MAT 1320 Lecture 1

Review: Algebraic Manipulations.

• Simplify: Ex. $\dfrac{\sqrt{x^{\frac{1}{2}}y^{5}}}{x^{5}y^{\frac{1}{4}}} = \dfrac{\left(x^{\frac{1}{2}}y^{5}\right)^{\frac{1}{2}}}{x^{5}y^{\frac{1}{4}}} = \dfrac{x^{\frac{1}{4}}y^{\frac{5}{2}}}{x^{5}y^{\frac{1}{4}}}$

$= x^{\frac{1}{4}-5}y^{\frac{5}{2}-\frac{1}{4}} = x^{-\frac{19}{4}}y^{\frac{9}{4}}$

• Rationalize the denominator: Ex. $\dfrac{1}{\sqrt{10}-3} = \dfrac{\sqrt{10}+3}{(\sqrt{10}-3)(\sqrt{10}+3)}$

$= \dfrac{\sqrt{10}+3}{10-9} = \sqrt{10}+3$

Ex. $\dfrac{\sqrt{6}-\sqrt{8}}{\sqrt{6}+\sqrt{8}} = \dfrac{(\sqrt{6}-\sqrt{8})(\sqrt{6}-\sqrt{8})}{(\sqrt{6}+\sqrt{8})(\sqrt{6}-\sqrt{8})} = \dfrac{(\sqrt{6}-\sqrt{8})^{2}}{6-8} = -\dfrac{(\sqrt{6}-\sqrt{8})^{2}}{2}$

• Solve for x: ★ Idea: regard x as a variable but regard others as constants.

Ex. $\dfrac{1}{x}+\dfrac{2}{y} = \dfrac{5}{z} \Rightarrow \dfrac{1}{x} = \dfrac{5}{z}-\dfrac{2}{y} = \dfrac{5y-2z}{zy}$

$\Rightarrow zy = x(5y-2z) \Rightarrow x = \dfrac{zy}{5y-2z}$

• Absolute value

Definition: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Since the definition is in two pieces, equations with absolute values often require two cases.

Ex. $|x-3| = 6$

There are two different cases by definition:

$x-3 \geq 0 \Rightarrow x \geq 3$ or $x-3 < 0 \Rightarrow x < 3$

Consider the first case:

If $x \geq 3$, then $|x-3| = x-3 = 6 \Rightarrow x = 9$.

Now we need to check that whether $x=9$ falls into our assumption $x \geq 3$. Indeed, $x = 9 \geq 3$. Therefore, $x=9$ is a solution.

If $x<3$, then $|x-3| = -(x-3) = 3-x = 6 \Rightarrow x = -3$.

Again, check: $x=-3<3$ $\langle \checkmark \rangle$. So $x=-3$ is also a solution

• The same ideas work with inequalities:

<u>Ex</u>. $|x^2-12| > 3$.

Split into two cases:

$\langle a \rangle$ If $x^2-12 \geq 0 \Longleftrightarrow x^2 \geq 12 \Longleftrightarrow x > 2\sqrt{3}$ or $x < -2\sqrt{3}$

Then $|x^2-12| = x^2-12>3 \Rightarrow x^2 > 15 \Rightarrow x > \sqrt{15}$ or $x < -\sqrt{15}$

Remember to check:

Indeed, $-\sqrt{15} < -\sqrt{12}$ and $\sqrt{15} > \sqrt{12}$

Therefore, $x > \sqrt{15}$ or $x < -\sqrt{15}$ is part of the solution.

$\langle b \rangle$ If $x^2-12<0 \Longleftrightarrow -\sqrt{12} < x < \sqrt{12}$

Then $|x^2-12| = -(x^2-12) = 12-x^2 > 3 \Rightarrow 9 > x^2 \Rightarrow -3<x<3$

Again, check $-\sqrt{12} < -3 < x < 3 < \sqrt{12}$. Therefore, $-3<x<3$ is also a part of the solution.

The whole solution set: $\{x<-\sqrt{15}\} \cup \{-3<x<3\} \cup \{x>\sqrt{15}\}$

• Note that when we solve inequalities, sometimes, the question naturally requires to split into different cases to consider.

③

<There is no "absolute" symbol in the inequalities>.

Ex. $\frac{3}{x}+4<-1 \Rightarrow \frac{3}{x}<-5$

If $x>0$, then $\frac{3}{x}<-5 \Rightarrow 3<-5x \Rightarrow -\frac{3}{5}>x$

Now check: $x<-\frac{3}{5}$ is incompatible with $x>0$. Therefore,

$x<-\frac{3}{5}$ is not part of the solution.

If $x<0$, then $3>-5x \Rightarrow x>-\frac{3}{5}$

Again check: $-\frac{3}{5}<x<0$. <✓>.

Therefore, $-\frac{3}{5}<x<0$ is our solution.