# COMP 1805 – Discrete Structures I
# Tutorial 1

## Week of January 11

1. Are the following sentences propositions? Why or why not?

   (a) Always eat your vegetables.

   **Solution:** This is an imperative statement that is neither true nor false. Therefore, it is not a proposition.

   (b) Is this program correct?

   **Solution:** This is a question, not a statement. Therefore, it is not a proposition.

   (c) The cable is defective.

   **Solution:** This is a declarative statement that is either true or false. Therefore, it is a proposition.

   (d) $2 + 9 = 7$

   **Solution:** This statement is a proposition with a truth value of false.

   (e) $x + 12 = 7$

   **Solution:** This statement can be either true or false depending on the assignment of the variable $x$. Therefore, it is not a proposition.

2. Which of the following implications are true?

   (a) If $1 + 11 = 12$, then $2 + 2 = 5$.

   **Solution:** This proposition is logically equivalent to $T \to F$, which is false.

   (b) If $6 + 3 = 9$, then $2 + 2 = 4$.

   **Solution:** This proposition is logically equivalent to $T \to T$, which is true.

   (c) If $12 + 4 = 7$, then $2 + 2 = 4$.

   **Solution:** This proposition is logically equivalent to $F \to T$, which is true.

   (d) If $7 + 2 = 4$, then $2 + 2 = 5$.

   **Solution:** This proposition is logically equivalent to $F \to F$, which is true.

3. Translate the following sentences in English into propositions. $T$ denotes “Tom is a good tennis player”, $P$ denotes “Tom plays every day”, and $S$ denotes “Tom practices his serve”.

   (a) Tom is a good tennis player who practices his serve but does not play every day.

   **Solution:** This sentence is stating facts about Tom, so it is written as $T \wedge S \wedge \neg P$.

   (b) In order for Tom to be a good tennis player he must play every day and practice his serve.

   **Solution:** This sentence does not say whether Tom is or isn’t a good tennis player, just that if he is a good tennis player, then it must be because he plays every day and practices his serve. This is written in propositional logic as $T \rightarrow (P \wedge S)$.

4. Translate the following propositions into English. $A$ denotes “The computer in the lab uses Linux”, $B$ denotes “A hacker breaks into the computer”, and $C$ denotes “The data on the computer is lost”.

   (a) $(A \rightarrow \neg B) \wedge (\neg B \rightarrow \neg C)$

   **Solution:** “If the computer in the lab uses Linux, then no hacker will break into it, and if no hacker breaks into it, then no data on the computer will be lost.”

   (b) $C \leftrightarrow (\neg A \wedge B)$

   **Solution:** “The data on the computer is lost if and only if the computer in the lab doesn’t use Linux and a hacker breaks into it.”

5. What is the negation of the proposition “It’s snowing but it isn’t cold outside”?

   **Solution:** Let $A$ denote “It’s snowing” and let $B$ denote “It’s cold outside.” Then the sentence can be expressed in propositional logic as $A \wedge \neg B$.

   We need to find a proposition that is true when $A \wedge \neg B$ is false, and false when $A \wedge \neg B$ is true. The first solution is $\neg A \vee B$, which reads as “It isn’t snowing or it’s cold outside.” Another solution is $A \rightarrow B$, which reads as “If it’s snowing then it’s cold outside.”

   These solutions can be verified using a truth table:

| $A$ | $B$ | $A \wedge \neg B$ | $\neg A \vee B$ | $A \rightarrow B$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | T | F | F |
| F | T | F | T | T |
| F | F | F | T | T |

6. A fictional island is inhabited by two types of people: Knights, who always tell the truth, and knaves, who always lie. You meet two inhabitants of this island, $A$ and $B$, who greet you with statements about each other. Based on these statements, is it possible to determine their dispositions?

   (a) $A$ says "$B$ is a knight."

   $B$ says "The two of us are opposite types."

   **Solution:** Assume that $A$ is a knight. Then $A$ is telling the truth, so $B$ must also be a knight. Since $B$ is a knight, $A$ must be a knave, since $A$ and $B$ are opposite types. This is a contradiction because we assumed $A$ to be a knight. This proves that $A$ cannot be a knight.

   Now assume that $A$ is a knave. Then $A$ is lying, so $B$ must be a knave. Since $B$ is a knave, the statement "the two of us are opposite types" is a lie, meaning that both $A$ and $B$ are the same type. This is not a contradiction because we assumed $A$ to be a knave.

   Therefore, both $A$ and $B$ are knaves.

   (b) $A$ says "$B$ is a knave."

   $B$ says "I am a knight or $A$ is a knight."

   **Solution:** $A$ is a knave and $B$ is a knight.

   These puzzles, as well as 378 more, can be found here: `http://philosophy.hku.hk/think/logic/knights.php`. 