MATH 1004 - LECTURE 3

Objective (chap 2)
- Key limits of trig fcns
- limits @ infinity

SANDWHICH THEOREM (SQUEEZE)

If. $f(x) \leq g(x) \leq h(x)$

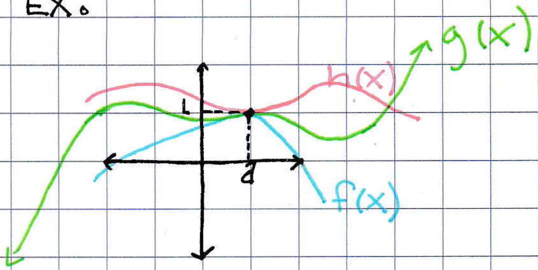
when x near a (except possibly at a),

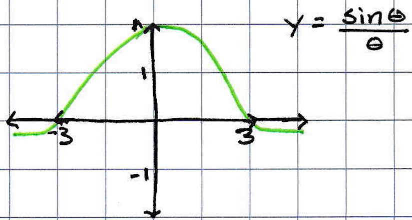
and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$

*fcn must be continuous near a

__THEN:__ $\lim_{x \to a} g(x) = L$

EX:





*page 51 //: $\cos\theta < \frac{\sin\theta}{\theta} < 1$

$\hookrightarrow \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \rightsquigarrow \lim_{\theta \to 0} \frac{1-\cos\theta}{\theta} = 0$

*If $1 \leq \alpha(x) \leq x^2 + 2x + 2$

get $\lim_{x \to -1} \alpha(x) = 1$ //

* $\lim_{x \to 0} \left(x^2 \sin\left(\frac{1}{x}\right)\right)$

Now: $\lim_{x \to 0} \sin\left(\frac{1}{x}\right) \nexists$

$\lim_{\theta \to \infty} \sin\theta \nexists$

But using sandwich theorem:

Note: $\left|\sin\left(\frac{1}{x}\right)\right| \leq 1$

$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$

multiply by x^2

$-x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$

FINAL EXAM

EX: $\lim_{\alpha \to 0} \frac{\sin(7\alpha)}{5\alpha}$

$= \frac{1}{5} \lim_{\alpha \to 0} \frac{\sin(7\alpha)}{\alpha}$

$= \frac{7}{5} \lim_{\alpha \to 0} \frac{\sin(7\alpha)}{7\alpha}$

$= \left(\frac{7}{5}\right)(1)$

$\therefore \lim_{\alpha \to 0} = \frac{7}{5}$

EX: $\lim_{x \to 0} \frac{\cos(3x-1)}{x}$

$= 3 \lim_{x \to 0} \frac{\cos(3x-1)}{3x}$

$= 0$

EX: $\lim_{\theta \to \pi} \frac{\sin \theta}{\theta - \pi}$ } change of variable

$\beta = \theta - \pi$ $\qquad \theta = \beta + \pi$

$\lim_{\beta \to 0} \frac{\sin(\beta + \pi)}{\beta}$ ← *SUB INTO LIMIT

REMEMBER:

$\sin(A+B) = \sin A \cos B + \sin B \cos A$

$\therefore \lim_{\beta \to 0} = -\sin\beta + 0$ $\qquad$ * $\sin \pi = 1$

$= -\lim_{\beta \to 0} = \frac{\sin\beta}{\beta}$

$= -1$

$\lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta}$

$= 0$

EX: $\lim_{x \to \frac{\pi}{2}} \frac{2x - \pi}{\cos x}$

$2 \lim \frac{x - \frac{\pi}{2}}{\cos x}$

SET: $\alpha = x - \frac{\pi}{2}$

$2 \lim_{\alpha \to 0} \frac{\alpha}{\cos(\alpha + \pi/2)}$

REMEMBE:

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$2 \lim_{\alpha \to 0} \frac{\alpha}{\cancel{\cos\alpha} \cdot 0 - \sin\alpha(1)}$

$-2 \lim_{\alpha \to 0} \frac{\alpha}{-\sin\alpha}$

$= -2 \lim_{\alpha \to 0} \frac{1}{\frac{\sin\alpha}{\alpha}}$ } limit laws

$= -2\left(\frac{1}{1}\right)$

$= -2$