the night before the given class)**

During each lecture two more problems from the list will be solved (on average). The remaining problems are left for students as practice. 90% of the suggested problems are from the textbook. The solutions to the suggested problems will not be provided online (so don't ask!)

How to study physics? Survival manual.

What to expect in this course?

- How does this course compare with the high-school physics
- Advice from experienced instructor.
- Advice from your students who took this class in the past.
- Ground rules.

How does this course compare with the high-school physics

In the first semester of introductory course of Physics I am used to spending up to 40% to 50% of the semester (this year I am planning to use 9 lectures only) on the review of classical mechanics that students know from high-school grade 11 and grade 12 physics.

(It is pretty much all of the mechanics that is studied in grade 12 high-school physics, there are some significant differences as illustrated by tables below:

Table 1:

Typical differences between the HS and First Year University class:

high-school (class size: under 30)	University (class size under 300)
Teacher knows students	Teacher does not know students
Students know each other	Students don't know each other
Slower pace	Faster pace (10 times faster!)
Many tests focused on small parts of the material:	Fewer tests covering larger parts of the material
Credit recovery schemes, additional assignments	Marks are final and rarely disputed

Table 2

What do we understand by feedback on the assignments and tests

high-school (class size: under 30)	university (class size under 300)
Grade is split into the Understanding and Knowledge, Application, Communication	One grade to rule them all!
The graded paper includes personalized comments (always full of tenderness and care): One may read something like this: "Dear Jimmy you show a great talent in drawing small triangles, however your calculations of the trigonometric functions require some refining... Keep working on it!☺ mark:20%"	The grade is the feedback! There will be no comments, or maybe something along the lines: "20%! F- Please see me immediately!"
Some parents successfully intervene with principals to rise their children grades.	Grades are confidential and may not be discussed with third parties!

WARNING: PACE OF LECTURES

I get the following a lot:

- "At first we were moving at nice slow pace and I was able to keep up; but then we just rushed through a lot of material very quickly!"
- I did not speed up! I actually moved at the same pace in the second half of the semester: 1 week per chapter (12 chapters all together)
- Naturally if the same amount of time is spend on the relatively familiar material as it is on the content that has never been introduced before, the latter will appear to be rushed through!

Why Physics is perceived as "hard?"

- Many students will claim that Physics is harder than other subjects studied by them.
- Almost all students will admit that success in Physics requires more work than similar success in other disciplines
- In the past I asked students if they agree with these statements, and asked them to try to think why is it so?

Here is what I think...

- 1 University physics is perceived as hard as it requires operational knowledge of large areas of simple math.
- 2 **Majority of students are familiar with the relevant math content itself, but they have not internalized it.**
(The mathematical ideas are not second nature to students in the same way as knowledge of English language is).
As result majority of students need much more time to comprehend simple physical content expressed in the language of mathematics.
- 3 **Physics grades are established predominantly based on problem solving.**
Even the students who were "very good in math" in HS (received 80% or better) would find the "text problems," to be the most difficult in the particular class.
(Fortunately such text problems typically make up only up to 20% of any given math test).
- 4 Guess what...
In Physics all of the problems are really "applied math text problems!"
(Lots of "applied math" problems are from kinematics!)

This is an opinion commonly presented by a students in physics classes

- "When I am in class I understand everything perfectly, I can follow the instructor without any problems. I also understand the textbook. It is only when I try to solve problems on my own that I have difficulty!"
- Guess what?
Your grade is based almost entirely on your ability to solve problems!
- That means that you should spend 90% of your study time outside of the classroom on solving problems.
- In the words of immortal Richard Feynman:
"You don't know anything until you practice..."

This leads us to answer to the key question: How to study physics?

- It all depends on what do you want from this course both in terms of a grade as well as your own satisfaction and knowledge.
- Doing just the assignments and re-doing the problems from the lecture notes should give a solid base to pass a course with the grade in a C-B range.
- To get really strong in physics (A- or higher) one will need to have a lot of practice.
- The end of the chapter (EOC) problems are great for practice. Every odd problem has the numerical solution at the end of the textbook. These problems should be practiced constantly by serious students.

As a professor I strive to give my students greatest possible exposure to many problems, therefore:

- There are about 100 problems listed in the lecture notes, some of which will be solved or discussed during the lectures.
- I do not commit to solve all of them but they represent absolute minimum!
- There are additional 60 problems to be discussed in DGDs with your TA
- There will be 2 tests in which you will be asked to solve about 14 MC problems and 10 long problems in total
- There will be 60 assignment problems for you to solve
- This will amount to about 250 problems solved in the course of the semester! (not counting the quizzes etc.)
- There are about 80 online quiz questions I will ask of you.
- Also there are past midterms and past final exam for you to get use to the format and difficulty level of the tests.
- You may also use them as an evaluation tool before your tests.
- Any student who'll do (re-do) all of the above before the final will not be surprised by the exam!

How much time will it all take?

Research indicates that what is needed is 2 to 3 hrs per one hour of in-class instruction/per week.

In other words to do well in this class (B+ or better) one would need to put in 7 to 12 hrs of studying per week (does not include the labs).

CIVILITY AND RESPECT

1 Please be mindful of other students needs!

Limit to minimum the disruptive behavior:

- Being late,
- Talking,
- Taking trips to the washroom.

Please switch off the phones and other electronic devices.

2 Rules regarding Food in the classroom

As a general rule do not bring food to the class-room, please!

Food staples to be avoided in particular:

- Extremely hot coffee (safety!)
- Full 4-course breakfast!
- Loud crunchy food (crackers, celeries and carrots, chips, plastic wrapped candy, popcorn)
- Aromatic food (pasta, oranges, nachos, pizza, onions, popcorn).

If you really have to have something to eat, I suggest soft-bread sandwich eaten discretely, somewhere in the back seats of the classroom.

WHY DO WE NEED TO KNOW BASIC PHYSICS?

- 5 Educated people should know a little bit about everything. This includes physics.
- 4 Everyday practice of all of natural sciences requires using of the very advanced physical instruments.
All natural sciences use physical methods and models.
As a result of this, our understanding of physics will make us better biologists, chemists, geologists, and physicians.
- 3 Physics is the most fundamental knowledge about the material world. Laws of physics apply to every system. It means that on the most fundamental level, all of the natural sciences: chemistry, biology, geology, meteorology, medicine are in fact some form of physics.
ALL SCIENTISTS SHOULD KNOW PHYSICS AT SOME FUNDAMENTAL LEVEL.
- 2 As science graduates, some of us might one day need to teach Physics to high-school students. It would be best if we were competent...
- 1 Physics is a fascinating subject worth knowing.
(It is cool!)

HIGH SCHOOL MATHEMATICS EVERYBODY IS EXPECTED TO KNOW

This course requires certain mathematical proficiency. Below is a list of skills/ knowledge that we expect every student to possess to be successful in this class:

1. Solving linear equations
2. Solving quadratic equations
3. Solving systems of linear/quadratic equations
4. Trigonometry
5. Geometry
6. Vectors
7. Knowledge of all the fundamental functions and corresponding graphs:
Linear quadratic polynomial trigonometric logarithmic, exponential
8. Differential Calculus

At its end, the textbook contains appendices on the fundamental math. Please look them up!

Also there is a whole chapter on vectors that will not be discussed in class (but reviewed in the first DGD)

Make sure that you mastered this material ASAP.

It will make huge difference, as we will be using this simple math all the time.

INTRODUCTION TO THERMODYNAMICS

WHAT IS THERMODYNAMICS?

State Variables

Macroscopic vs. Microscopic

0th Law of Thermodynamics

Temperature and Pressure

Thermal expansion of solids and fluids

(Reading Assignment)

DEMO 1: ATMOSPHERIC PRESSURE: CAN

DEMO 2: Guernicke Experiment

DEMO 3: Oil + water in the U-Tube

DEMO 4: Thermal Expansion of Solids

4 Mid-lecture Quiz Questions

7 Suggested Problems

READING ASSIGNMENT: Chapter 19

State Variables

Macroscopic vs. Microscopic

A state variable is an element of the set of variables that describe the state of a dynamical system.

Temperature, pressure, internal energy, enthalpy, entropy are examples of state variables in a thermodynamics system.

These are macroscopic variables
(attributable to the large ensembles of atoms/molecules)
In case of simple mechanical systems, position coordinates and their derivatives are typical state variables.

These are microscopic variables
(attributable to single atoms/molecules)

ONE OF THE MAJOR ACCOMPLISHMENTS OF THE
THEORY OF STATISTICAL MECHANICS WAS
OBTAINING STATE VARIABLES FROM MICROSCOPIC
VARIABLES DESCRIBING AVERAGE SINGLE
MOLECULE!

• Zeroth Law of Thermodynamics

- If objects A and B are separately in thermal equilibrium with a third object C, then A and B are in thermal equilibrium with each other

- Object C (thermometer) is contact with A until they achieve thermal equilibrium

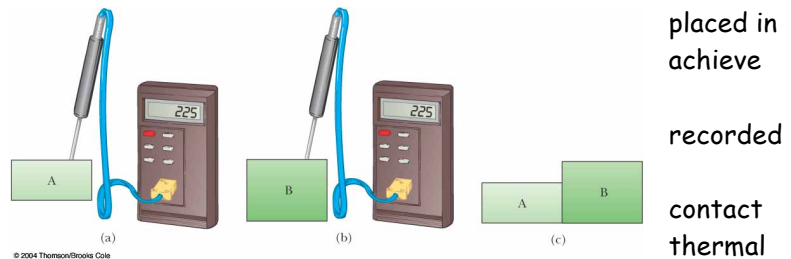
- The reading on C is

- Object C is then placed in with object B until they achieve equilibrium

- The reading on C is recorded again

- If the two readings are the same, A and B are also in thermal equilibrium

- IF TWO OBJECTS ARE IN THERMAL EQUILIBRIUM THERE IS NO ENERGY TRANSFER BETWEEN THEM (THEY HAVE THE SAME TEMPERATURE)



placed in
achieve
recorded
contact
thermal

Thermal Contact

- Two objects are in **thermal contact** with each other if energy can be exchanged between them
- The exchanges we will focus on will be in the form of heat or electromagnetic radiation
- The energy is exchanged due to a temperature difference

Thermal Equilibrium

- Thermal equilibrium is a situation in which two objects would not exchange energy by heat or electromagnetic radiation if they were placed in thermal contact
- The thermal contact does not have to also be physical contact

TEMPERATURE

Temperature

- We associate the concept of temperature with how "hot" or "cold" an objects feels
- Our senses provide us with a qualitative indication of temperature
- Our senses are unreliable for this purpose
- We need a technical definition of temperature

- **Temperature** can be thought of as the property that determines whether an object is in thermal equilibrium with other objects
- Two objects in thermal equilibrium with each other are at the same temperature
 - If two objects have different temperatures, they are not in thermal equilibrium with each other

Thermometers

- Thermometer is a device used to measure the temperature of a system.
- In order to be effective thermometer needs to fulfill following general conditions
 - Thermometer needs to have some linearly temperature dependant physical parameter
 - **Needs to have small "heat capacity"**
 - (by introducing it into the system we will change the system temperature - we want that change to be negligible!

- Thermometers are based on the principle that some physical property of a system changes as the system's temperature changes.
- These properties include:
 - The volume of a liquid
 - The dimensions of a solid
 - The pressure of a gas at a constant volume
 - The volume of a gas at a constant pressure
 - The electric resistance of a conductor
 - The color of an object
- A temperature scale can be established on the basis of any of these physical properties
- A thermometer can be calibrated by placing it in contact with some natural systems that remain at constant temperature
- Common systems involve water.
- A mixture of ice and water at atmospheric pressure called the **ice point** of water.

Liquid in glass Thermometers

- A common type of thermometer is a liquid-in-glass
- The material in the capillary tube expands as it is heated
- The liquid is usually mercury or alcohol
- Inaccurate far from the calibration points
- Useful only in the -30 to 80 C



Thermocouple Thermometers

- Commonly used in variety of environments from low -250C to +1900C.
- Utilize the thermoelectric effect:
 - when conductor is subjected to a thermal gradient, it will generate a voltage.
- Any attempt to measure this voltage necessarily involves connecting another conductor to the "hot" end. This additional conductor will then also experience the temperature gradient, and develop a voltage of its own which will oppose the original.
- The magnitude of the effect depends on the metal in use.
- Using a different metal to complete the circuit creates a circuit in which the two legs generate different voltages, leaving a small difference in voltage available for measurement. That difference increases with temperature, and can typically be between 1 and 70 microvolts per degree Celsius ($\mu V/^\circ C$) for the modern range of available metal combinations.
- Certain combinations have become popular as industry standards, driven by cost, availability, convenience, melting point, chemical properties, stability, and output. This coupling of two metals gives the thermocouple its name.



TEMPERATURE SCALES

Celsius Scale

- The ice point of water is defined to be 0° C
- The steam point of water is defined to be 100° C
- The length of the column between these two points is divided into 100 increments, called degrees

Fahrenheit Scale

- A common scale in everyday use in the US
- Named for Daniel Fahrenheit
- Temperature of the ice point is 32 °F
- Temperature of the steam point is 212 °F
- There are 180 divisions (degrees) between the two reference points

Kelvin (absolute) Scale

- Absolute zero is used as the basis of the absolute temperature scale
- The size of the degree on the absolute scale is the same as the size of the degree on the Celsius scale
- The absolute scale is also called the kelvin scale
- Named for William Thomson, Lord Kelvin
- The triple point temperature is 273.16 K
- No degree symbol is used with kelvins
- The kelvin is defined as 1/273.16 of the difference between absolute zero and the temperature of the triple point of water

Conversions:

- Celsius and Kelvin have the same size degrees, but different starting points

$$T_C = T - 273.15$$

- Celsius and Fahrenheit have different sized degrees and different starting points

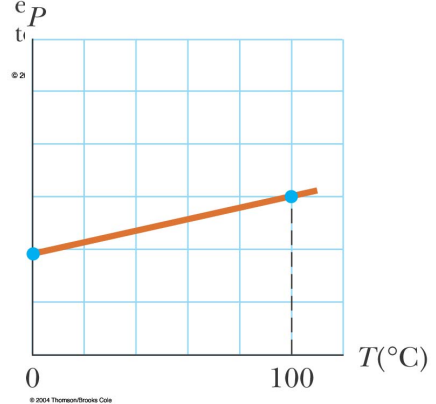
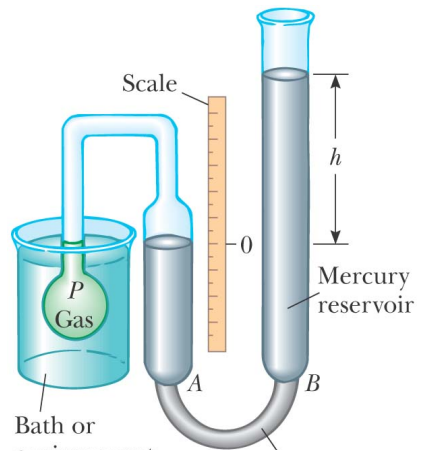
$$T_F = \frac{9}{5} T_C + 32$$

Ice point temperatures: $0^\circ\text{C} = 273.15 \text{ K} = 32^\circ \text{F}$

Steam point temperatures: $100^\circ\text{C} = 373.15 \text{ K} = 212^\circ \text{F}$

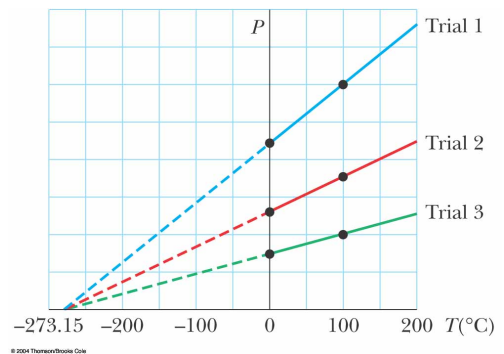
Constant Volume Gas Thermometer

- The physical change exploited is the variation of pressure of a fixed volume gas as its temperature changes
- The volume of the gas is kept constant by raising or lowering the reservoir B to keep the mercury level at A constant
- The thermometer is calibrated by using a ice water bath and a steam water bath
- The pressures of the mercury under each situation are recorded
 - The volume is kept constant by adjusting A
- The information is plotted
- To find the temperature of a substance, the gas flask is placed in thermal contact with the substance
- The pressure is found on the graph
- The temperature is read from the graph



Absolute Zero

- The thermometer readings are virtually independent of the gas used
- If the lines for various gases are extended, the pressure is always zero when the temperature is -273.15°C
- This temperature is called **absolute zero**
- The absolute temperature scale is now based on two new fixed points (adopted by in 1954 by the International Committee on Weights and Measures)
 - One point is absolute zero
 - The other point is the triple point of water
- This is the combination of temperature and pressure where ice, water, and steam can all coexist
- The triple point of water occurs at 0.01°C and 4.58 mm of mercury
- This temperature was set to be 273.16 on the absolute temperature scale
 - This made the old absolute scale agree closely with the new one
 - The units of the absolute scale are **kelvins**
- The figure at right gives some absolute temperatures at which various physical processes occur
- The scale is logarithmic
- The temperature of absolute zero cannot be achieved
 - Experiments have come close

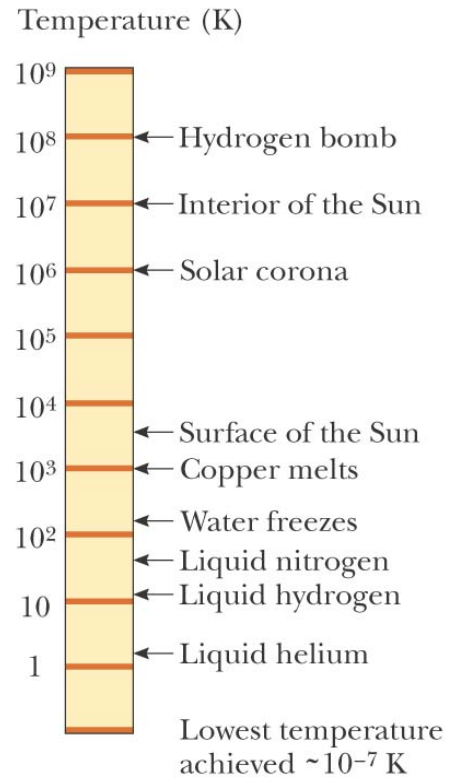


Energy at Absolute Zero

- According to classical physics, the kinetic energy of the gas molecules would become zero at absolute zero
- The molecular motion would cease
 - Therefore, the molecules would settle out on the bottom of the container
- Quantum theory modifies this and shows some residual energy would remain
 - This energy is called the **zero-point** energy

BIG QUESTION :

How does one measure $T=10^6$ K or higher?

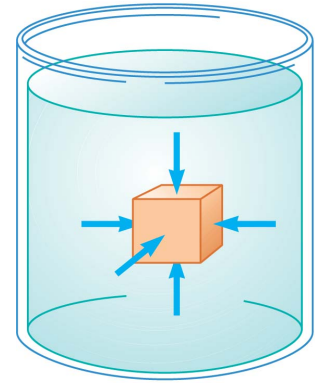


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PRESSURE

- The **pressure** P of the fluid at the level to which the device has been submerged is the ratio of the force to the area

$$p = \frac{F}{A}$$



- Pressure is a scalar quantity
 - Because it is proportional to the magnitude of the force
- If the pressure varies over an area, evaluate dF on a surface of area dA as $dF = P dA$
- Unit of pressure is **pascal** (Pa)

$$1\text{Pa} = \frac{1\text{N}}{1\text{m}^2}$$

Pressure in the uniform static fluid is constant at the same level

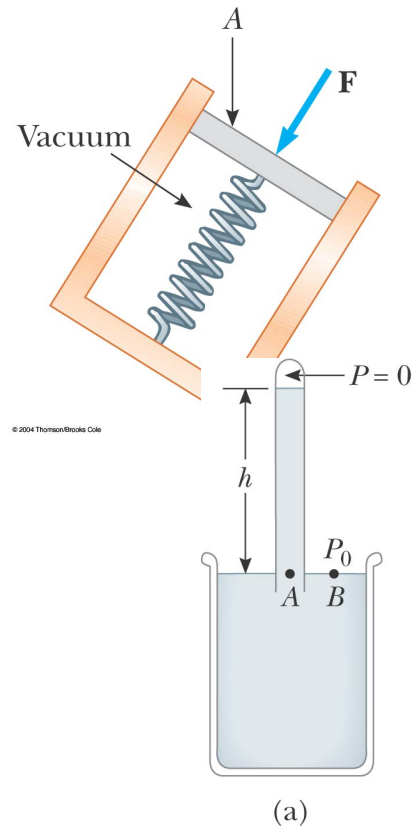
The pressure p at a depth h below a point in the liquid at which the pressure is p_0 is greater by an amount ρgh .

$$p = p_0 + \rho gh$$

- The liquid has a density of ρ ./ We assume the density is the same throughout the fluid./
- If the liquid is open to the atmosphere, and P_0 is the pressure at the surface of the liquid, then P_0 is *atmospheric pressure*
- $P_0 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$

Measuring Pressure

- The spring is calibrated by a known force
- The force due to the fluid presses on the top of the piston and compresses the spring
- The force the fluid exerts on the piston is then measured

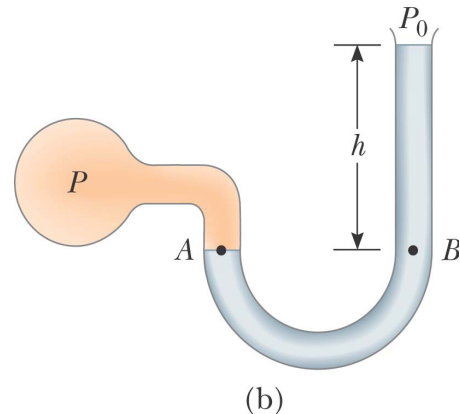


Barometer

- Invented by Torricelli
- A long closed tube is filled with mercury and inverted in a dish of mercury
 - The closed end is nearly a vacuum
- Measures atmospheric pressure as
- One 1 atm = 0.760 m (of Hg)

Manometer

- A device for measuring the pressure of a gas contained in a vessel
- One end of the U-shaped tube is open to the atmosphere
- The other end is connected to the pressure to be measured
- Pressure at B is $P_0 + \rho gh$
- $P = P_0 + \rho gh$
- P is the **absolute pressure**
- The **gauge pressure** is $P - P_0$
 - This is also ρgh
 - This is what you measure in your tires



IMPORTANT QUESTIONS:

1What is the total force acting on human body associated with the atmospheric pressure?

2What is the origin of atmospheric pressure?

THERMAL EXPANSION OF SOLIDES AND LIQUIDS: READING ASSIGNMENT

Length:

$$\Delta L = L_f - L_i = \beta(T_f - T_i)L_i = \beta L_i \Delta T$$

Volume:

$$\Delta V = V_f - V_i = \beta(T_f - T_i)V_i = \beta V_i \Delta T$$

Area:

$$\Delta A = A_f - A_i = \gamma(T_f - T_i)A_i = \beta A_i \Delta T$$

Read the relevant material in the textbook

Find out the meaning of all the equations and variables above

Answer the following questions:

- 1 How do α (linear expansion coefficient) and β (average coefficient of volume expansion) relate to each other?
- 2 How do α (linear expansion coefficient) and γ (average coefficient of area expansion) relate to each other?

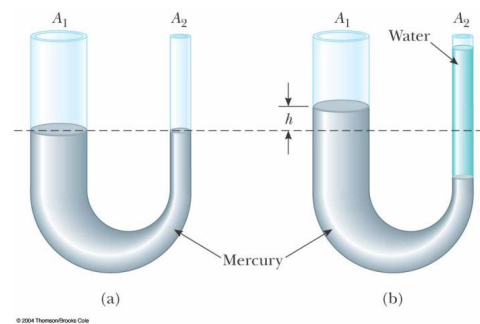
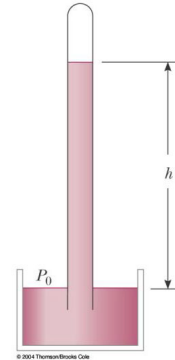
SUGGESTED PROBLEMS

Temperature Scales

- At what absolute temperature the temperatures measured using Celsius scale and Fahrenheit Scale are equal?
- On a Strange temperature scale, the freezing point of water is -15.0°S and the boiling point is $+60.0^{\circ}\text{S}$. Develop a *linear* conversion equation between this temperature scale and the Celsius scale. **2S_JS_E 6**

Pressure:

- Blaise Pascal duplicated Torricelli's barometer using a red Bordeaux wine, of density 984 kg/m^3 , as the working liquid (Fig. P14.17). What was the height h of the wine column for normal atmospheric pressure? Would you expect the vacuum above the column to be as good as for mercury?
- Mercury is poured into a U-tube as in Figure P14.18a. The left arm of the tube has cross-sectional area A_1 of 10.0 cm^2 , and the right arm has a cross-sectional area A_2 of 5.00 cm^2 . One hundred grams of water are then poured into the right arm as in Figure
 - Determine the length of the water column in the right arm of the U-tube.
 - Given that the density of mercury is 13.6 g/cm^3 , what distance h does the mercury rise in the left arm?



Change of length with temperature

- A certain telescope forms an image of part of a cluster of stars on a square silicon charge-coupled detector (CCD) chip 2.00 cm on each side. A star field is focused on the CCD when it is first turned on and its temperature is 20.0°C . The star field contains $5\,342$ stars scattered uniformly. To make the detector more sensitive, it is cooled to -100°C . How many star images then fit onto the chip? The average coefficient of linear expansion of silicon is $4.68 \times 10^{-6}\text{ (}^{\circ}\text{C)}^{-1}$.
- A copper wire and a lead wire are joined together, end to end. The compound wire has an effective coefficient of linear expansion of $20.0 \times 10^{-6}\text{ (}^{\circ}\text{C)}^{-1}$. What fraction of the length of the compound wire is copper?
- A steel rod undergoes a stretching force of 500 N . Its cross-sectional area is 2.00 cm^2 . Find the change in temperature that would elongate the rod by the same amount as the 500-N force does. Consult the tables for proper values of expansion coefficients.

IDEAL GAS LAW

What is the ideal gas?

Ideal Gas Law

pV Diagrams for Ideal Gas

pV Diagrams for Various Gas Transformations

DEMO 1: Atmospheric Pressure: Can

DEMO 2: Liquid Nitrogen + Helium filled balloon.

4 Mid-lecture Quiz Questions

8 Suggested Problems

READING ASSIGNMENT: 19

IDEAL GAS

- An ideal gas is a theoretical gas composed of a set of randomly-moving point particles that interact only through elastic collisions.
- The ideal gas concept is useful because it obeys the ideal gas law, a simplified equation of state, and it yields to analysis under statistical mechanics.
- It is a gas in which the interatomic forces within the gas are very weak
 - We can imagine these forces to be nonexistent
- Note that there is no equilibrium separation for the atoms
 - Thus, no "standard" volume at a given temperature
 - For a gas, the volume is entirely determined by the container holding the gas
- Equations involving gases will contain the volume, V , as a variable

Equation of State

- It is useful to know how the volume, pressure and temperature of the gas of mass m are related
- The equation that interrelates these quantities is called the **equation of state**
 - These are generally quite complicated
 - If the gas is maintained at a low pressure, the equation of state becomes much easier
 - This type of a low density gas is commonly referred to as an **ideal gas**

Gas Laws

- When a gas is kept at a constant temperature, its pressure is inversely proportional to its volume (Boyle's law)
- When a gas is kept at a constant pressure, its volume is directly proportional to its temperature (Charles and Gay-Lussac's law)

Ideal Gas Law

- The equation of state for an ideal gas combines and summarizes the other gas laws

$$PV=nRT$$

This is known as the **ideal gas law**

- P pressure
- V volume
- n number of moles of substance $n=N/N_A$
- T temperature
- R is a constant, called the Universal Gas Constant
- $R = 8.314 \text{ J/mol} \cdot \text{K} = 0.08214 \text{ L} \cdot \text{atm/mol} \cdot \text{K}$
- From this, you can determine that 1 mole of any gas at atmospheric pressure and at 0°C is taking
-
- 22.4 liters of volume.

The Mole

- The amount of gas in a given volume is conveniently expressed in terms of the number of moles
- One **mole** of any substance is that amount of the substance that contains **Avogadro's number** of constituent particles
 - Avogadro's number $N_A = 6.022 \times 10^{23}$
 - The constituent particles can be atoms or molecules
- The number of moles can be determined from the mass of the substance: $n = m / M$
 - M is the molar mass of the substance
 - m is the mass of the sample
 - n is the number of moles

IDEAL GAS EQUATION IN ALTERNATIVE FORM

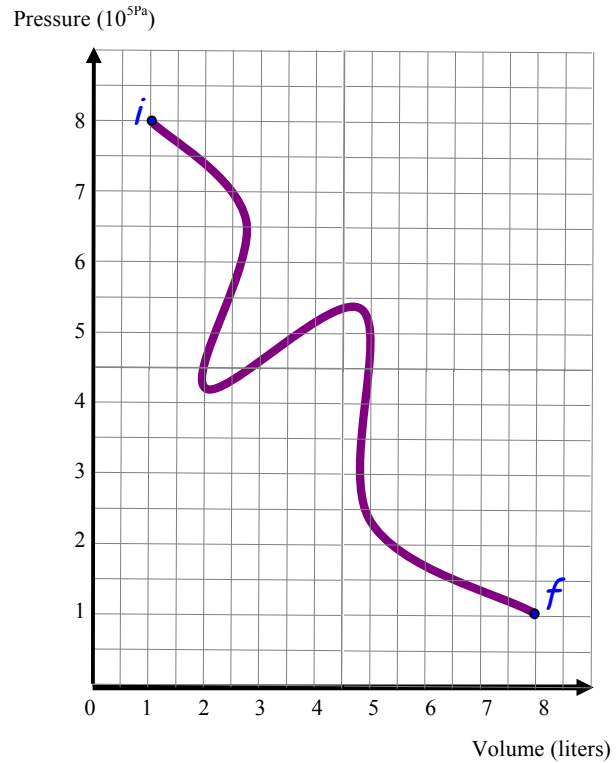
- The ideal gas law is often expressed in terms of the total number of molecules, N , present in the sample:

$$PV = nRT = (N/N_A) RT = Nk_B T$$

- k_B is Boltzmann's constant
- $k_B = 1.38 \times 10^{-23} \text{ J/K}$
- It is common to call P , V , and T the **thermodynamic variables** of an ideal gas

Phase Space and PV Diagrams

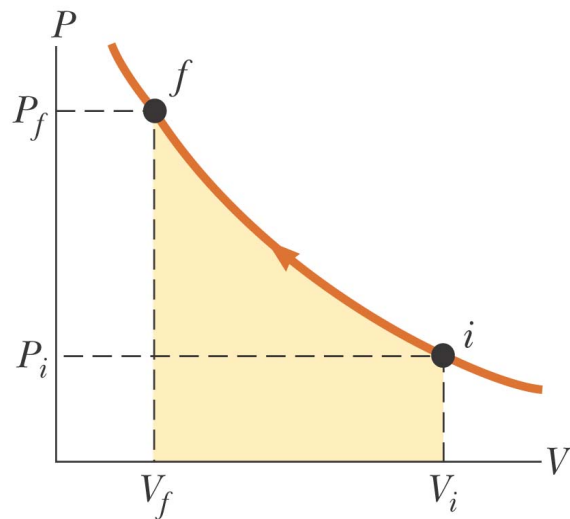
- If the system has well-defined Pressure, Volume and Temperature, we are able to represent its state at every moment in time as a point on a P-V, P-T or V-T diagram.
- The change of these parameters corresponds to the change of the coordinates in such "phase space diagram"
- PV diagram is used when the pressure and volume are known at each step of the process.
- The state of the gas at each step can be plotted on a graph called a **PV diagram**
 - This allows us to visualize the process through which the gas is progressing
- The curve is called the *path*



Pressure-volume diagram is used to describe a thermal cycle involving the following two variables:
 Volume (on the X axis) Pressure (on the Y axis)

This is in fact enough information to fully describe a simple system from a thermodynamic standpoint. Each point on the diagram has coordinates (V, p) which fully describe the ideal gas system at any point of time.

The diagrams are useful when one wants to calculate the work done by the system, the integral of the pressure with respect to volume. One can quickly calculate this using the PV diagram as simply the area enclosed by the cycle.



often it is

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Suggested Problems:

Ideal Gas Equation

- 1 Show that 1 mole of any gas at atmospheric pressure and at 0°C is taking 22.4 liters of volume
- 2 100 grams of oxygen and 100 grams of hydrogen gas occupy separate, equal sections of 200 liter tank. The divide is removed and the gases are allowed to mix and react with each other. The temperature is kept constant at 110°C , throughout the process
 - a) find the pressure of each gas in the separate containers
 - b) find the pressure after the reaction ends.
- 3 A popular brand of cola contains 6.50 g of carbon dioxide dissolved in 1.00 L of soft drink. If the evaporating carbon dioxide is trapped in a cylinder at 1.00 atm and 20.0°C , what volume does the gas occupy?
- 4 The mass of a hot-air balloon and its cargo (not including the air inside) is 200 kg. The air outside is at 10.0°C and 101 kPa. The volume of the balloon is 400 m^3 . To what temperature must the air in the balloon be heated before the balloon will lift off? (Air density at 10.0°C is 1.25 kg/m^3 .)
- 5 Just 9.00 g of water is placed in a 2.00-L pressure cooker and heated to 500°C . What is the pressure inside the container?
- 6 A tank having a volume of 0.100 m^3 contains helium gas at 150 atm. How many balloons can the tank blow up if each filled balloon is a sphere 0.300 m in diameter at an absolute pressure of 1.20 atm?
- 7 At 25.0 m below the surface of the sea ($\rho = 1025\text{ kg/m}^3$), where the temperature is 5.00°C , a diver exhales an air bubble having a volume of 1.00 cm^3 . If the surface temperature of the sea is 20.0°C , what is the volume of the bubble just before it breaks the surface?
- 8 Long-term space missions require reclamation of the oxygen in the carbon dioxide exhaled by the crew. In one method of reclamation, 1.00 mol of carbon dioxide produces 1.00 mol of oxygen and 1.00 mol of methane as a byproduct. The methane is stored in a tank under pressure and is available to control the attitude of the spacecraft by controlled venting. A single astronaut exhales 1.09 kg of carbon dioxide each day. If the methane generated in the respiration recycling of three astronauts during one week of flight is stored in an originally empty 150-L tank at -45.0°C , what is the final pressure in the tank?

FIRST LAW OF THERMODYNAMICS

Work

Heat

Internal Energy

Conservation of Energy

First Law of Thermodynamics

DEMO 1 Heating of the drill

DEMO 2 Heat -work equivalence experiment

2 Mid-lecture quiz questions

4 Suggested Problems

READING ASSIGNMENT: Chapter 20

The Law of Conservation of Energy:

In the isolated system, the total energy (sum of all forms of energy) is constant.

$$E = \text{const.}$$

CONSERVATION OF MECHANICAL ENERGY

If we dealing with a system that conserves Mechanical Energy (Which means that there are no losses) we can state

The Law of Conservation of Mechanical Energy:

In the absence of losses. the Total Mechanical Energy of the system is constant.

The only two forms of Mechanical Energy available to macroscopic bodies are Potential Energy(U) and Kinetic Energy (K)

Thus we can state our Conservation Law in the following form:

$$K+U=E=\text{const.}$$

WORK in MECHANICS

Work is an energy transfer associated with the change of mechanical energy.

For simple body work is defined as product of acting Force and displacement of that body.

$$dW = Fdx$$

Work = Change in Mechanical Energy

$$W = \Delta K + \Delta U$$

Everyday experience makes us aware of unavoidable nature of losses.

Effects such as: friction, drag, viscosity dissipate energy and diminish amount of mechanical energy of the system, eventually bringing all motion to a stop.

The loss of mechanical energy is accounted for if one adds heat and the increase of the system's internal energy to the balance.

We will now find out more about these Heat and Internal Energy

HEAT:

- **Heat** is a byproduct of friction. Whenever the Kinetic Energy is lost due to friction losses we will observe the system and the surroundings to "heat up."
-
- **Heat** is defined as the transfer of energy across the boundary of a system due to a temperature difference between the system and its surroundings.
- The term *heat* will also be used to represent the *amount* of energy transferred by this method

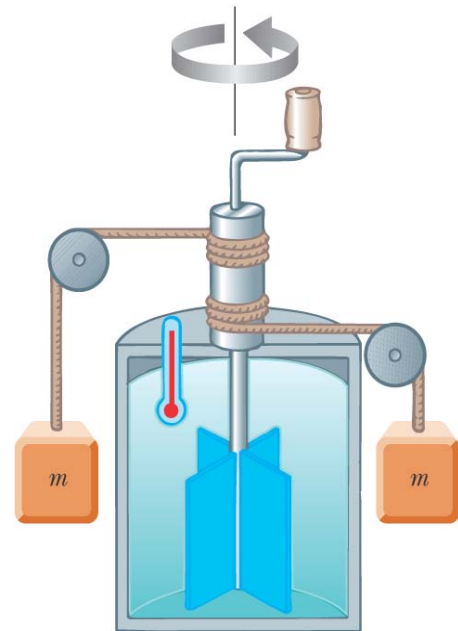
SINCE BOTH: WORK AND HEAT ARE FORMS OF ENERGY TRANSFER
(THEIR UNITS MUST NBE THE SAME AS ENERGY!)

INTERNAL ENERGY

- **Internal energy** is all the energy of a system that is associated with its internal structure (its microscopic components).
- **Today we know that on fundamental level** these components are its atoms and molecules.
- The system is viewed from a reference frame at rest with respect to the center of mass of the system.
- The kinetic energy due to its motion through space is **not** included.
- Internal energy does include kinetic energies due to:
 - Random translational motion
 - Rotational motion
 - Vibrational motion
- Internal energy also includes potential energy between molecules.

Mechanical Equivalent of Heat

- Joule established the equivalence between mechanical energy and internal energy
- His experimental setup is shown at right
- The loss in potential energy associated with the blocks equals the work done by the paddle wheel on the water.
- Joule found that it took approximately 4.18 J of mechanical energy to raise temperature of 1g of water by 1°C.
- Later, more precise, measurements determined the amount of mechanical energy needed to raise the temperature of 1g of water from 14.5°C to 15.5°C
- 1 cal = 4.186 J
 - This is known as the **mechanical equivalent of heat**
 - The "Calorie" used for food is actually 1 kilocalorie
- In the US Customary system, the unit is a BTU (British Thermal Unit)



Thermal insulator

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- One BTU is the amount of energy transfer necessary to raise the temperature of 1 lb of water from 63oF to 64oF
- The standard in the text is to use Joules
- Thermodynamics and mechanics were considered to be separate branches
 - Until about 1850
 - Experiments by James Joule and others showed a connection between them
- A connection was found between the transfer of energy by **heat** in thermal processes and the transfer of energy by **work** in mechanical processes
- The concept of energy was generalized to include internal energy
- The Law of Conservation of Energy emerged as a universal law of nature

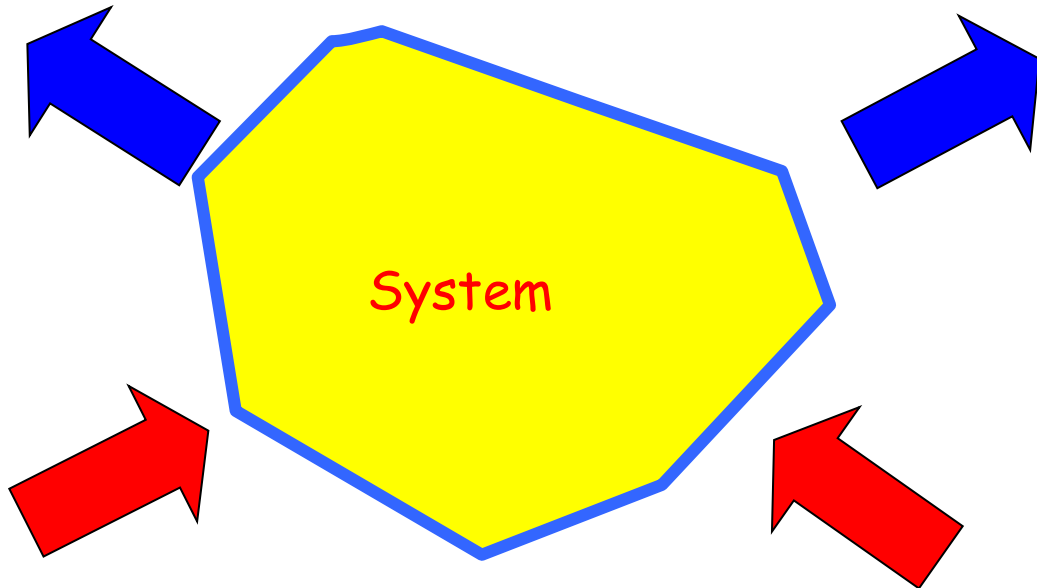
The First Law of Thermodynamics:

Expresses the conservation of energy for thermodynamic systems.
Change of the Internal Energy(E_{int}) is due to two transfer mechanisms:
Heat(Q) and Work (W)

$$\Delta E_{int} = W + Q$$

Work transferred
out of the system

Heat transferred
out of the system



Heat transferred
into the system

Work transferred
into the system

$$\Delta E_{int} = \Delta W + \Delta Q$$

First Law of Thermodynamics in Differential Form:

For small incremental quasi-static transfer

$$dE_{int} = dQ + dW$$

NOTE:

First law in its differential form is very useful in obtaining of the thermodynamic relations.

We will use it a lot in this course!

UNITS of ENERGY HEAT AND WORK

SI unit of energy is Joule:

$$1J = 1Nm \text{ (Newton times meter)}$$

Heat and work are measured using the same units as energy

Other important and widely used energy units are:

$$1Btu = 1054J$$

Calorie-cal - used in thermodynamics, chemistry and biomechanics

Electron Volt - eV - used in quantum physics and chemistry, spectroscopy

Wavenumber - cm^{-1} -(reciprocal centimeter) used in widely in spectroscopy

HOMEWORK:

Fill the conversion table below

	J	cal	eV	cm ⁻¹
J	1			
cal	4.186	1		
eV	1.6×10^{-19}		1	
cm ⁻¹				1

$$1 \text{ cal} = 4.186 \text{ J}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

Suggested Problems:

Note : some of the problems below may require violation of the II law of thermodynamics

- 1 The system performed 70J of work on the outside environment while receiving the 80J of heat from it and expelling 20J of heat. What is the net change of the internal energy of the system?
- 2 The thermal engine (system operating in a cyclic fashion) performs 150 J of work while receiving 500 J of heat from heat reservoir. How much heat does the engine expel to the cold reservoir?
- 3 One liter cylinder contains 1 mole of the ideal gas molecules having the average kinetic energy of 0.1eV. What is the total internal energy of this gas?
- 4 1 mole of ideal gas has total internal energy of 200J. What is the average kinetic energy (in eV) of one molecule of this gas?

HEAT

Heat

Specific Heats and Molar Specific Heats

Phase Changes and Latent Heat

Calorimetry

Adiabatic Process in the Ideal Gas

Heat Transfer Mechanisms

Demo 1: Calorimetric Measurements

Demo 2: Radiation Convection and Conductance

Demo 3: Radiating Bodies

12 Suggested Problems

READING ASSIGNMENT: chapter20

- **Heat** is defined as the transfer of energy across the boundary of a system due to a temperature difference between the system and its surroundings
- The term *heat* will also be used to represent the *amount* of energy transferred by this method
- Historically, the **calorie** was the unit used for heat
 - One calorie is the amount of energy transfer necessary to raise the temperature of 1 g of water from 14.5°C to 15.5°C
 - The “Calorie” used for food is actually 1 kilocalorie
 -
- In the US Customary system, the unit is a BTU (British Thermal Unit)
 - One BTU is the amount of energy transfer necessary to raise the temperature of 1 lb of water from 63°F to 64°F
- The standard in the text is to use Joules

The Heat Capacities and Specific Heats

Various bodies require different amounts of energy to be heated by the same temperature difference. Heat capacity reports on that property of the body .

- The **heat capacity**, C , of a particular sample is defined as the amount of energy needed to raise the temperature of that sample by 1°C
- If energy Q produces a change of temperature of ΔT , then

$$Q = c \Delta T$$

Specific Heat

- **Specific heat**, c , is the heat capacity per unit mass
- If energy Q transfers to a sample of a substance of mass m and the temperature changes by ΔT , then the specific heat is
- The specific heat is essentially a measure of how insensitive a substance is to the addition of energy
 - The greater the substance's specific heat, the more energy that must be added to cause a particular temperature change
- The equation is often written in terms of Q :

$$Q = m c \Delta T$$

Specific Heat versus Molar Specific Heat

$$Q = m c \Delta T$$

specific heat expression (m mass in kg) c specific heat

$$Q = n M c \Delta T$$

I am using $n = \frac{N_A}{N_A}$ (number of moles) and $M = N_A m_1$ (molar mass)

$$Q = n C \Delta T$$

specific molar heat expression (n amount of substance in moles)

$C = M c$ is a molar specific heat of the substance

$$Q = n C \Delta T$$

This expression is very useful in discussion of the ideal gas thermodynamics

Molar Specific Heat of Gases

It has been found experimentally that during the isobaric and isovolumetric gas transitions two gases exhibit two different molar heat capacities C_p and C_V

Consequently two different expressions for heat need to be used in these transitions

$$Q = nC_V \Delta T \text{ when } V = \text{const.}$$

$$Q = nC_p \Delta T \text{ when } p = \text{const.}$$

$Q = mc \Delta T$ SIGN CONVENTION

- If the temperature increases:
 Q and ΔT are positive
Energy transfers into the system
- If the temperature decreases:
 Q and ΔT are negative
Energy transfers out of the system

Specific Heat Variation with Temperature

- Technically, the specific heat varies with temperature
- The corrected equation is $Q = m \int_{T_i}^{T_f} c dT$
- However, if the temperature intervals are not too large, the variation can be ignored and c can be treated as a constant
For most substances there is only about a 1% variation between 0° and 100°C

Specific Heat of Water

- Water has the highest specific heat of common materials
- This is responsible for many weather phenomena
 - Moderate temperatures near large bodies of water

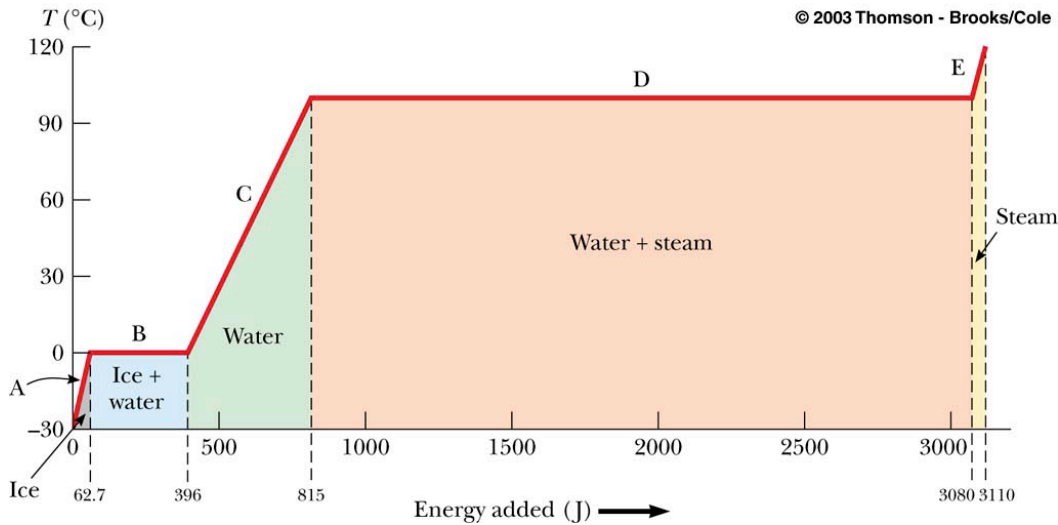
- Global wind systems
- Land and sea breezes

Phase Changes and Latent Heats of Transformation

- A **phase change** is when a substance changes from one form to another
 - Two common phase changes are
 - Solid to liquid (melting)
 - Liquid to gas (boiling)
- During a phase change, there is no change in temperature of the substance
- Different substances react differently to the energy added or removed during a phase change
 - Due to their different molecular arrangements
- The amount of energy also depends on the mass of the sample
- If an amount of energy Q is required to change the phase of a sample of mass m ,
- $L = Q / m$
- The quantity L is called the **latent heat** of the material
 - Latent means "hidden"
 - The value of L depends on the substance as well as the actual phase change
- The energy required to change the phase is $Q = \pm mL$

- The *latent heat of fusion* is used when the phase change is from solid to liquid
- The *latent heat of vaporization* is used when the phase change is from liquid to gas
- The positive sign is used when the energy is transferred into the system
 - This will result in melting or boiling
- The negative sign is used when energy is transferred out of the system
 - This will result in freezing or condensation

Graph of Ice to Steam



How much energy is needed to convert one gram of ice at -30.0°C to 1g of steam at 120°C ?

Part A (Warming up the Ice)

- During phase A, the temperature of the ice changes from -30.0°C to 0°C
- Use $Q = m_i c_i \Delta T$ In this case, 62.7 J of energy are added

Part B (Melting: Phase change)

- Once at 0°C , the phase change (melting) starts
- The temperature stays the same although energy is still being added
- Use $Q = m_i L_f$
 - The energy required is 333 J the total energy moves from 62.7 J to 396 J

Part C (Warming up the water)

- Between 0°C and 100°C , the material is liquid and no phase changes take place
- Energy added increases the temperature
- Use $Q = m_w c_w \Delta T$
 - 419 J are added The total is now 815 J

Part D- Boiling (phase change)

- At 100°C , a phase change occurs (boiling)
- Temperature does not change
- Use $Q = m_w L_v$
 - This requires 2260 J The total is now 3070 J

Part E- Warming up the steam

- After all the water is converted to steam, the steam will heat up
- No phase change occurs
- The added energy goes to increasing the temperature
- Use $Q = m_s c_s \Delta T$
 - In this case, 40.2 J are needed
 - The temperature is going to 120°C The total is now 3110 J

Molecular View of Phase Changes

Phase changes can be described in terms of the rearrangement of molecules (or atoms in an elemental substance)

- Liquid to Gas phase change
 - Molecules in a liquid are close together
 - The forces between them are stronger than those in a gas
 - Work must be done to separate the molecules
 - The latent heat of vaporization is the energy per unit mass needed to accomplish this separation
- Solid to Liquid phase change
 - The addition of energy will cause the amplitude of the vibration of the molecules about their equilibrium position to increase
 - At the melting point, the amplitude is great enough to break apart bonds between the molecules
 - The molecules can move to new positions
 - The molecules in the liquid are bound together less strongly than those of the solid
 - The latent heat of fusion is the energy per unit mass required to go from the solid-type to the liquid-type bonds
- The latent heat of vaporization is greater than the latent heat of fusion
 - In the liquid-to-gas phase change, the liquid-type bonds are broken
 - The gas molecules are essentially not bonded to each other
- It takes more energy to completely break the bonds than to change the type of bonds

Calorimetry

- The technique for measuring specific heat involves heating a material, adding it to a sample of water, and recording the final temperature is known as **calorimetry**,
- A calorimeter is a device in which this energy transfer takes place
- The system of the sample and the water is isolated
- Conservation of energy requires that the amount of energy that leaves the sample equals the amount of energy that enters the water
 - Conservation of Energy Gives a Mathematical Expression of This:

$$Q_{\text{cold}} + Q_{\text{hot}} = 0$$

The negative sign in the equation is critical for consistency with the established sign convention
Since each $Q = mc\Delta T$, c_{sample} can be found by:

$$c_s = \frac{m_w c_w (T_f - T_w)}{m_s (T_s - T_f)}$$

- technically, the mass of the container should be included, but if $m_w \gg m_{\text{container}}$ it can be neglected

Adiabatic Process in the Ideal Gas

We are ready to discuss the last important process of ideal gas.

Adiabatic Process Is the process in which no heat is exchanged with the surroundings!

It may be proven that in such process

$$pV^\gamma = \text{const where } \gamma > 1$$

We will demonstrate this in the course of the class

Heat Transfer Mechanisms: Reading Assignment (Self study!)

Thermal Conduction:

Energy transfer by the heat (on microscopic level it is caused by the exchange of kinetic energies of molecules of two bodies in thermal contact)

$$P = kA \frac{dT}{dx}$$

Radiation:

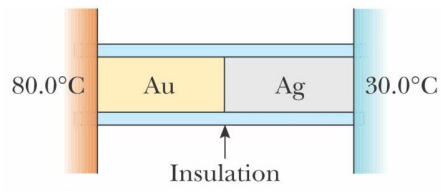
energy transfer by the electromagnetic radiation emitted from the body

$$P = \sigma A e T^4$$

Convection:

energy transfer by the movement of high T substance into low T substance. Takes form of convection currents.

Suggested Problems:

- 1 A 2.5 kg aluminum is heated to 92°C and then dropped into 8 kg of water at 5°C . Assuming that water metal system is thermally isolated what is the system equilibrium temperature?
- 2 A 20g copper calorimeter contains 100g of water at 30°C . Into this is poured a 40 g of glass beads which have been heated to a temperature of 100°C . The final equilibrium temperature of the whole system is 40°C . What is the specific heat for the glass?
- 3 A water heater is operated by solar power. If solar collector has an area of 6.00m^2 , and the Intensity delivered by sunlight is $550\text{W}/\text{m}^2$, how long does it take to increase the temperature of 1m^3 of water from 20°C to 60°C ?
- 4 A 1kg of ice at -30°C is added to 10 kg of steam at 500°C . answer the following questions:
 - a) What is the phase of the system of ice + steam if no heat escaped from it.
 - b) What is the final temperature when the equilibrium is established
- 5 A 5g lead bullet travelling with $v=1200\text{m}/\text{s}$ embeds in the 1kg block of ice at (-1°C) . Knowing that practically all of the kinetic energy of the bullet is converted to heat find the final temperature of the bullet + ice system. How much water was produced during the melting?
- 6 Two speeding lead bullets each of mass 5 g and temperature of 20°C collide head on at speeds of $500\text{m}/\text{s}$ each. Assuming a perfectly inelastic collision and no loss of energy to the atmospheres, describe the final state of the two bullet system.
7. The *Nova* laser at Lawrence Livermore National Laboratory in California is used in studies of initiating controlled nuclear fusion (Section 45.4). It can deliver a power of $1.60 \times 10^{13}\text{W}$ over a time interval of 2.50 ns. Compare its energy output in one such time interval to the energy required to make a pot of tea by warming 0.800 kg of water from 20.0°C to 100°C .
8. A combination of 0.250 kg of water at 20.0°C , 0.400 kg of aluminum at 26.0°C , and 0.100 kg of copper at 100°C is mixed in an insulated container and allowed to come to thermal equilibrium. Ignore any energy transfer to or from the container and determine the final temperature of the mixture.
- 9 A bar of gold is in thermal contact with a bar of silver of the same length and area (Fig.). One end of the compound bar is maintained at 80.0°C while the opposite end is at 30.0°C . When the energy transfer reaches steady state, what is the temperature at the junction?

The diagram shows a horizontal compound bar consisting of two sections: a yellow section labeled 'Au' (gold) on the left and a grey section labeled 'Ag' (silver) on the right. The left end of the bar is exposed to a vertical orange bar representing a heat reservoir at 80.0°C . The right end is exposed to a vertical blue bar representing a heat reservoir at 30.0°C . Below the compound bar, a horizontal blue bar is labeled 'Insulation' with an upward-pointing arrow, indicating that the compound bar is thermally isolated from its surroundings.
- 10 A glass window pane has an area of 3.00m^2 and a thickness of 0.600cm . If the temperature difference between its faces is 25.0°C , what is the rate of energy transfer by conduction through the window.
11. The surface of the Sun has a temperature of about $5\,800\text{K}$. Radius of the Sun is $6.96 \times 10^8\text{m}$. Calculate the total energy radiated by the Sun each second. Assume that the emissivity is 0.965.
- 12 The tungsten filament of a certain 100-W light bulb radiates 2.00W of light. (The other 98W is carried away by convection and conduction.) The filament has a surface area of 0.250mm^2 and an emissivity of 0.950. Find the filament's temperature. (The melting point of tungsten is $3\,683\text{K}$.)

HOW TO DEAL WITH INTEGRALS IN PHYSICS?

Only small part of the material presented here will be actually used in our physics class. Also, the proper discussion of integrals will be done in your calculus class in this and the next semester.

The basic idea as outlined here is very useful in understanding of many fine points outlined in the textbook.

Finally: nobody ever failed physics due to a lack of full understanding of integrals. Also, nobody missed an A+ for the same reason!

The key concept is that of the value of the linear integral of the function as the area under the curve!

It is expected that everybody to be able to use geometry to find simple integrals this way! Anything beyond this that you might learn from me will be a nice extra!

HOW TO DEAL WITH INTEGRALS IN PHYSICS?

Calculus is integral part of Physics.

Differential and Integral Calculus were invented by Sir. Isaac Newton and Gottfried Wilhelm Leibniz as a language of Mechanics. (XVII/XVIII century)

In the XIXth century, calculus was put on a much more rigorous footing by mathematicians such as Gauss, Cauchy, Riemann, and Weierstrass.

From its emergence in XVII century calculus continues to make various concepts in physics easier to comprehend.

Trying to avoid calculus in physics typically leads to confusion.

Fortunately, all students in this class already know basic differential calculus from high-school.

It is somewhat different story when it comes to integral calculus.

The proper introduction of the integrals with all necessary tricks and practicing, is left to your calculus class.

In this class we will need only the key ideas, and the simplest integrals. What follows is an attempt to deliver this content.

Everything that you will ever need to know
about integrals in first year Physics:

1 Indefinite Integrals as the anti-derivatives

2 Geometric Interpretation of the Definite Integral of a Function

3 Fundamental Theorem of Calculus

4 Examples

5 Integrals frequently encountered in Physics:

Path, Work, Moment of Inertia, Change in Entropy,

THE FIRST FUNDAMENTAL THEOREM OF CALCULUS

Integrals as the anti-derivatives

If we know how to find derivative of the fundamental function, we should be able to find the integral

If:

$$f = \frac{dF}{dt}$$

$$F = \int f dt$$

We are looking for such function F (called fundamental function) that the function f is its derivative !

Example:

Find integral $\int t^3 dt$

SOLUTION:

$$f(t) = t^3$$

What is the function of $F(t)$ such that $\frac{dF}{dt} = t^3$?

$$\text{Let's try } F(t) = t^4 \quad \frac{dF}{dt} = 4t^3$$

not quite what we are looking for! (we want t^3 not $4t^3$!)

So...

$$\text{Let's try } F(t) = \frac{1}{4}t^4$$

and we get:

$$\int t^3 dt = \frac{1}{4}t^4 + \text{const.}$$

Function $f(t)$	$F = \int f dt$
A	$At + \text{const.}$
at^n	$a \frac{1}{n+1} t^{n+1} + \text{const.}$
$\cos(t)$	$\sin(t) + \text{const.}$
$\sin(t)$	$-\cos(t) + \text{const.}$
$a \frac{1}{t}$	$a \ln(t) + \text{const.}$
ae^{bt}	$\frac{a}{b} e^{bt} + \text{const.}$

Example: Find $\int f dt$ in following cases:

a) $f(t) = \frac{1}{4} t^5$

b) $f(t) = 3t^7 - 4t^5 - 3t^2 + 7t + 3$

c) $f(t) = e^{-3t}$

d) $f(t) = 3e^{3t} - 4e^{-t}$

e) $f(t) = \frac{1}{4t^5}$

f) $f(t) = \frac{1}{4t}$

g) $f(t) = \frac{5t}{t^2}$

h) $f(t) = \frac{g'(t)}{g(t)}$

i) $f(t) = 5$

j) $f(t) = x$

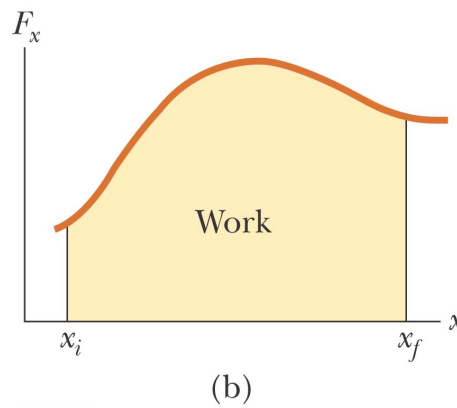
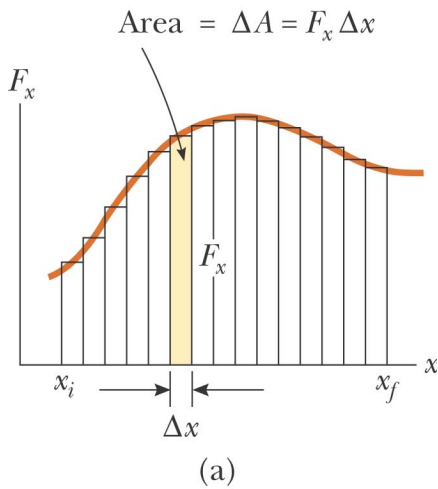
Geometrical Interpretation of Integral

Many important physics quantities are defined as integrals

For example:

$$W = \int_{x_i}^{x_f} F_x dx$$

Work done by acting force F_x between initial (x_i) and final position (x_f) is defined as as the area under the curve $F_x(x)$:



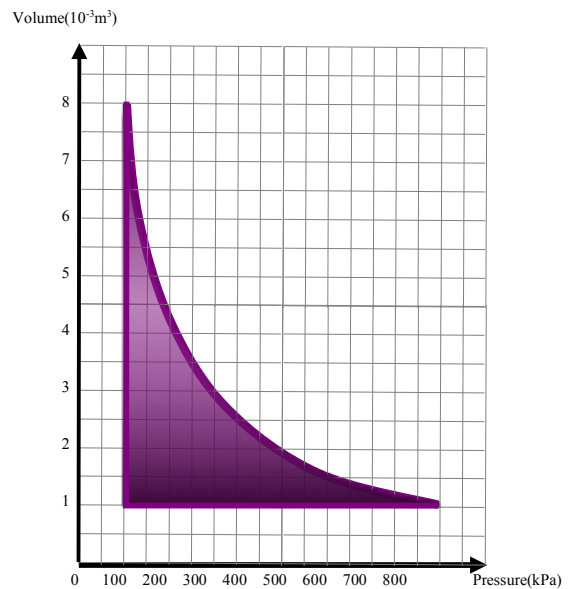
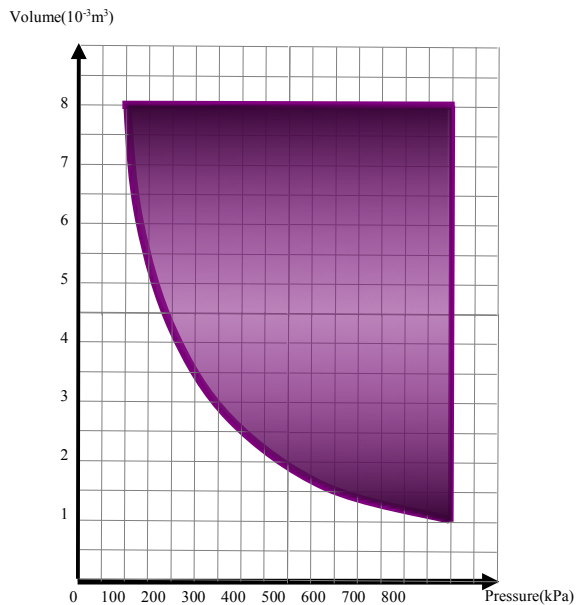
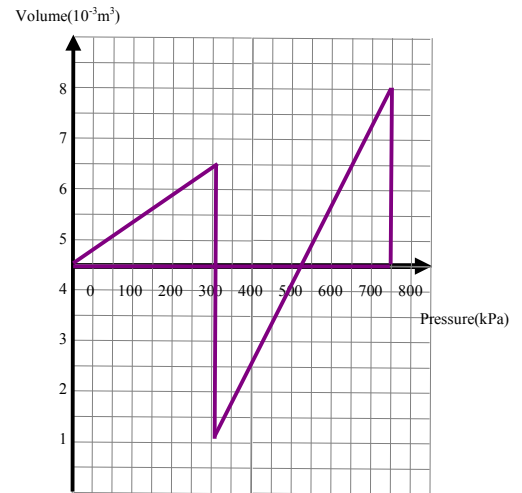
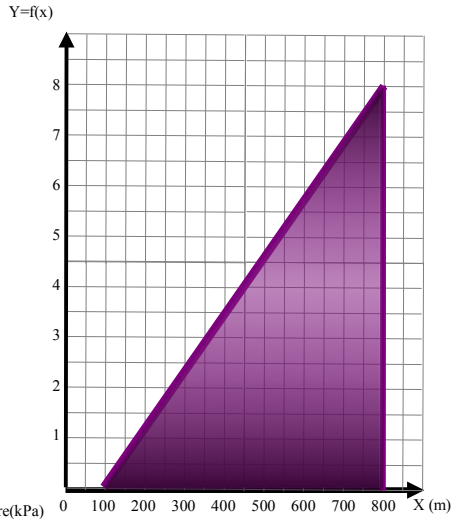
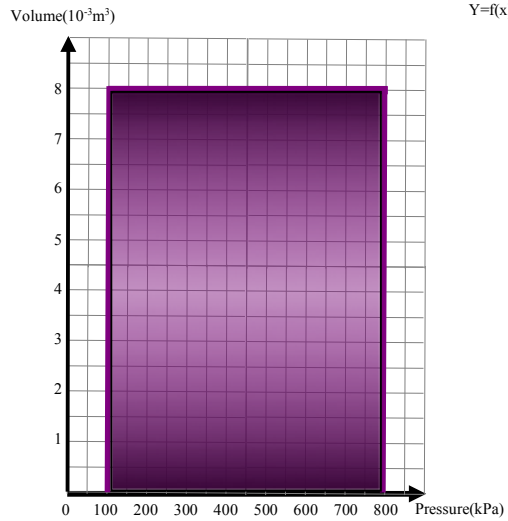
Geometric Interpretation of Integral implies the following'

$$\int_{x_1}^{x_2} F_x dx + \int_{x_2}^{x_3} F_x dx = \int_{x_1}^{x_3} F_x dx$$

$$\int_{x_1}^{x_2} F_x dx = - \int_{x_2}^{x_1} F_x dx$$

Examples:

Find the area under the curve and identify the integral in following cases:



THE SECOND FUNDAMENTAL THEOREM OF CALCULUS

Let's

$$f = \frac{dF}{dt}$$

So that:

$$F = \int f dt$$

$$\int_{t_i}^{t_f} f dt = [F(t)]_{t_i}^{t_f} = F(t) \Big|_{t_i}^{t_f} = F(t_f) - F(t_i)$$

To find the value of the definite integral one only needs to find the fundamental function $F(t)$ (antiderivative of $f(t)$!) and then simply substitute the upper and lower limits of integration and subtract them.

Examples:

Find $\int_{t_i}^{t_f} f dt$ in following cases:

a) $f(t) = \frac{1}{4}t^5$ $t_i = 0$ $t_f = 2$

b) $f(t) = 3t^7 - 4t^5 - 3t^2 + 7t + 3$ $t_i = 0$ $t_f = 2$

c) $f(t) = e^{-3t}$ $t_i = +\infty$ $t_f = 0$

d) $f(t) = 3e^{3t} - 4e^{-t}$ $t_i = 1$ $t_f = 2$

e) $f(t) = \frac{1}{4t^5}$ $t_i = 0$ $t_f = 2$

f) $f(t) = \frac{1}{4t}$ $t_i = 1$ $t_f = 2$

Find following

$$\int_1^4 t^2 dt$$

$$\int_0^3 (2t^2 - 1) dt$$

$$\int_{-1}^3 \frac{1}{3t^2} dt$$

$$\int_0^3 \frac{dt}{t}$$

$$3 \int_{t_i}^{t_f} dt$$

$$\int_0^3 (2e^{2t} - 1) dt$$

$$\int_{100}^{350} \frac{dT}{T^2}$$

$$nRT \int_{V_i}^{V_f} \frac{dV}{V}$$

$$\int_0^t (a) dt'$$

$$\int_0^t (at' + v_0) dt'$$

LIST OF SOME OF PHYSICAL QUANTITIES DEFINED

AS INTEGRALS:

Function	Corresponding Integral
W (work)	$W = \int_{x_i}^{x_f} F dx$
I (Impulse) (change of linear momentum)	$I = \Delta p = \int_{t_i}^{t_f} F dt$
Moment of Inertia	$I = \int_{Vol} R^2 dm$
Volume	$V = \int_{Vol} dx dy dz$
Speed	$v = \int_{t_i}^{t_f} a dt$
Distance traveled	$\Delta x = \int_{t_i}^{t_f} v dt$
Entropy Change	$\Delta S = \int_{T_i}^{T_f} \frac{dQ}{T}$

Final remarks:

This is a small part of what students will be learning in introductory calculus classes.

We focused on simplest integrals and avoided substitutions, integrating by parts etc.

Students will learn that even though it is always possible to find derivative of a function, finding of the integral is frequently tricky or impossible. (An arbitrary integral may be found using numerical methods only!)

This content will be also discussed during part of the DGD.

Back to physics...

WORK IN THERMODYNAMIC PROCESSES

Work as an Integral.
Work in PV Diagrams

Demo 1: Finding work in compression
Demo 2: Finding work in Joule's Experiment

4 Mid-lecture quiz questions
7 Suggested Problems

READING ASSIGNMENT: chapter20

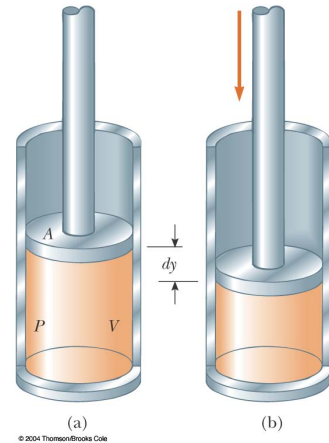
Work in Thermodynamics

- Work can be done on a deformable system, such as a gas
- Consider a cylinder with a moveable piston
- A force is applied to slowly compress the gas
 - The compression is slow enough for all the system to remain essentially in thermal equilibrium
 - This is said to occur **quasi-statically**

- The piston is pushed downward by a force \mathbf{F} through a displacement of $d\mathbf{r}$:

$$dW = \vec{F} \cdot d\vec{r} = -F\hat{j} \cdot dy\hat{j} = -Fdy = -pA dy = -pdV$$

- $A \cdot dy$ is the change in volume of the gas, dV
- Therefore, the work done on the gas is $dW = -P dV$



Interpreting $dW = -P dV$

- If the gas is compressed, dV is negative and the work done on the gas is positive
- If the gas expands, dV is positive and the work done on the gas is negative
- If the volume remains constant, the work done is zero
- The total work done is:

$$W = \int_i^f dW = \int_i^f -pdV$$

Work in PV Diagrams

- The work done on a gas in a quasi-static process that takes the gas from an initial state to a final state is the negative of the area under the curve on the PV diagram, evaluated between the initial and final states
 - This is true whether or not the pressure stays constant

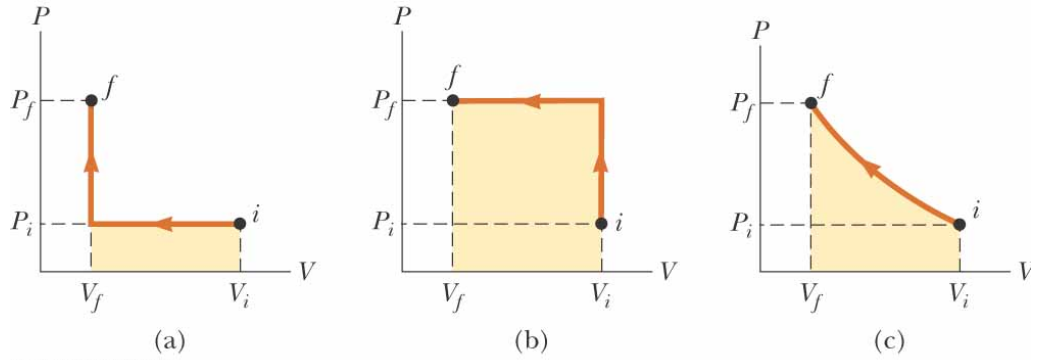
The work done **does** depend on the path taken!

Work Done By Various Paths between the same i and f

Each of these processes has the same initial and final states
 The work done differs in each process

The work done depends on the path

Example:

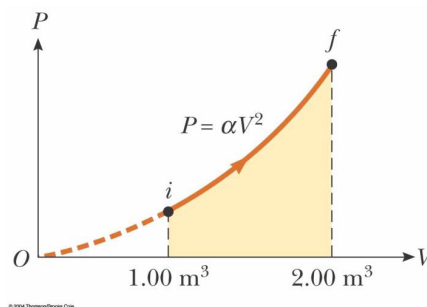


Find the expressions for work in ideal gas transformations:

- | | |
|-----------------------|--------------------|
| Isobaric Process | $p = \text{const}$ |
| Isovolumetric Process | $V = \text{const}$ |
| Isothermal Process | $T = \text{const}$ |
| Adiabatic Process | $Q = 0$ |

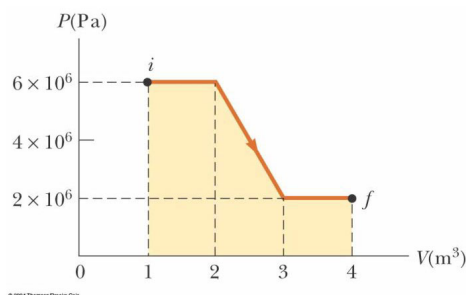
Suggested Problems

1 A sample of ideal gas is expanded to twice its original volume of 1.00 m^3 in a quasi-static process for which $P = \alpha V^2$, with $\alpha = 5.00 \text{ atm/m}^6$, as shown in Figure P20.23. How much work is done on the expanding gas?



2 One mole of an ideal gas does $3\,000 \text{ J}$ of work on its surroundings as it expands isothermally to a final pressure of 1.00 atm and volume of 25.0 L . Determine (a) the initial volume and (b) the temperature of the gas.

3 (a) Determine the work done on a fluid that expands from i to f as indicated in Figure P20.24. (b) What If? How much work is performed on the fluid if it is compressed from f to i along the same path?

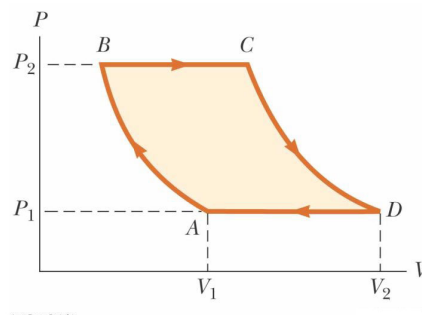


4 A 4 liter sample of a diatomic gas with $\gamma = 1.4$ confined to a cylinder, is carried through a closed cycle. The gas is initially at 1.00 atm and 300 K . First, its pressure is tripled under constant volume. Then it expands adiabatically to its original pressure. Finally the gas is compressed isobarically to its original volume.

- draw pV diagram of this cycle
- Determine the volume of the end of the adiabatic expansion
- Find the temperature of the gas at the start of the adiabatic expansion
- Find the temperature at the end of the cycle
- What was the net work done on the gas for this cycle

5 An ideal gas is carried through a thermodynamic cycle consisting of two isobaric and two isothermal processes as shown in Figure P20.69. Show that the net work done on the gas in the entire cycle is given by

$$W_{\text{net}} = -P_1(V_2 - V_1) \ln \frac{P_2}{P_1}$$



6 A cylinder containing n mol of an ideal gas undergoes an adiabatic process. (a) Starting with the expression $W = -\int P dV$ and using the condition $PV^\gamma = \text{constant}$, show that the work done on the gas is:

$$W = \left(\frac{1}{\gamma - 1} \right) (P_f V_f - P_i V_i)$$

(b) Starting with the first law of thermodynamics in differential form, prove that the work done on the gas is also equal to $nC_V(T_f - T_i)$. Show that this result is consistent with the equation in part (a).

7 One mole of an ideal gas is heated slowly so that it goes from the PV state (P_0, V_0) , to $(3P_0, 3V_0)$, in such a way that the pressure is directly proportional to the volume. (a) How much work is done on the gas in the process? (b) How is the temperature of the gas related to its volume during this process?

Summary of Thermodynamics of Gas Transformations I

Using the First Law of Thermodynamics together with definition of work and internal energy we should be able to fill a following table:

Process	W	Q	ΔE
V = const.			
P = const.			
T = const.			
Q = 0			

Three important questions remain:

- 1 What is the relation between C_v and C_p ?
- 2 What is the relation between C_v , (or C_p), and R in the ideal gas?
- 3 What is γ (gamma) in adiabatic transformation and how come

$$pV^\gamma = \text{const.}?$$