CONCORDIA UNIVERSITY

FACULTY OF ENGINEERING AND COMPUTER SCIENCE

COURSE		NUMBER	SECTION
NUMERICAL METHODS IN ENGINEERING		**ENGR391**	**AA - AB**
EXAMINATION	DATE	TIME	
MIDTERM	May 25th, 2017	6:30 p.m. – 8:00 p.m.	
PROFESSOR			
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SPECIAL INSTRUCTIONS:

ENCS calculators are permitted. Closed book, no notes or textbook permitted.

Read each question carefully

Answer all questions

Write your solutions on the question sheet

You *MUST* show *all* your steps in solving all problems!

Question 1	
Question 2	
Question 3	
Total	

QUESTION 1:

a) Approximate $f(2.5)$ for $f(x) = \ln(x)$ using the zero, first, second and third order Taylor series expansions and employing a base point at $x = 1$. Provide the truncation error associated with each approximation. Use 8 digits in your calculations.
b) Compute $f(2.5)$ using 8 digits with your calculator
c) Compute the true percent relative error for each approximation found in a) using your answer from b) as a true solution.
d) Discuss how you would improve convergence to the true solution of $f(2.5)$

4.6 True value: $f(2.5) = \ln(2.5) = 0.916291\ldots$

zero order:

$f(2.5) = f(1) = 0$ $\qquad \varepsilon_t = \left|\frac{0.916291 - 0}{0.916291}\right| \times 100\% = 100\%$

first order:

$f(2.5) = f(1) + f'(1)(2.5-1) = 0 + 1(1.5) = 1.5$ $\qquad \varepsilon_t = \left|\frac{0.916291 - 1.5}{0.916291}\right| \times 100\% = 63.704\%$

second order:

$f(2.5) = 1.5 + \frac{f''(1)}{2}(2.5-1)^2 = 1.5 + \frac{-1}{2}1.5^2 = 0.375$ $\qquad \varepsilon_t = \left|\frac{0.916291 - 0.375}{0.916291}\right| \times 100\% = 59.074\%$

third order:

$f(2.5) = 0.375 + \frac{f^{(3)}(1)}{6}(2.5-1)^3 = 0.375 + \frac{2}{6}1.5^3 = 1.5$ $\qquad \varepsilon_t = \left|\frac{0.916291 - 1.5}{0.916291}\right| \times 100\% = 63.704\%$

fourth order:

$f(2.5) = 1.5 + \frac{f^{(4)}(1)}{24}(2.5-1)^4 = 1.5 + \frac{-6}{24}1.5^4 = 0.234375$ $\qquad \varepsilon_t = \left|\frac{0.916291 - 0.234375}{0.916291}\right| \times 100\% = 74.421\%$

Thus, the process seems to be diverging suggesting that a smaller step would be required for convergence.

Final answers:

a)	0^{th} order approximation = 1^{st} order approximation = 2^{nd} order approximation = 3^{rd} order approximation =
b)	$f(2.5) =$
c)	$\varepsilon_t =$ $\varepsilon_t =$ $\varepsilon_t =$ $\varepsilon_t =$

QUESTION 2:

Consider the following linear system of equations

$$\begin{aligned} 5x_1 - 2x_2 + 3x_3 &= 6 \\ x_1 - 4x_2 + 3x_3 &= 0 \\ 2x_1 - x_2 + 9x_3 &= 10 \end{aligned}$$

a) Write the system as a matrix equation: $[A]\{x\} = \{b\}$

b) Decompose the matrix into a set of one lower triangular (L) and one upper triangular (U) matrices using the LU method of your choice.

c) Solve the system $[A]\{x\} = \{b\}$ using L and U matrices found in b)

Use sufficient digits in your calculations such that your final answer has an accuracy of 5 significant digits

a)

$$\begin{bmatrix} 5 & -2 & 3 \\ 1 & -4 & 3 \\ 2 & -1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 10 \end{bmatrix}$$

b)

$$\begin{bmatrix} 5 & -2 & 3 \\ 1 & -4 & 3 \\ 2 & -1 & 9 \end{bmatrix} \begin{matrix} (1) \\ (2) \\ (3) \end{matrix} \qquad m_{21} = \frac{1}{5} = 0.2 \qquad m_{31} = \frac{2}{5} = 0.4$$

$$\begin{matrix} \\ (2) - m_{21}(1) \\ (3) - m_{31}(1) \end{matrix} \begin{bmatrix} 5 & -2 & 3 \\ 0 & -3.6 & 2.4 \\ 0 & -0.2 & 7.8 \end{bmatrix} \begin{matrix} (1) \\ (2') \\ (3') \end{matrix} \qquad m_{32} = \frac{0.2}{3.6} = \frac{1}{18} = 0.05555$$

$$\begin{matrix} \\ \\ (3') - m_{32}(2') \end{matrix} \begin{bmatrix} 5 & -2 & 3 \\ 0 & -3.6 & 2.4 \\ 0 & 0 & 7.66667 \end{bmatrix} = [U] \qquad [L] = \begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1 & 0 \\ 2/5 & 1/18 & 1 \end{bmatrix}$$

c)
$[U]\{x\} = \{z\}$
$[L]\{z\} = \{b\}$
Or

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 1 & 0 \\ 0.4 & 0.05555 & 1 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 0 \\ 10 \end{Bmatrix}$$

Then,
$z_1 = 6$
Forward substitutions yield
$z_2 = -0.2 \times 6 = -1.2$
And
$z_3 = 10 - 0.4 \times 6 - 0.05555 \times (-1.2) = 7.66667$

$$\begin{bmatrix} 5 & -2 & 3 \\ 0 & -3.6 & 2.4 \\ 0 & 0 & 7.66667 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ -1.2 \\ 7.66667 \end{Bmatrix}$$

Then,
$x_3 = \dfrac{7.66667}{7.66667} = 1$
Backward substitution
$x_2 = 1$
$x_1 = 1$

Double check:

$$\begin{bmatrix} 5 & -2 & 3 \\ 1 & -4 & 3 \\ 2 & -1 & 9 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 0 \\ 10 \end{Bmatrix}$$

Final answers:

b)	$[L] =$	$[U] =$
c)	$\{x\} =$	

QUESTION 3:

The following equation $f(x) = -2x^6 - 1.5x^4 + 10x + 1$

In order to determine the ***maximum*** of this function, we must first differentiate it.

a) Use the bisection method to solve $f'(x) = 0$. Employ initial guesses of $a = x_L = 0$ and $b = x_U = 1$. Perform iterations until the approximate relative error falls below 11%.

b) Now solve $f'(x) = 0$ using Newton-Raphson and an initial guess $x_0 = 1$. Perform iterations until the approximate relative error falls below 11%.

a)

$$f'(x) = -12x^5 - 6x^3 + 10$$

The root of this function represents an extremum. Using bisection and the recommended initial guesses gives:

i	x_l	x_u	x_r	$f(x_l)$	$f(x_r)$	$f(x_l)\times f(x_r)$	ε_a
1	0.00000	1.00000	0.50000	10.00000	8.87500	88.75000	
2	0.50000	1.00000	0.75000	8.87500	4.62109	41.01221	33.33%
3	0.75000	1.00000	0.87500	4.62109	-0.17444	-0.80610	14.29%
4	0.75000	0.87500	0.81250	4.62109	2.53263	11.70351	7.69%

b)

$$x_{i+1} = x_i - \frac{f'(x_i)}{f''(x_i)}$$

$$x_{i+1} = x_i - \frac{-12x^5 - 6x^3 + 10}{-60x^4 - 18x^2}$$

Iteration 1

$x_1 = 0.897435 \quad \varepsilon_{a1} = 11.428\%$

$x_2 = 0.872682 \quad \varepsilon_{a2} = 2.83\%$

Final answers:

a)	Using bisection $x_r =$
b)	Using Newton Raphson $x_r =$