Concordia University
Engineering and Computer Science

Department of Mechanical and Industrial Engineering

FACULTY OF ENGINEERING AND COMPUTER SCIENCE

DEPARTMENT OF MECHANICAL AND INDUSTRIAL ENGINEERING

COURSE		NUMBER	SECTION
ANALYSIS AND DESIGN OF CONTROL SYSTEMS		MECH 371/4	T, X
EXAMINATION FINAL EXAM	DATE April 19, 2013	TIME & PLACE Room:	# OF PAGES 4+ 1 cover page
PROFESSORS W.F. Xie, B. Gordon		LAB INSTRUCTOR	
MATERIALS ALLOWED ■ NO O YES (PLEASE SPECIFY) CALCULATORS ALLOWED O NO ■ YES		CLOSED BOOK EXAM NO MATERIALS ALLOWED *NONPROGRAMABLE CALCULATORS*	

SPECIAL INSTRUCTIONS:

ANSWER ALL FIVE (5) QUESTIONS.

ALL RELEVANT PROCEDURE STEPS MUST BE SHOWN CLEARLY. OTHERWISE ZERO MARKS WILL BE GIVEN.

PLEASE **RETURN EXAM QUESTIONAIRE** ***WITH YOUR ANSWER BOOKLET.***

Name: ______________________________ **I.D.:** ______________________________

Surname, given names

QUESTION #	MARKS per QUESTION	MARKS
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

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UNIVERSITÉ Concordia UNIVERSITY
1455 De Maisonneuve Blvd. West, EV004.139, Montreal, Quebec, Canada H3G 1M8
Tel 514-848-2424 ext. 3125 Fax 514-848-3175 www.mie.concordia.ca

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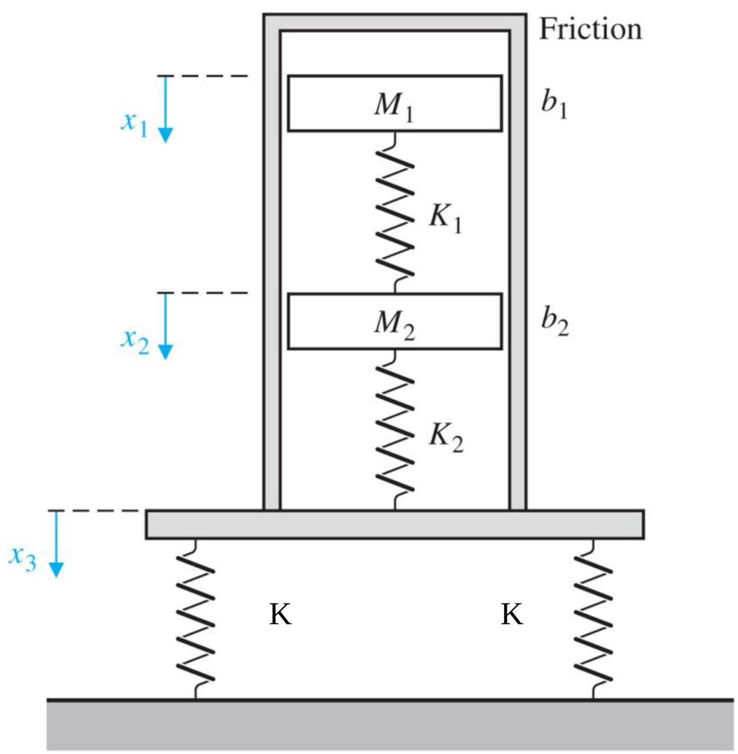
Problem #1 (20 marks—(a)-8marks, (b)-4marks, (c)-8marks,)

A mechanical system is shown in the following figure, which is subjected to a known displacement $x_3(t)$ with respect to the reference.

(a) Determine the two independent equations of motion with regard to $x_1(t)$ and $x_2(t)$.
(b) Obtain the equations of motion in terms of the Laplace transforms, assuming that the initial conditions are zero.

(c) Obtain the relationship $T_{13}(s) = \dfrac{X_1(s)}{X_3(s)}$



Problem #2 (20mark—(a)-10marks, (b)-3marks, (c)-3marks, (d)-4marks)

a) Plot the Nyquist diagram of the following system for k=1.

$$GH(s) = \frac{0.5k(s+1+j)(s+1-j)}{s(s-2)}$$

b) Investigate the stability of the system based on the Nyquist diagram for k=1.

c) Using the Nyquist diagram determine the range of k where the closed-loop system is stable.

d) If GH(s) is multiplied by e^{-Ts}, what is the maximum time delay T allowed before the closed-loop system becomes unstable, assuming k is the minimum value allowed in part c).

Problem #3 (20marks—(a)-15marks, (b)-5marks)

A unity feedback system has the process

$$G(s) = \frac{K(s+10)}{s(s+5)}$$

a). Sketch the root locus of the system as $0 < K < \infty$.
b). Prove the points $s_{1,2} = -5 \pm 5j$ are located on the root locus and determine the gain K at the points.

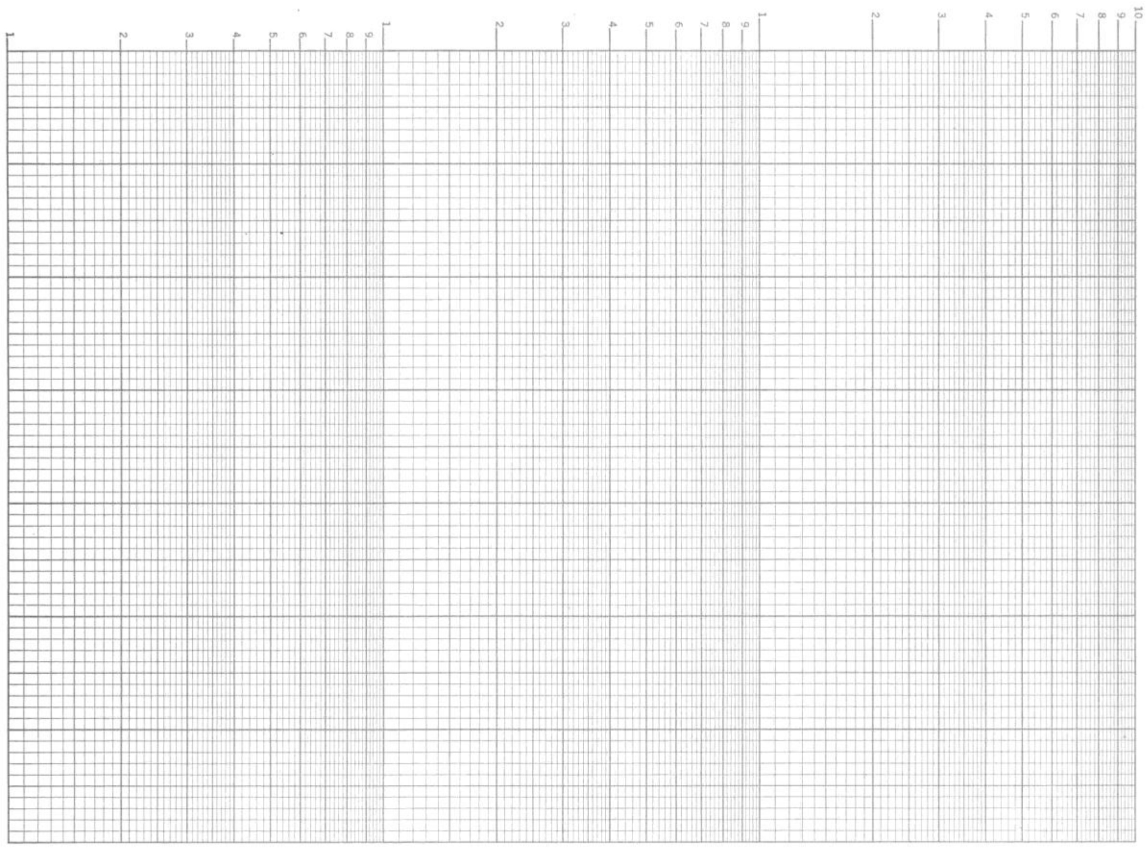
Problem #4 (20 marks—(a)-10marks, (b)-4marks, (c)-4marks, (c)-2marks)
A unity feedback system has a plant transfer function given by

$$G(s) = \frac{0.0065(s^2+9s+49)}{s^2(s+1)}$$

a) On the semi-log paper provided, plot the asymptotic Bode diagram for G(s). Show all relevant steps. Indicate the relevant slopes on the diagram.

b) Determine the asymptotic gains at the corner frequencies and list them in a table.

c) From the asymptotic Bode diagram determine the crossover frequencies, the gain margin, and the phase margin. Indicate these values on the diagram clearly.
d) Is the closed loop system stable or unstable? Explain your answer clearly.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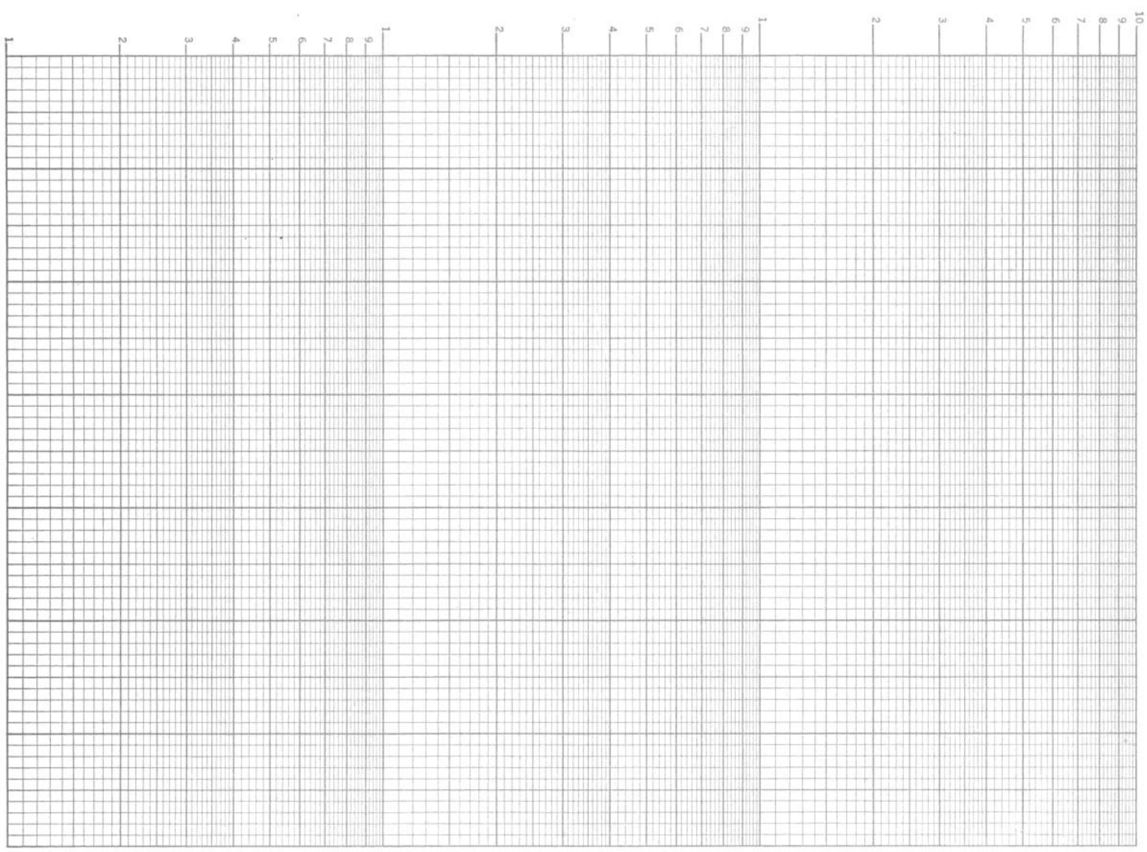
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Problem #5 (20 marks—(a)-10marks, (b)-10marks)

A unity negative feedback control system has the plant transfer function given by:

$$G_P = \frac{20}{(s+3)(s+5)}$$

(a) Using the root locus method, design a proportional controller so that the system transient response due to a step input *r(t)* has exactly time to peak T_P =1.57sec.

(b) Using the root locus method, design a proper controller so that the system transient response due to a step input *r(t)* has exactly 4.3 % overshoot, time to peak $T_P = 1.57sec$, and the steady state error e_{ss} due to the step input is zero.

Given information:

Percent overshoot $P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

Time to peak $T_P = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

Settling time $T_s = \frac{4}{\zeta\omega_n}$

where ω_n, ω_d and ζ is the system natural frequency, system damped natural frequency, and system damping ratio, respectively.