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Instructor's Name (**Print**)

Mingjue Ni
Student's Name (**Print**)

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THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENTS OF APPLIED MATHEMATICS AND MATHEMATICS

Calculus 1000A Midterm Examination

Friday, October 17, 2008 **Code 222** 7:00 p.m. -9:30 p.m.

INSTRUCTIONS

1. The first part of the exam (PART A) is MULTIPLE CHOICE. This part is to be answered on the SCANTRON answer sheet. As well, make sure that you circle your selected answer in the examination booklet. The answer you give on the scantron sheet is taken as being your intended choice in the event that the answer in the examination booklet is not the same.
2. Print your name on the SCANTRON answer sheet. Sign the answer sheet, and mark your student number and exam code on the answer sheet. USE AN HB PENCIL and mark your answers on the SCANTRON answer sheet.
3. ALL QUESTIONS IN PART B MUST BE ANSWERED IN THE SPACE PROVIDED. Be sure to answer each part of a question in the space provided for that part of the question. INDICATE YOUR ANSWER CLEARLY.
4. DO NOT UNSTAPLE THE BOOKLET.
5. SHOW ALL YOUR WORK FOR PROBLEMS IN PART B. All results must be justified unless you are instructed otherwise. Unjustified answers will receive little or no credit.
6. Calculators and other aids are NOT allowed.
7. Questions start on Page 1 and continue to Page 17. Should you require extra space for any answer, Page 18 is provided for this purpose. Be sure that your booklet is complete.
8. TOTAL MARKS = 100.
9. Fill in the top of this page. Circle your section in the list below.

001	P. Oman	006	X. Zou
002	R.N. Bryan	007	M. Khalkhali
003	M. Khalkhali	008	M. Franz
004	A. MacIsaac	009	N. Kiriushcheva
005	A. Buchel	010	R. Corless
		570KC	R.N. Bryan

PART A (50 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET.

2 marks A1. Find a formula for the inverse function $f^{-1}(x)$ where $f(x) = 1 + e^{2x}$.

B

$f(x) = y = 1 + e^{2x}$

$x = 1 + e^{2y}$

$x - 1 = e^{2y}$

$y = \frac{1}{2}\ln(x-1)$

A: $2\ln(x-1)$	B: $\frac{1}{2}\ln(x-1)$	C: $2\ln(x+1)$	D: $\frac{1}{2}\ln(x+1)$	E: $\ln(2x-1)$

2 marks A2. Find the exact value of $\log_5 10 + \log_5 20 - 3\log_5 2$.

C

$= \log_5 10 + \log_5 20 - \log_5 2^3$

$= \log_5 \frac{10 \times 20}{8}$

$= \log_5 25$

$= 2$

A: 32	B: 24	C: 2	D: 0	E: 25

2 marks A3. Find the exponential function $f(x) = Ca^x$, where $a > 0$, whose graph passes through the points with coordinates $(0, 3)$ and $(2, 48)$.

C

$f(x) = Ca^x$

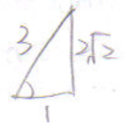
$f(0) = Ca^0 = 3$

$f(2) = Ca^2 = 48$

$\begin{cases} C = 3 \\ a = 4 \end{cases}$

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET.

2 marks A4. Find the exact value of $\cos\left[2\arccos\left(\frac{1}{3}\right)\right]$.



C

Suppose $\arccos(\frac{1}{3}) = x$

$\cos 2x = \cos^2 x - \sin^2 x$ $\quad (2\cos^2 x - 1 = 2\times(\frac{1}{3})^2 - 1 = \frac{-7}{9}$

$= (\frac{1}{3})^2 - (\frac{2\sqrt{2}}{3})^2$

$= -\frac{7}{9}$

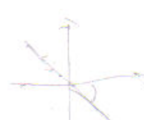
A: $\frac{2}{3}$	B: $\frac{7}{9}$	C: $-\frac{7}{9}$	D: $-\frac{2}{3}$	E: $\frac{2}{9}$

2 marks A5. Determine $\lim_{x\to 1^-} \frac{|x-1|}{x-1}$.

B $\quad = \lim_{x\to 1^-} \frac{-(x-1)}{x-1} = -1$

A: 1	B: -1	C: does not exist	D: 0	E: -2

2 marks A6. Find the exact value of $\arctan\left[\tan\left(\frac{3\pi}{4}\right)\right]$.



E $\quad R(y=\arctan x) = (-\frac{\pi}{2}, \frac{\pi}{2})$

$\arctan[\tan(\frac{3\pi}{4})] = \arctan[\tan(-\frac{\pi}{4})] = -\frac{\pi}{4}$

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET.

2 marks A7. If, for all values of x, $\cos(x+\pi) \le f(x) \le x^2 - 1$, find $\lim_{x\to 0} f(x)$.

C $\lim_{x\to 0} \cos(x+\pi) = \cos\pi = -1$

$\lim_{x\to 0} x^2-1 = -1$. When $x=0$, $\cos(x+\pi) = x^2-1$

By the Squeeze Thm, $\lim_{x\to 0} f(x) = -1$.

A: 0	B: 1	C: -1	D: 2	E: does not exist

2 marks A8. Determine $\lim_{x\to -\infty} \tan^{-1}(x^2 - x)$.

A As $x \to -\infty$, $x^2 \to \infty$, $-x \to \infty$, $x^2 + (-x) \to \infty$

$\lim_{x\to\infty} \tan^{-1}(x^2-x) = \frac{\pi}{2}$

A: $\frac{\pi}{2}$	B: $-\frac{\pi}{2}$	C: ∞	D: $-\infty$	E: 0

2 marks A9. Determine $\lim_{x\to -2^-} \frac{10}{x+2}$.

D As $x \to -2^-$, $x+2 \to 0^-$ $x+2<0$

$\lim_{x\to -2^-} \frac{10}{x+2} = -\infty$

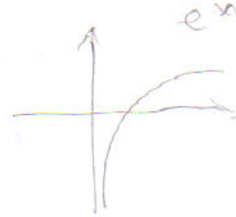
NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET.

2 marks A10. Determine $\lim_{x\to 0^+} \ln(e^x - 1)$.

B

As $x \to 0^+$, $e^x \to 1^+$, $e^x - 1 \to 0^+$

$\lim_{x\to 0^+} \ln(e^x - 1) = -\infty$



A: ∞	B: $-\infty$	C: 0	D: 1	E: e

2 marks A11. If $y = \cos(e^x)$, find $\frac{dy}{dx}$.

C

$\frac{d}{dx} y = -\sin(e^x) \cdot e^x$

A: $e^x \cos(e^x)$	B: $e^x \sin(e^x)$	C: $-e^x \sin(e^x)$	D: $\sin(e^x)$	E: $-\sin(e^x)$

2 marks A12. Determine $\lim_{x\to -\infty} \frac{\sqrt{9x^2+1}}{x-3}$.

$x \to -\infty$, $x = -\sqrt{x^2}$

D

$\lim_{x\to\infty} \frac{\sqrt{9x^2+1}}{x-1} = \lim_{x\to-\infty} \frac{-\frac{1}{\sqrt{x^2}}\sqrt{9x^2+1}}{\frac{1}{x}(x-3)} = \lim_{x\to-\infty} \frac{-\sqrt{9+\frac{1}{x^2}}}{1-\frac{3}{x}} = -3$

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET.

2 marks A13. If $y = \sin(\cos x)$, find $\frac{dy}{dx}$.

D $\frac{d}{dx}y = \cos(\cos x)\cdot(-\sin x)$

A: $\cos(\cos x)$	B: $\sin(-\sin x)$	C: $\sin x \cos(\cos x)$	D: $(-\sin x)\cos(\cos x)$	E: $\cos(-\sin x)$

2 marks A14. If $y = \arcsin(2x^2)$, find $\frac{dy}{dx}$. $2x^2$

A $\frac{d}{dx}y = \frac{1}{\sqrt{1-(2x^2)^2}} \cdot 4x = \frac{4x}{\sqrt{1-4x^4}}$

A: $\frac{4x}{\sqrt{1-4x^4}}$	B: $\frac{4x}{\sqrt{1-4x^2}}$	C: $\frac{4x}{\sqrt{1+4x^2}}$	D: $\frac{4x}{1+4x^4}$	E: $8x[\arcsin(2x^2)][\arccos(2x^2)]$

2 marks A15. If $f(x) = e^{\sec x}$, find $f'(x)$.

C $f'(x) = e^{\sec x} \cdot \tan x \sec x$

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET.

2 marks A16. Suppose $F(x) = f(g(x))$ and $f(0) = 1$, $f(6) = 3$, $f'(0) = 2$, $f'(6) = -1$, $g(0) = 6$, $g(6) = 5$, $g'(0) = 2$ and $g'(6) = 0$. Find $F'(0)$.

A

$g(0)=6$

$F'(x) = f'(g(x)) \cdot g'(x) = f'(g(0)) \cdot g'(0) = f'(6) \cdot g'(0)$

$= -1 \cdot 2 = -2$

A: -2	B: 30	C: -6	D: 18	E: 15

2 marks A17. Suppose $f(x) = 10^{\tan x}$. Find $f'\left(\frac{\pi}{4}\right)$.

A

$y = 10^u$, $u = \tan x$

$f'(x) = 10^{\tan x} \ln 10 \cdot \sec^2 x$

$= 10 \ln 10 \cdot \underbrace{\sec^2 x}_{(\sqrt{2})^2} = 20 \ln 10$

A: $20 \ln 10$	B: $10 \ln 10$	C: 1	D: $\ln 10$	E: 10

2 marks A18. If $y = \tan^{-1}(e^{3x})$, find $\frac{dy}{dx}$.

E

$\frac{d}{dx} y = \frac{1}{1+(e^{3x})^2} \cdot e^{3x} \cdot 3$

$= \frac{3e^{3x}}{1+e^{6x}}$

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET.

2 marks A19. Determine $\frac{d}{dx}(e^2)$. $=0$

B

constant

A: $2e$	B: 0	C: e^2	D: 1	E: e

2 marks A20. Determine $\lim_{x \to 0} \frac{\tan 4x}{x}$.

D

$$= \lim_{x\to 0} \frac{\sin 4x}{x} \cdot \frac{1}{\cos 4x}$$

$$= \lim_{x\to 0} \frac{\sin 4x}{4x} \cdot \frac{4}{\cos 4x}$$

$$= \lim_{x\to 0} \frac{\sin 4x}{4x} \cdot \lim_{x\to 0} \frac{4}{\cos 4x}$$

$$= 1 \times 4 = 4$$

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET.

A21 - A25. For each of the following, choose the letter which labels the graph below and transfer each of your answers to the scantron sheet.

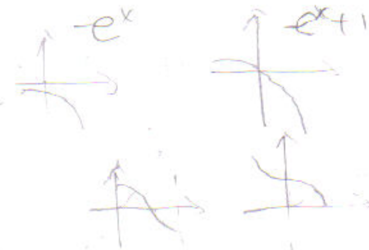
2 marks A21. The graph of $y = \arctan x$ is answer on the scantron sheet. B

2 marks A22. The graph of $y = 2^{-x}$ is answer on the scantron sheet. E $(\frac{1}{2})^x$

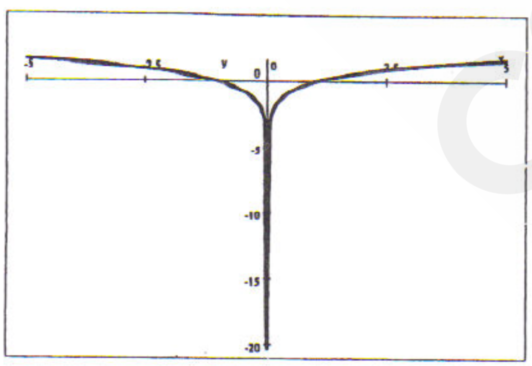
2 marks A23. The graph of $y = \ln|x|$ is answer on the scantron sheet. A

2 marks A24. The graph of $y = 1 - e^x$ is answer on the scantron sheet. C $-e^x$ $-e^x+1$

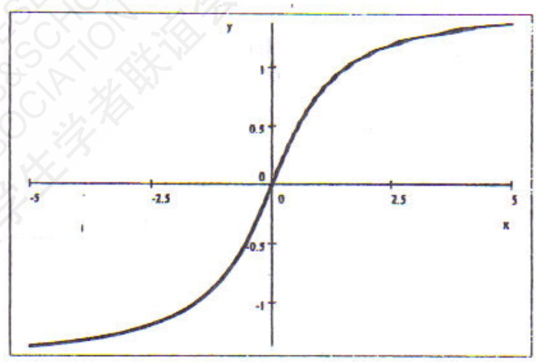
2 marks A25. The graph of $y = \cos^{-1} x$ is answer on the scantron sheet. D



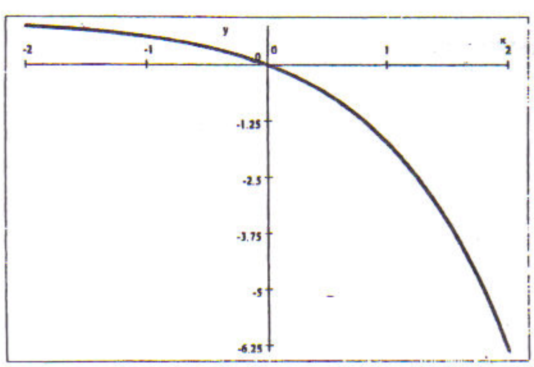
A



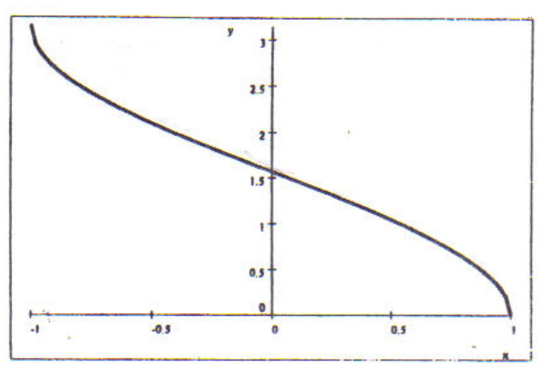
B



C



D



PART B (50 marks)

8 marks B26. Let $f(x)$ be the function given by

$$f(x) = \begin{cases} 1 + \ln(-x) & \text{if } x \le -1 \\ e^{x+1} & \text{if } -1 < x \le 0 \\ \cos x & \text{if } x > 0 \end{cases}$$

(a) State the value of the indicated limit, if it exists, in the space provided. If a limit does not exist, write DNE.

(i) $\lim_{x \to -1^-} f(x) =$ 1

(ii) $\lim_{x \to -1^+} f(x) =$ 1

(iii) $\lim_{x \to -1} f(x) =$ 1

(iv) $\lim_{x \to 0^-} f(x) =$ e

(v) $\lim_{x \to 0^+} f(x) =$ 1

(vi) $\lim_{x \to 0} f(x)$ DNE

(b) Is f continuous at -1? Justify your answer.

From a, $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) = 1 = \lim_{x \to -1} f(x)$

and $f(-1) = 1 + \ln 1 = 1 = \lim_{x\to -1} f(x)$

Thus, f is continuous at -1

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

4 marks B27. For what value of the constant c is the function f, given below, continuous at 3? Justify your answer.

$$f(x) = \begin{cases} 2c - x & \text{if } x < 3 \\ cx^2 - c & \text{if } x \geq 3 \end{cases}$$

f(x) is continuous on $(-\infty, 3)$ and $(3, +\infty)$

Solution: $\lim_{x \to 3^-} f(x) = 2C - 3$

$\lim_{x \to 3^+} f(x) = C \cdot 3^2 - C = 8C$

When there exists $\lim_{x \to 3} f(x)$, f is continuous at 3

So for C, $\lim_{x \to 3^-} = 2C - 3 = 8C = \lim_{x \to 3^+}$, f is continuous at 3.

$\Rightarrow 6C = -3 \quad C = -\frac{1}{2}$ +4

Thus when $C = -\frac{1}{2}$, f is continuous at 3.

NOTE: SHOW ALL YOUR WORK FOR PART (ii) OF THE PROBLEM ON THIS PAGE.

4 marks B28. The limit $\lim_{x\to 0} \frac{\arccos(x) - \frac{\pi}{2}}{x}$ is the derivative of some function f at some number a.

(*i*) What are $f(x)$ and a? (Answers alone are sufficient.)

Answers: $f(x) =$ arccos x

$a =$ 0

2

(*ii*) Compute (evaluate) $\lim_{x\to 0} \frac{\arccos(x) - \frac{\pi}{2}}{x}$ by finding $f'(x)$ and then evaluating $f'(a)$, where f and a are the answers you gave in part (i).

Solution: $f(x) = \arccos x$

$f'(x) = -\frac{1}{\sqrt{1-x^2}}$ $\quad a = 0$

$f'(a) = -\frac{1}{\sqrt{1-0}} = -1$

2

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

4 marks B29. Find $\frac{dy}{dx}$ if $y = \tan[\sec(x^2)]$. Do not simplify your answer.

Solution: $y = \tan u \quad u = \sec v \quad v = x^2$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} = \sec^2 u \cdot (\sec v \cdot \tan v) \cdot 2x$$

$$\frac{dy}{dx} = \sec^2[\sec(x^2)] \cdot \sec(x^2) \cdot \tan(x^2) \cdot 2x$$

4

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

6 marks B30. Use the Intermediate Value Theorem to show that there is a root of (i.e., a solution of) the equation $\tan 2x = \cos x$ in the interval $\left(0, \frac{\pi}{6}\right)$.

Solution: Suppose $f(x) = \tan 2x - \cos x$

$f(0) = \tan 0 - \cos 0 = 0 - 1 = -1$

$f(\frac{\pi}{6}) = \tan(2 \times \frac{\pi}{6}) - \cos\frac{\pi}{6} = \sqrt{3} - \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

By the Intermediate Value Theorem, since $f(0) < 0 < f(\frac{\pi}{2})$
f(x) is cont.
There must exist a number C, $C \in (0, \frac{\pi}{6})$, such that $f(C) = 0$, $\tan 2C = 2\cos C$.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

4 marks B31. Find $\frac{dy}{dx}$ if $y = 3\sin^2(\sqrt{x^2+1})$. Do not simplify your answer.

Solution: $y = 3u^2 \quad u = \sin v \quad v = \sqrt{w} \quad w = x^2+1$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

$$\frac{dy}{dx} = 6u \cdot \cos v \cdot \frac{1}{2} w^{-\frac{1}{2}} \cdot 2x$$

$$= 6\sin(\sqrt{x^2+1}) \cdot \cos(\sqrt{x^2+1}) \cdot \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x$$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

6 marks B32. Find $\frac{dy}{dx}$ if $y = \cos^2[\sin^{-1}(e^x)]$. <u>Do not simplify your answer</u>.

Solution: $y = \cos^2[\sin^{-1}(e^x)]$

$y = u^2 \quad u = \cos v \quad v = \sin^{-1} w \quad w = e^x$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

$$\frac{dy}{dx} = 2u \cdot (-\sin v) \cdot \frac{1}{\sqrt{1-w^2}} \cdot e^x$$

$$= 2\cos[\sin^{-1}(e^x)] \cdot \{-\sin[\sin^{-1}(e^x)]\} \cdot \frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x$$

6

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

right side limit !!!!

6 marks B33. Determine $\lim_{x\to 0} \boxed{\ln}[\cos^{-1}(1-x^2)]$.

Solution: As $x \to 0$, $x^2 \to 0$, $1-x^2 \to 1$ V

! $\cos^{-1} u \in [0, \pi]$, so as $1-x^2 \to 1$, $\cos^{-1}(1-x^2) \to 0^+$ V

As $\cos^{-1}(1-x^2) \to 0$, $\ln[\cos^{-1}(1-x^2)] \to -\infty$ V

So $\lim_{x\to 0} \ln[\cos^{-1}(1-x^2)] = -\infty$ V

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

8 marks B34. Find an equation of the tangent line to the graph of

$$\tan^{-1}\left(\frac{2x}{y}\right) = \frac{\pi x}{y^2}$$

at the point $(1, 2)$.

Solution. $\tan^{-1}\left(\frac{2x}{y}\right) = \frac{\pi x}{y^2}$

$$\Rightarrow \left[\tan^{-1}\left(\frac{2x}{y}\right)\right]' = \left(\frac{\pi x}{y^2}\right)'$$

$$\frac{1}{1+\left(\frac{2x}{y}\right)^2} \times \frac{y \cdot 2 + 2x \cdot y'}{y^2} = \frac{y^2 \cdot \pi + \pi x \cdot 2y \cdot y'}{(y^2)^2}$$

$$\frac{2y + 2x \cdot y'}{y^2 + 4x^2} = \frac{\pi y + 2\pi x \cdot y'}{y^3}$$

at the point $(1, 2)$

$$\frac{2 \times 2 + 2 \cdot y'}{4+4} = \frac{\pi(2 + 2y')}{2^3}$$

$$y' = \frac{2-\pi}{\pi - 1}$$

So the slope of the tangent line is $\frac{2-\pi}{\pi-1}$

the equation of the tangent line is

$$y - 2 = \frac{2-\pi}{\pi-1}(x-1)$$

$$y = \frac{2-\pi}{\pi-1}(x-1) + 2$$

8