CVG 2571

Examen 2012 : Solution

Question #1

$A_{ire} = 10\,362{,}3\ m^2$

$A = (0, 0)$
$B = (90, 0)$
$C = (X_c, Y_c)$
$D = (57{,}735 \times \cos(60°), 57{,}735 \times \sin(6$
$= (28{,}8675, 50)$

Station	X	Y	$X_1 Y_2$	$Y_1 X_2$
A	0	0	0	0
B	90	0	$90 Y_c$	0
C	X_c	Y_c	$50 X_c$	$28{,}8675 Y_c$
D	28,8675	50	0	0
A	0	0		
		$\Sigma =$	$90Y_c + 50X_c$	$28{,}8675\, Y_c$

Cependant : $\tan(180° - 150°) = Y_c / (X_c - 90)$

$$Y_c = \tan(30°) \times (X_c - 90)$$

$$2\text{Aire} = 90 Y_c + 50 X_c - 28{,}8675\, Y_c = 50 X_c + 61{,}1325\, Y_c$$

$$2\text{Aire} = 2 \times 10362{,}3\,m^2 = 50 X_c + 61{,}1325\,(X_c - 90)\tan(30°)$$
$$20\,724{,}6\ m^2 = (50 + 35{,}2949) X_c - 3176{,}5\ m^2$$

$$X_c = \frac{23901{,}1}{85{,}2949} = 280{,}2172 = 280{,}217$$

$$Y_c = (280{,}2172 - 90)\tan(30°) = 109{,}8220$$

$$\overline{BC} = \sqrt{(280{,}2172 - 90)^2 + 109{,}8220^2} = 219{,}644\ m$$

$$\alpha_{DC} = \arctan\left(\frac{(280{,}2172 - 288{,}675)}{(109{,}822 - 50)}\right) = 76°\,36'\,45''$$

$$\alpha_{DA} = 180° + 30° = 210° = \alpha_{DC} + D$$

$$D = \alpha_{DA} - \alpha_{DC} = 210° - 76°36'45'' = 133°23'15''$$

$$\therefore \boxed{\overline{BC} = 219{,}644 \text{ m} \quad , \angle D = 133°23'15''}$$

Question #2

$I = 11°56'52''$ @ 18+681,542 $R = 1500\ m$

a) $R = \frac{1746,37° m}{D} \Rightarrow D = \frac{1746,37° m}{R} = \frac{1746,37° m}{1500 m} = 1,1642$

b) $T = R \tan I/2 = 1500 m \tan\left(\frac{11°56'52''}{2}\right) = 156,965\ m$

$L = R\theta = 1500 m \cdot (11°56'52'') \times \frac{\pi}{180} = 312,792\ m$

Station du début = PI − T = 18+681,542
− 0+156,965
BC = 18+524,577

Station de la fin = BC + L = 18+524,577
+ 0+312,792
18+837,369

c) S_a: 18+600,000
− 18+524,577
S_a = 75,423

$S_b = 100$

S_c: 18+837,369
− 18+800
37,369

$S_a = R d_a$
$d_a = S_a / R \times \frac{180°}{\pi}$
$= \frac{75,423}{1500} \times \frac{180°}{\pi}$
$d_a = 2,8809°$
$\delta_a = d_a/2 = 1,4404$
$\delta_a = 1°26'26''$

$S_b = R d_b$
$d_b = S_b / R \times \frac{180°}{\pi}$
$= \frac{100}{1500} \times \frac{180}{\pi}$
$= 3,819°$
$\delta_b = 1,9098°$
$\delta_b = 1°54'35''$

$S_c = R d_c$
$d_c = S_c / R \times \frac{180°}{\pi}$
$= \frac{37,369}{1500} \times \frac{180°}{\pi}$
$= 1,427391$
$\delta_c = 0,713695°$
$\delta_c = 0°42'49''$

$C_a = 2R\sin\delta_a$
$= 2 \times 1500 \times \sin(1°26'26'')$
$= 3000 \times 0{,}02514$
$= 75{,}420$

$C_b = 2R\sin\delta_b$
$= 2 \times 1500 \times \sin(1°54'35'')$
$= 3000 \times 0{,}033325$
$= 99{,}975$ m

$C_c = 2R\sin\delta_c$
$= 2 \times 1500 \times \sin(0°4$
$= 3000 \times 0{,}01249$
$= 37{,}36$ m

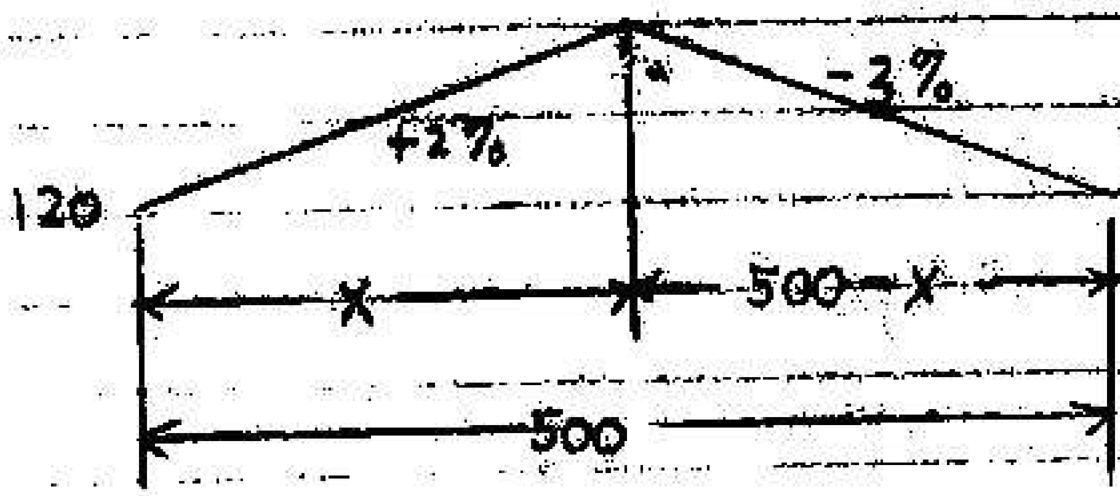
Station	S_a	C_a	δ_a	$\Sigma\delta_a$
18+524,577				
18+600	75,423	75,420	1°26'26''	1°26'26''
18+700	100,000	99,975	1°54'35''	3°21'01''
18+800	100,000	99,975	1°54'35''	5°15'36''
18+837,369	37,369	37,364	0°42'49''	5°58'25''
			5°58'25''	

$\frac{I}{2} = 11°56'52'' = 5°58'26''$

Question #3

$g_1 = +2\%$ @ 18+000, 120
$g_2 = -3\%$ @ 18+500, 120

a)



$$120 + X \cdot 0{,}02 = 120 + (500 - X) \times 0{,}03$$
$$(0{,}02 + 0{,}03)X = 120 + 0{,}03 \times 500 - 120$$
$$X = \frac{15}{0{,}05} = 300 \text{ m}$$

∴ Station du point d'intersection: 18+000
+ 0+300
= 18+300

élévation au point d'intersection:
$$Y_{PVI} = 120 + 0{,}02 \times 300 = 126 \text{ m}$$

b) Point sur la courbe: 18+225,5 ; 122,970

$$y = \frac{r}{2} x^2 + g_1 x + Y_{BVC} \quad ; \quad Y_{BVC} = Y_{PVI} - g_1 \cdot \frac{L}{2}$$
$$y = \frac{1}{2} \frac{g_2 - g_1}{L} x^2 + g_1 X + Y_{PVI} - g_1 \cdot \frac{L}{2}$$

$x = L/2 + \Delta$ (Δ = distance depuis PVI)
$l = L/2 \Rightarrow x = l + \Delta$

$\Delta_{PF} = \text{Station}_{\text{point Fixe}} - \text{Station}_{PVI}$ $= 18+225,5 - 18+300,0 = -74,5$

$x_{PF} = l + \Delta_{PF}$

$y = \frac{1}{2} r x^2 + g_1 x + Y_{BVC}$

$L = 2l$

$y_{PF} = \frac{1}{2}\left(\frac{g_2 - g_1}{2l}\right)(l + \Delta_{PF})^2 + g_1(l + \Delta_{PF}) + Y_{PVI} - g_1 \cdot l$

$y_{PF} = \frac{1}{2}\left(\frac{g_2 - g_1}{2l}\right)(l^2 + 2l\Delta_{PF} + \Delta_{PF}^2) + g_1 l + g_1 \Delta_{PF} + Y_{PVI} - g_1 \cdot l$

$y_{PF} = \frac{1}{4}(g_2 - g_1) l + \frac{1}{2}(g_2 - g_1) \cdot \Delta_{PF} + \frac{1}{4}\frac{(g_2 - g_1)}{l}\Delta_{PF}^2 + g_1 l + g_1 \Delta_{PF} + Y_{PVI} - g$

$y_{PF} \cdot l = \frac{1}{4}(g_2 - g_1) l^2 + \frac{1}{2}(g_2 - g_1)\Delta_{PF} \cdot l + \frac{1}{4}(g_2 - g_1)\Delta_{PF}^2 + \cancel{g_1 l^2} + g_1 \cdot \Delta_{PF} l - \cancel{g_1}$

$\left[\frac{1}{4}(g_2 - g_1)\right] l^2 + \left[\frac{1}{2}(g_2 - g_1)\Delta_{PF} + g_1 \Delta_{PF} + Y_{PVI} - y_{PF}\right] l + \frac{(g_2 - g_1)}{4}\Delta_{PF}^2 = 0$

$\frac{1}{4}(g_2 - g_1) l^2 + \left(\frac{g_2 + g_1}{2}\Delta_{PF} + Y_{PVI} - y_{PF}\right) l + \frac{1}{4}(g_2 - g_1)\Delta_{PF}^2 = 0$

$g_1 = 0,02$; $g_2 = -0,03$; $\Delta_{PF} = -74,5$; $y_{PF} = 122,970$; $Y_{PVI} = 126$

$\frac{1}{4}(-0,03 - 0,02) l^2 + \left(\frac{-0,03 + 0,02}{2}(-74,5) + 126 - 122,970\right) l + \frac{1}{4}(-0,03 - 0,02) \times (-7$

$-0,0125\, l^2 + 3,4025\, l - 69,378125 = 0$

$l = \dfrac{-3,4025 \pm \sqrt{3,4025^2 - 4 \times -0,0125 \times -69,378125}}{2 \times -0,0125}$

$l = \begin{cases} 2\cancel{2},20 < \Delta_{PF} \quad \therefore \text{hors courbe} \\ 250,00 \end{cases}$ $L = 2 \times l = 500\,m$

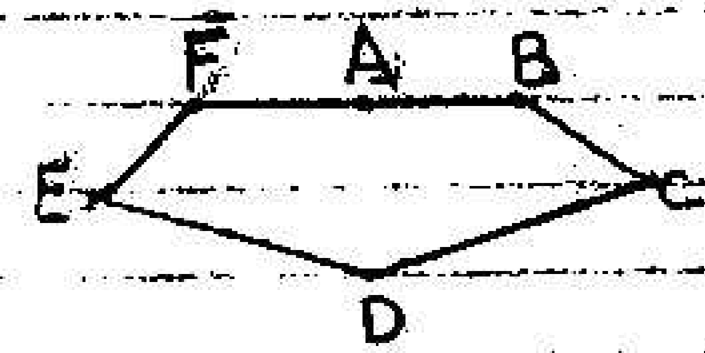
d) $x_{max} = \dfrac{g_1 \cdot L}{g_1 - g_2} = \dfrac{0,02 \times 500}{0,02 + 0,03} = 200\,m$

$18+300 - 250 = 18+050$ (BV

$+ 0+200 = 18+250$ (x_m

Question #4

Section typique:



Section 42+00:

b)

Station	X	Y	$X_1 Y_2$	$X_2 Y_1$
A	0	0		
B	25	0	0	0
C	63,2	-19,1	-477,5	0
D	0	-20,0	-1264	0
E	-68,2	-21,6	0	1364
F	-25	0	0	540
A	0	0	0	0
			-1741,5	1904,0
				+1741,5
			2A	3645,5
			$A_3 =$	1822,75

a) $m_{FE} = \frac{68,2-25}{21,6}$

$= 2h : 1V$

$m_{BC} = \frac{63,2-25}{19,1}$

$= 2h : 1V$

Section 43+00

Station	X	Y	$X_1 Y_2$	$X_2 Y_1$
A	0	0		
B	25	0	0	0
C	91,6	-33,3	-832,5	0
D	0	-15	-1374,0	0
E	-42,0	-8,5	0	630
F	-25	0	0	212,5
A	0	0	0	0
			-2206,5	842,5
				+2206,5
			2A =	3049,0

b)

a) $m_{FE} = \frac{42-25}{8,5}$

$= 2h : 1V$

$m_{BC} = \frac{91,6-25}{33,3}$

$= 2h : 1V$

$A_3 = 1524,5$

Section 44+00:

Station	X	Y	X_1Y_2	X_2Y_1
A	0	0		
B	25	0	0	0
C	91,6	-33,3	-832,5	0
D	0	-15,0	-1374,0	0
E	-91,6	-33,3	0	1374,0
F	-25	0	0	832,5
A	0	0	0	0
			-2206,5	2206,5
				+2206,5
			2A =	4412,5

$A_3 = 2206,5$

a) $m_{EF} = \frac{91,6-25}{33,3}$
$= 2h:1V$

$m_{BC} = \frac{91,6-25}{-33,3}$
$= 2h:1V$

b) $\overline{V_1} = \frac{1}{2}(A_1+A_2) \times 100\,ft = \frac{1}{2}(1822,75+1524,50)\times 100 = 167\,362,5\ ft^3$
$\overline{V_2} = \frac{1}{2}(A_2+A_3) \times 100\,ft = \frac{1}{2}(1524,5+2206,5)\times 100 = 186\,550,0\ ft^3$

$\overline{V_1} = 167362,5\ ft^3$
$\overline{V_2} = 186550,0\ ft^3$
$\overline{V} = 353912,5\ ft^3 \quad = 13\,107,87\ yd^3$

c) $C_{P_1} = \frac{100ft}{12}(20-15)((63,2+68,2)-(91,6+42,0)) = -91,7\ ft^3$
$C_{P_2} = \frac{100ft}{12}(15-15)((91,6+42,0))-(91,6+91,6)) = 0$

$V = \overline{V} + C_{P_1} + C_{P_2} = 353912,5 - 91,7 = 353820,8\ ft^3$
$= 13\,104,47\ yd^3$