Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

MAT 1341A – The Diagnostic Test (v.1) **Instructor: K. Zaynullin**

Last name: ____________________

First name: ____________________

Student number:

Please, read the following instructions carefully:

- You have 80 minutes to complete this test. **Do not detach** the pages of this examination. Read each question carefully. Where it is possible to check your work, do so.
- Answer all questions by choosing (crossing) the respective box. You can use the backs of the pages and the last page for computations.
- This is a closed book exam, and no notes of any kind are allowed. The use of programmable calculators, cell phones, laptops, pagers or any text storage or communication device is not permitted.

THIS SPACE IS RESERVED FOR THE MARKER:

Question	1	2	3	4	5	6	7	8	9	10	Total
Mark											
Out of	1	1	1	1	1	1	1	1	1	1	10

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1. Find an equation for the plane passing through the points $(1,-1,2)$ and (2,1,3), and parallel to the z-axis.

`cross (X) the correct answer:`

- [A] $7x-3y-2z=3$
- [B] $2y-z=5$
- [C] $-x+y=1$
- [D] $2x-z=3$
- [E] $x+y+z=2$
- [F] $-2x+y=-3$

Solution: Such a plane is parallel to the vectors $(2,1,3)-(1,-1,2)=(1,2,1)$ and $(0,0,1)$. Therefore, a normal vector is

$$\begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = (2,-1,0)$$

The only equation representing a plane with normal vector parallel to $(2,-1,0)$ in the list above is [F]. Substituting we see that this plain contains points $(1,-1,2)$ and $(2,1,3)$.

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2. Find an equation of the plane which passes through the point $(1,-7,8)$ and which is perpendicular to the line whose (scalar) parametric equations are:

$$x=2+2t, \quad y=7-4t, \quad z=-3+t, \quad t\in\mathbb{R}.$$

`cross (X) the correct answer:`

- [A] $-4x+2y+z=38$
- [B] $2x-4y+z=38$
- [C] $7x+2y-3z=-71$
- [D] $-4x+2y+z=-28$
- [E] $2x-4y+z=-10$
- [F] $2x-4y+z=10$

Solution: A normal vector for this plane will be the direction vector of the line above, namely, $(2, -4, 1)$. Note that as $(1, -7, 8)$ belongs to the plane, the only correct equation is $\boxed{\text{B}}$.

3. Find parametric equations of the line containing $(0, 1, -5)$ and which is parallel to the two planes $-4x + y + 2z = 0$ and $-3x - 2y + z = 1$.

`cross (X) the correct answer:`

- A $x = 3t, \quad y = 1 + 2t, \quad z = 5 + 11t, \quad t \in \mathbb{R}$
- B $x = -5t, \quad y = 1 - 10t, \quad z = -5 + 5t, \quad t \in \mathbb{R}$
- C $x = 0 \quad , y = t, \quad z = -5t, \quad t \in \mathbb{R}$
- D $x = 5t, \quad y = 1 - 2t, \quad z = -5 + 11t, \quad t \in \mathbb{R}$
- E $x = -3t, \quad y = 1 + 2t, \quad z = -5 + 11t, \quad t \in \mathbb{R}$
- F $x = 0, \quad y = t, \quad z = 5t, \quad t \in \mathbb{R}$

Solution: A direct vector d to this line must be perpendicular to both norm vectors of the planes above. We have

$$d = \begin{vmatrix} i & j & k \\ -4 & 1 & 2 \\ -3 & -2 & 1 \end{vmatrix} = (5, -2, 11)$$

which is the direction vector for the line from D. Note that it also contains a point $(0, 1, -5)$.

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4. Find an equation for the plane with vector parametric description

$$v = (1, 1, 1) + s(0, 0, -1) + t(1, 1, 0), \quad s, t \in \mathbb{R}.$$

`cross (X) the correct answer:`

- A $36x + 4y - 9z = 31$
- B $-2x + 9y + 5z = 14$
- C $18x + 9y - 11z = -40$
- D $2x + 9y - 2z = 5$
- E $-x + y = 0$
- F $-x + 3y + 2z = 0$

Solution: A normal vector is

$$(0, 0, -1) \times (1, 1, 0) = \begin{vmatrix} i & j & k \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix} = (1, -1, 0)$$

The only equation representing a plane with such a normal vector is E. It can be verified that it contains the point $(1, 1, 1)$.

5. Which two of the following are vector parametric descriptions for the plane with equation $-2x + y + z = 4$?

I. $v = (0, 0, 0) + s(1, 0, 2) + t(1, 2, 0), \quad s, t \in \mathbb{R}$

II. $v = (-2, 0, 0) + s(0, 1, -1) + t(1, 2, 0), \quad s, t \in \mathbb{R}$

III. $v = (0, 4, 0) + s(0, 1, -1) + t(1, 0, 2), \quad s, t \in \mathbb{R}$

IV. $v = (0, 4, 0) + s(1, 1, 1) + t(0, 1, 1), \quad s, t \in \mathbb{R}$

`cross (X) the correct answer`:

A I and II

B I and III

C I and IV

D II and III

E II and IV

F III and IV

Solution: Exclude (I) as the plane above does not contain $(0, 0, 0)$. When $v = P + sv_1 + tv_2$ is the vector parametric description of a plane, both vectors v_1 and v_2 must be perpendicular to any normal vector; in this case $(-2, 1, 1)$. Since $(0, 1, 1)$ is not perpendicular to $(-2, 1, 1)$, (IV) is excluded. So the correct is D

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6. If $P = (1, 2, 1)$, $Q = (2, 2, 1)$ and $R = (1 + \sqrt{3}, 3, 1)$, find the angle $\angle QPR$.

`cross (X) the correct answer`:

A $\pi/2$

B $\pi/3$

C $4\pi/3$

D $\pi/4$

E $3\pi/4$

F $\pi/6$

Solution: We have

$$\cos(\theta) = \frac{(Q-P)\cdot(R-P)}{||Q-P||||R-P||} = \frac{(1,0,0)\cdot(\sqrt{3},1,0)}{1\cdot 2} = \frac{\sqrt{3}}{2}.$$

Hence, $\theta = \pi/6$ and the correct answer is $\boxed{\text{F}}$.

7. Let $v = (1, 0, 1)$ and $u = (2, 2, 2)$. Find the projection (of v on u) $proj_u(v)$.

`cross (X) the correct answer:`

[A] $(2, 0, 2)$

[B] $\frac{\sqrt{2}}{2}(1, 0, 1)$

[C] $\frac{12}{7}(3, 3, 3)$

[D] $\frac{11}{7}(3, 3, 3)$

[E] $\frac{1}{3}(2, 2, 2)$

[F] $\frac{2\sqrt{3}}{3}(2, 2, 2)$

Solution: Using the formula we obtain

$$proj_u v = \frac{v \cdot u}{||u||^2} u = \frac{4}{12}(2, 2, 2) = \frac{1}{3}(2, 2, 2).$$

So the correct answer is [E].

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8. Find the volume of the parallelepiped determined by the vectors $u = (1, 1, -1)$, $v = (2, 0, 1)$ and $w = (1, -1, 3)$.

`cross (X) the correct answer:`

[A] -2

[B] 2

[C] 4

[D] 6

[E] 8

[F] 16

Solution: We have

$$v \times w = \begin{vmatrix} i & j & k \\ 2 & 0 & 1 \\ 1 & -1 & 3 \end{vmatrix} = (1, -5, -2).$$

Hence, the volume is

$$|u \cdot (v \times w)| = |(1, 1, -1) \cdot (1, -5, -2)| = 2.$$

So the correct is [B].

9. What is the area of the triangle with vertices $(5, 4, 3)$, $(1, 0, 1)$ and $(3, 9, 3)$?

`cross (X) the correct answer:`

- [A] 13
- [B] 15
- [C] 17
- [D] 20
- [E] 26
- [F] 30

Solution: Let $v = (1, 0, 1)$, $u = (5, 4, 3)$ and $w = (3, 9, 3)$. The area is

$$\frac{1}{2}||(u - v) \times (w - v)|| = \begin{vmatrix} i & j & k \\ 4 & 4 & 2 \\ 2 & 9 & 2 \end{vmatrix} = \frac{1}{2}||(-10, -4, 28)|| = ||(-5, -2, 14)|| = 15.$$

The correct answer is [B].

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10. Let L be the line passing through $(1, 0, 1)$ and $(3, 1, 2)$. Find the point of intersection of L with the plane $x - y + z = 1$.

`cross (X) the correct answer:`

- [A] $(0, -1/2, 1/2)$
- [B] $(1/2, 0, 1/2)$
- [C] $(0, 0, 1)$
- [D] $(1/2, -1/2, 0)$
- [E] $(1, 0, 0)$
- [F] $(0, -1, -1)$

Solution: A direction vector for L is $(3, 1, 2) - (1, 0, 1) = (2, 1, 1)$. Hence, parametric equations for L are

$$x = 1 + 2t, \quad y = t, \quad z = 1 + t, \quad t \in \mathbb{R}.$$

Thus,

$$x - y + z = 1 \iff (1 + 2t) - t + (1 + t) = 1 \iff 2t = -1 \iff t = -1/2.$$

Hence, the intersection point as at $t = -1/2$ and is $(x, y, z) = (0, -1/2, 1/2)$.

The correct answer is [A].

The last page (use it for computations)

$$\sin(\tfrac{\pi}{6}) = \cos(\tfrac{\pi}{3}) = \tfrac{1}{2}, \quad \sin(\tfrac{\pi}{3}) = \cos(\tfrac{\pi}{6}) = \tfrac{\sqrt{3}}{2}$$

$$\sin(\tfrac{\pi}{4}) = \cos(\tfrac{\pi}{4}) = \tfrac{\sqrt{2}}{2}, \quad \sin(0) = \cos(\tfrac{\pi}{2}) = 0, \quad \sin(\tfrac{\pi}{2}) = \cos(0) = 1.$$